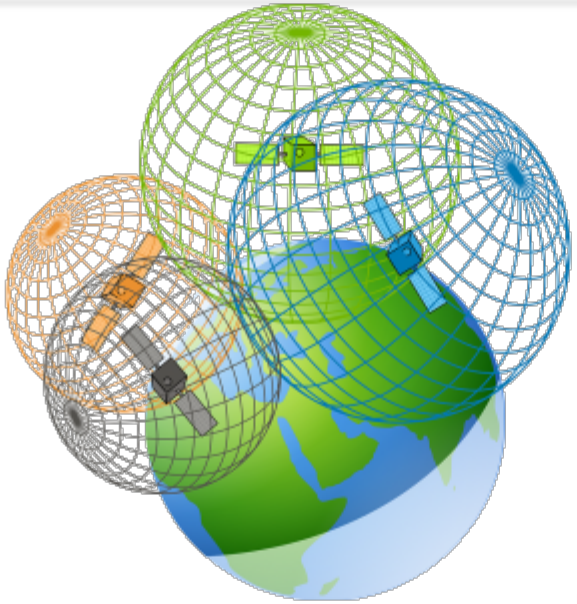


Welcome to EECS 16A!

Designing Information Devices and Systems I

Ana Claudia Arias and Miki Lustig

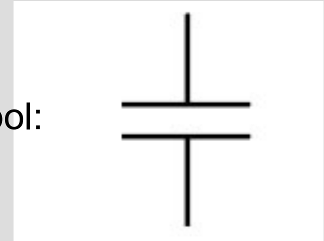
Fall 2021

Module 2
Lecture 7
Capacitors and Capacitive Touchscreens
(Note 17)

Greetings from Miki & Ana

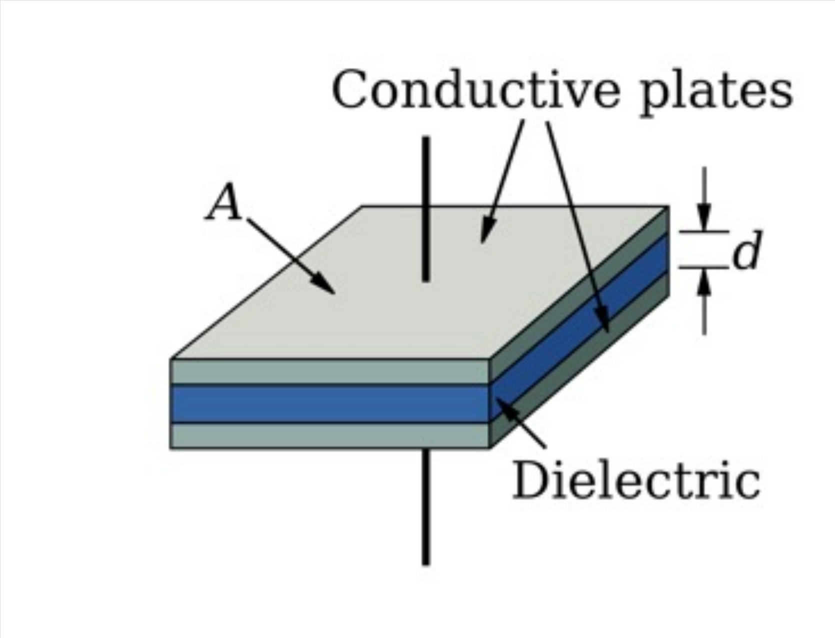
Last lecture: Capacitors

- Charge storage device (like a 'bucket' for charge)
- holds electric charge when we apply a voltage across it, and gives up the stored charge to the circuit when voltage removed

Symbol: 

Capacitance: C Units: Farads [F]

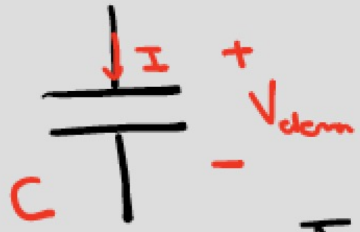
IV equation: $I = C \cdot \frac{dV}{dt}$

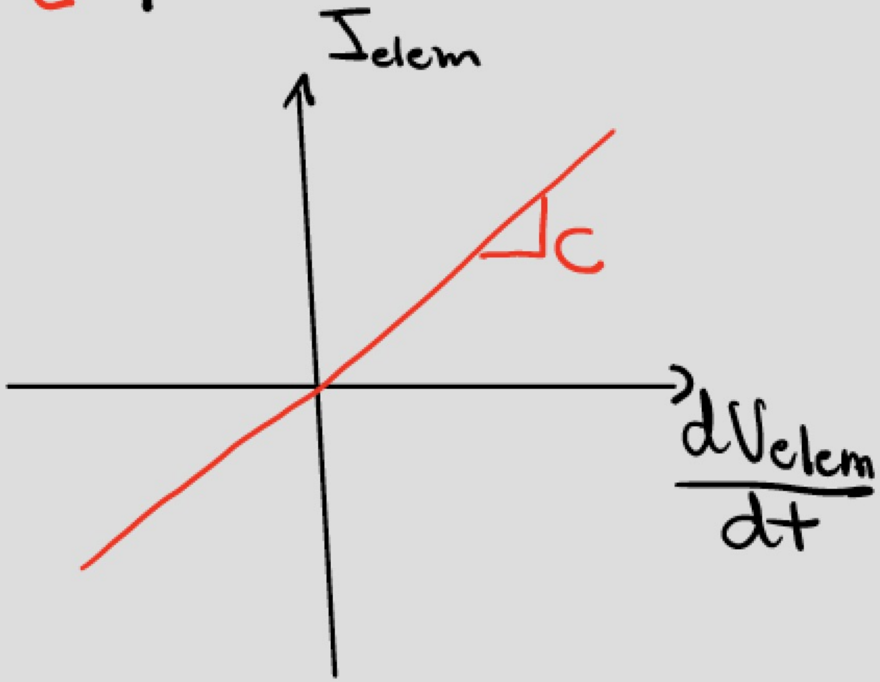


$$C = \varepsilon \cdot \frac{A}{d}$$

Circuit Model: IV relationship

Capacitor Symbol





$$\underset{[C]}{Q_{elem}} = \underset{\substack{[F] \\ \text{(Farad)}}}{C} \cdot \underset{[V]}{V_{elem}}$$

We know : $I_{elem} = \frac{dQ_{elem}}{dt}$

$$I_{elem} = \frac{d}{dt} C \cdot V_{elem}$$

C = constant over time

$$\boxed{I_{elem} = C \cdot \frac{dV_{elem}}{dt}}$$

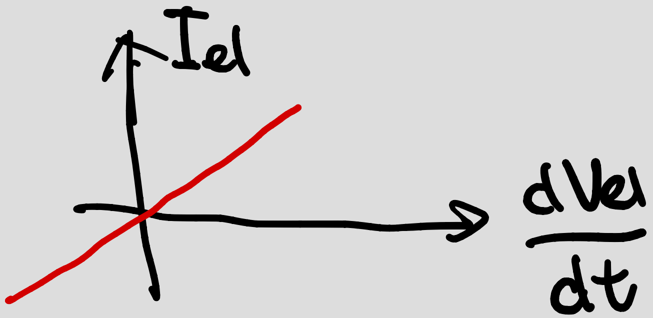
↳ Can use the same 7-step analysis.

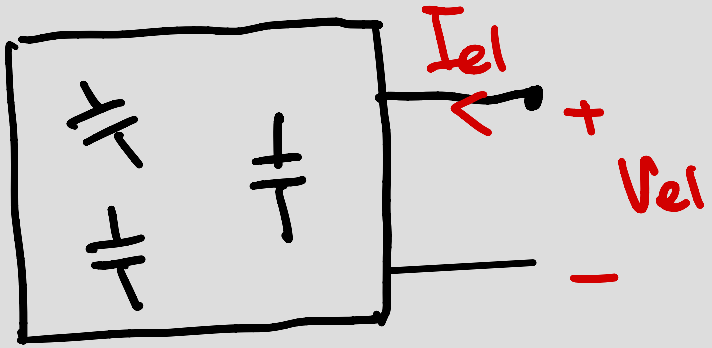
Equivalent Circuits with Capacitors

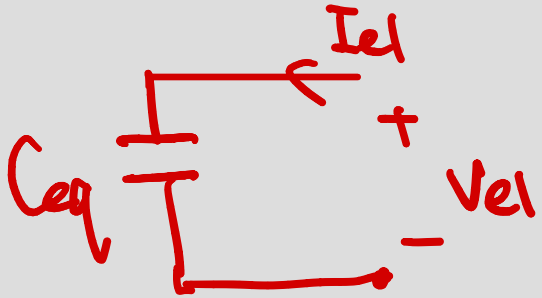
* Capacitor – only circuit

Step 1: ~~find Vth / Ino~~ no source

Step 2: $C_{eq} = \dfrac{I_{eq}}{\frac{dV_{el}}{dt}}$





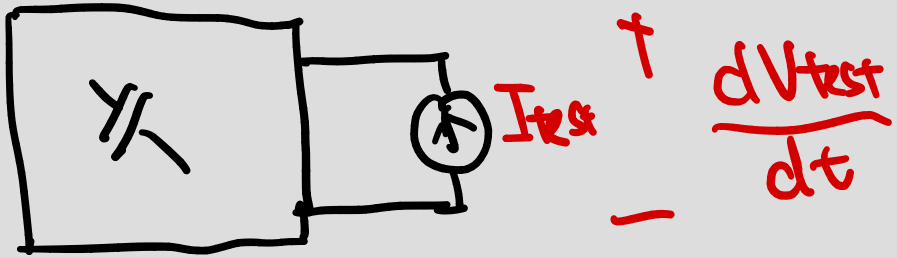


a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$

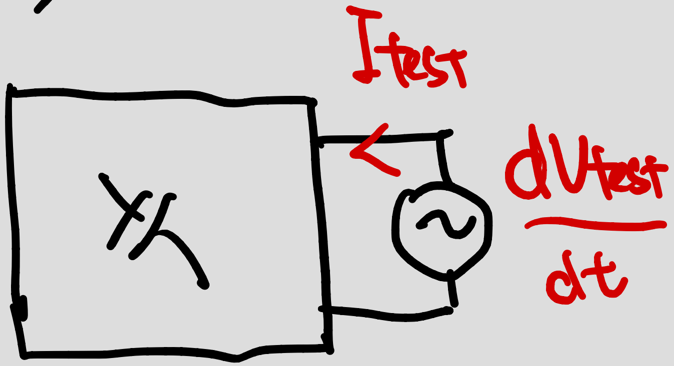
b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

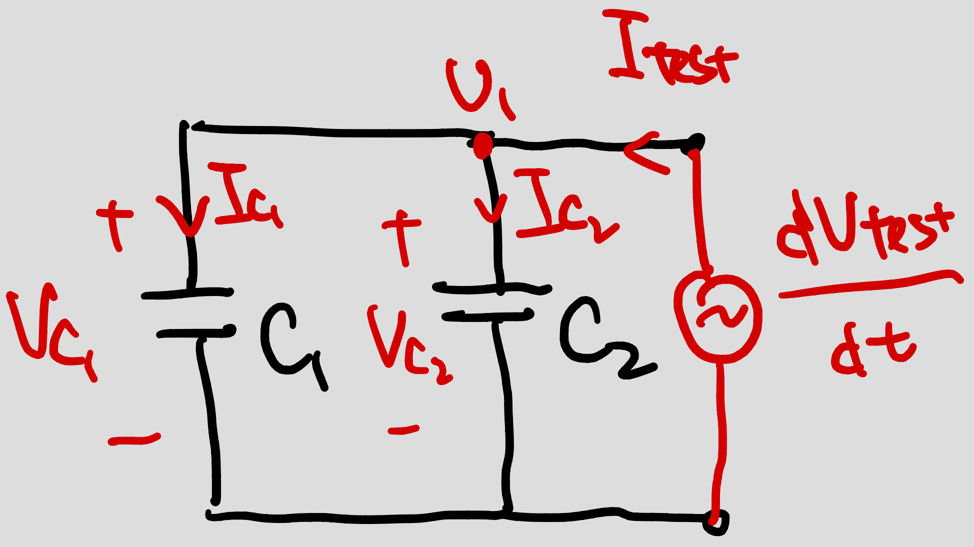
(a)



(b)



* These are methods for experiments



$$V_{C_1} = u_1 \quad , \quad V_{C_2} = u_1$$

$$u_1 = V_{test}$$

$$\frac{du_1}{dt} = \frac{dV_{test}}{dt}$$

Elem Definition: $I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$

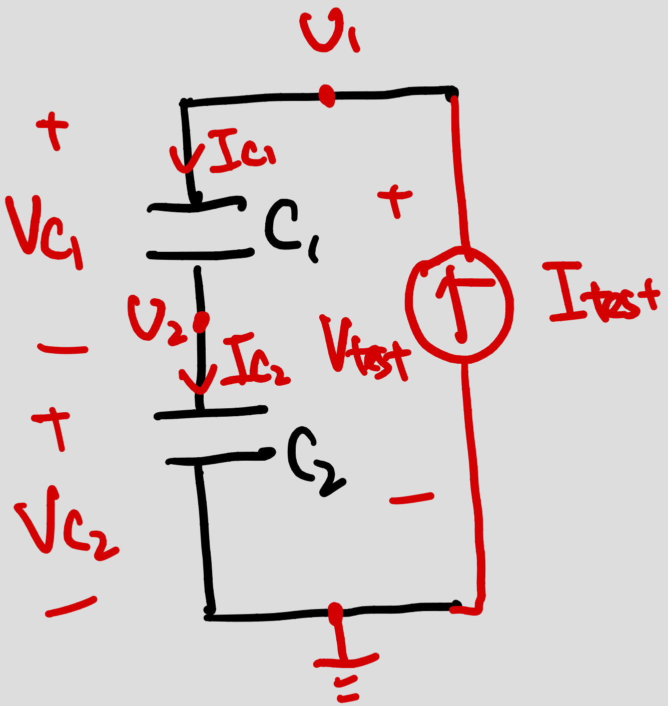
$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

KCL: $I_{test} = I_{C_1} + I_{C_2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt} = (C_1 + C_2) \frac{dV_{test}}{dt}$

$$I_{test} = (C_1 + C_2)\frac{dV_{test}}{dt}$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2$$

$\boxed{\text{parallel}}$



KCL: $I_{C_1} = I_{C_2} = I_{test}$

Elem Definition:

$$I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$$

$$I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$$

$$V_{C_2} = U_2 - 0 = U_2$$

$$V_{C_1} = U_1 - U_2$$

$$V_{test} = U_1$$

For V_{C_2}: $$I_{test} = I_{C_2} = C_2 \frac{dV_2}{dt} \Rightarrow \frac{dV_2}{dt} = \frac{I_{test}}{C_2}$$

For V_{C_1}: $$I_{C_1} = C_1 \frac{dV_1 - dV_2}{dt} \Rightarrow \frac{I_{C_1}}{C_1} = \frac{I_{test}}{C_1} = \frac{dV_1 - dV_2}{dt}$$

$$\frac{dV_1}{dt} = \frac{I_{test}}{C_1} + \frac{dV_2}{dt} \xrightarrow{\text{substitute}} \frac{dV_1}{dt} = \frac{I_{test}}{C_1} + \frac{I_{test}}{C_2}$$

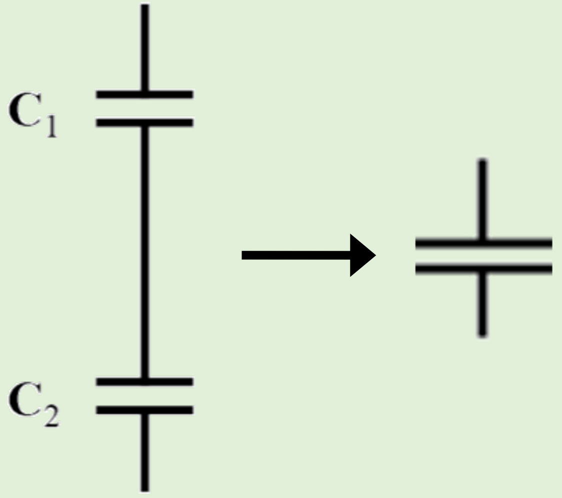
$$\frac{dV_{test}}{dt} = I_{test}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 // C_2$$

Series !

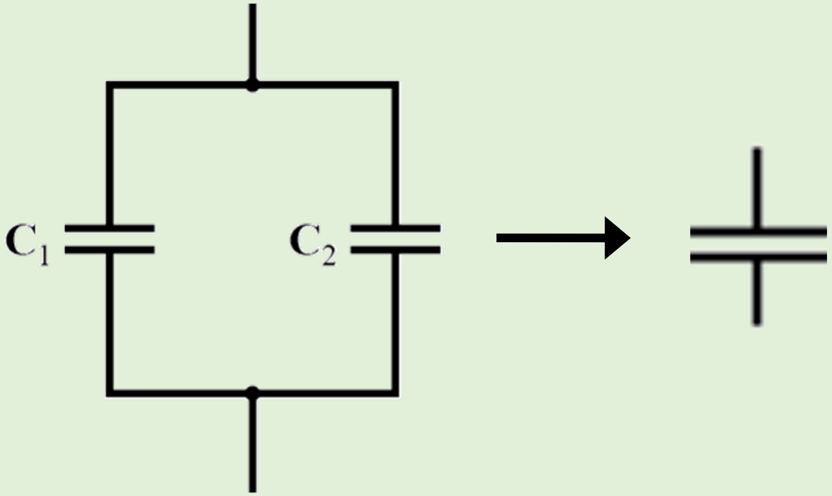
Equivalent capacitors

Capacitors in **Series**

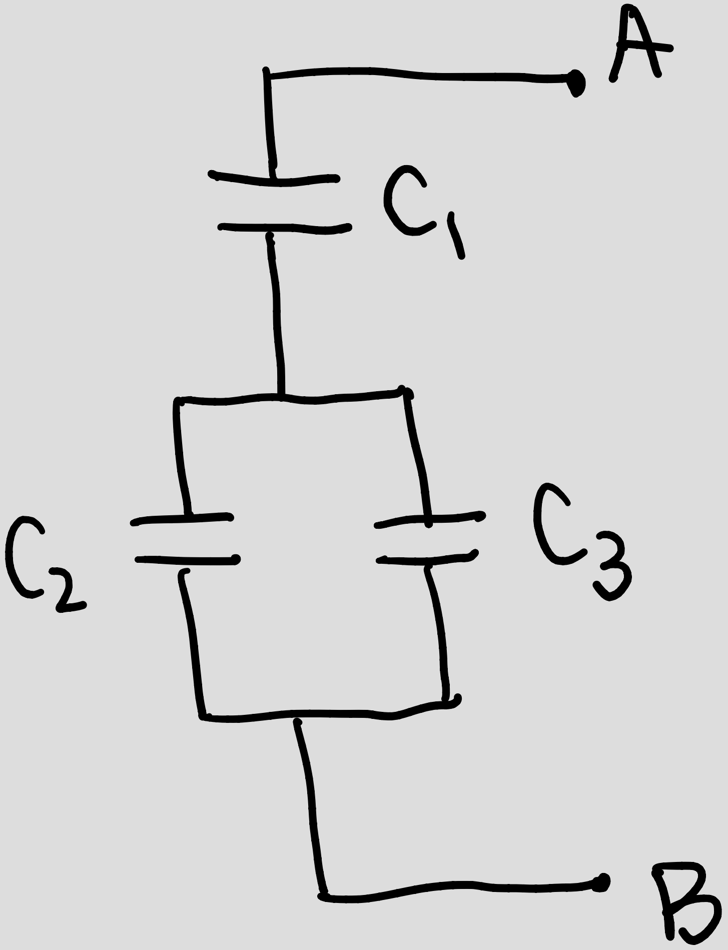


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

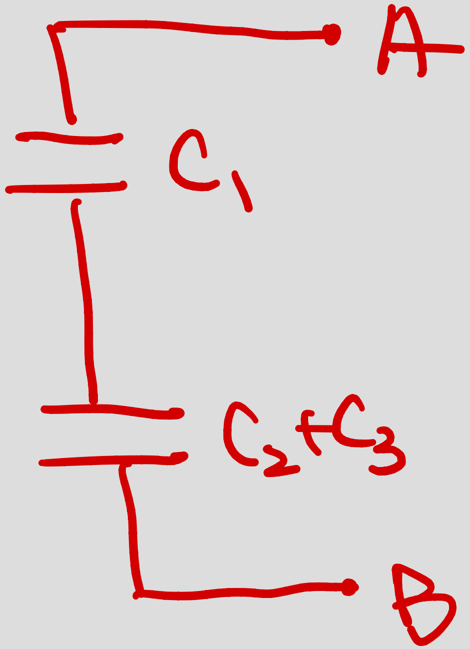
Capacitors in **Parallel**



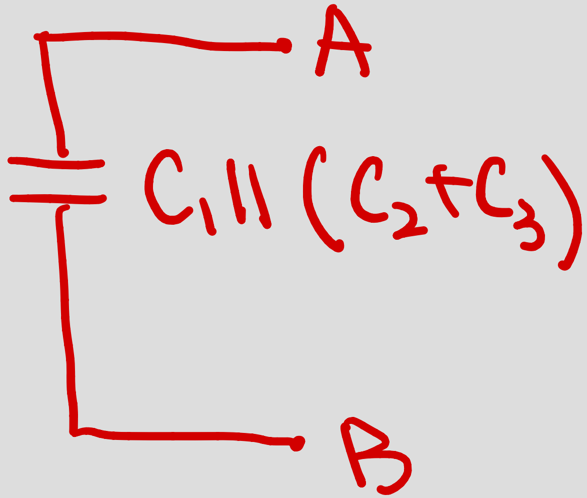
$$C_{eq} = C_1 + C_2$$



$\Rightarrow$

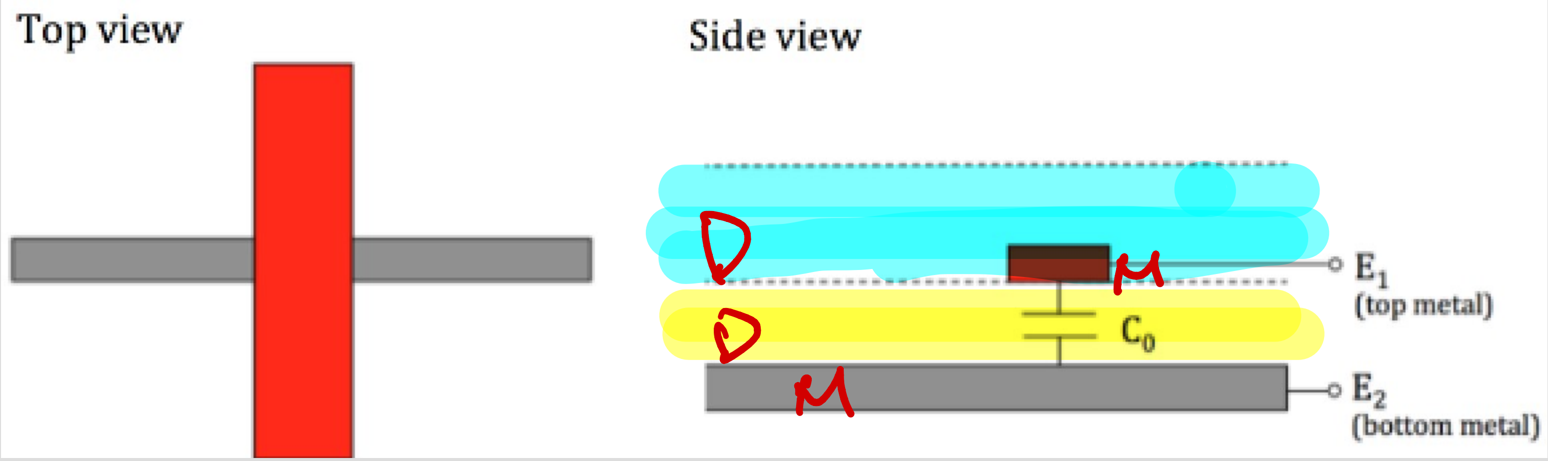


$\Rightarrow$



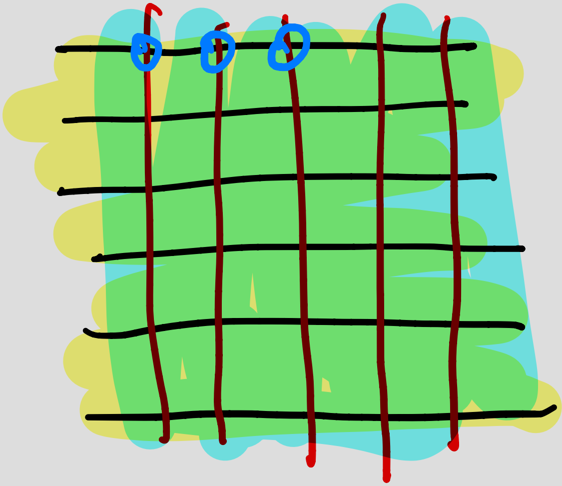
$$C_{eq} = C_1 \parallel (C_2 + C_3)$$

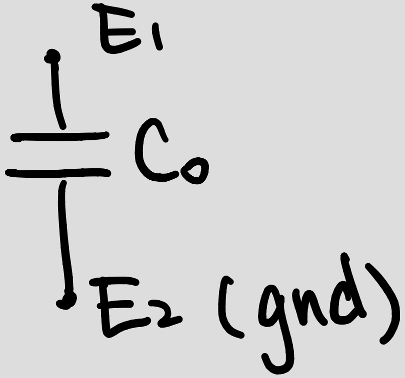
Capacitive Touchscreen – Model without touch



M – metal

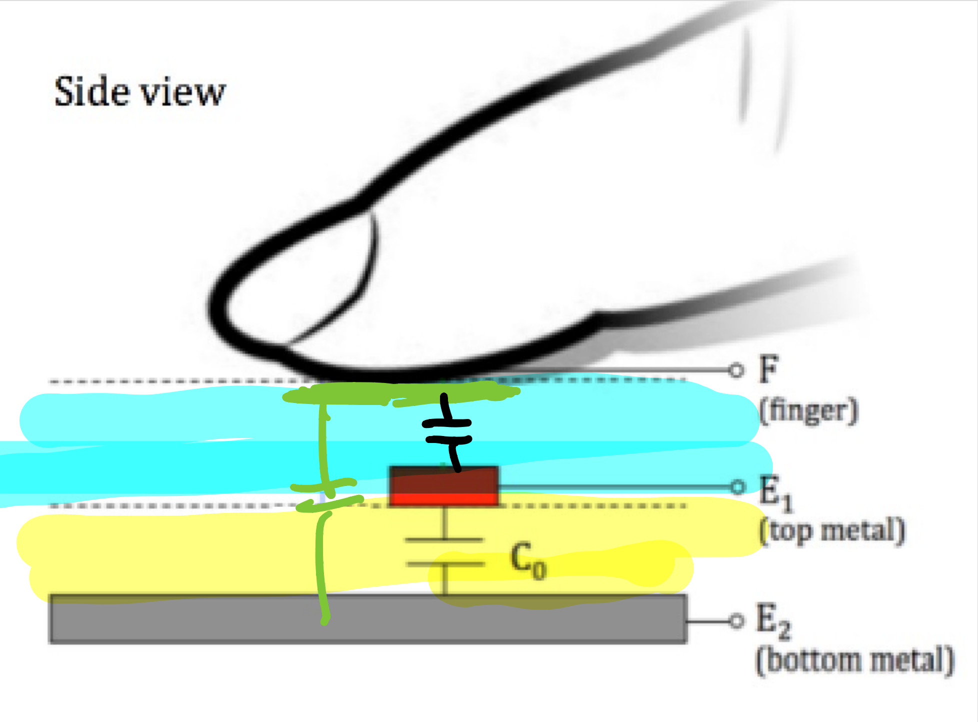
D – dielectric



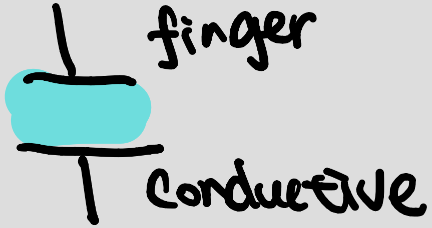


$$C_0 = \varepsilon \frac{A}{d}$$

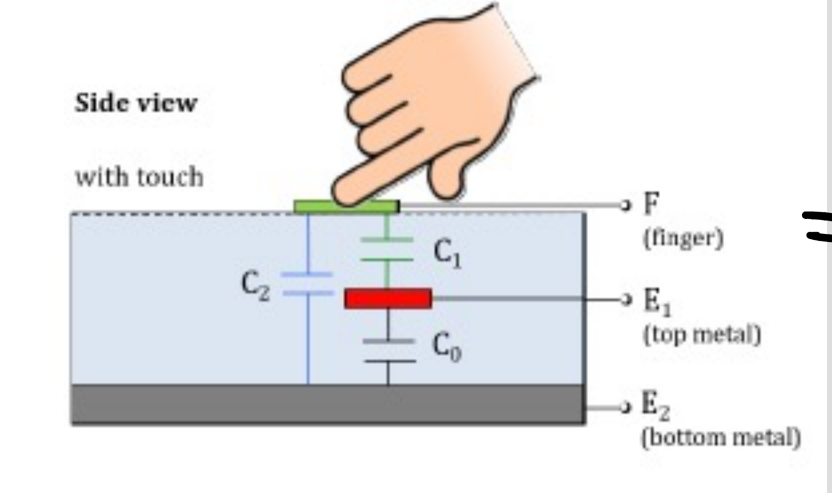
Capacitive Touchscreen – Model with touch



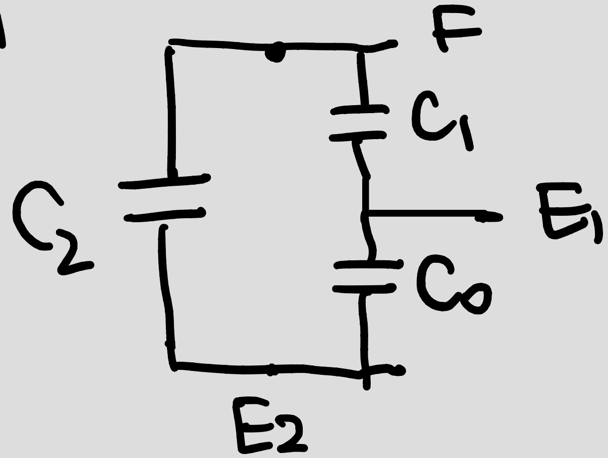
When there is touch, we form a capacitor



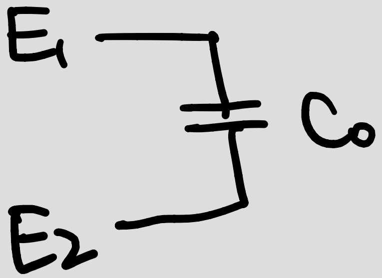
Problem: How can we measure V or I if our electrode is a finger?



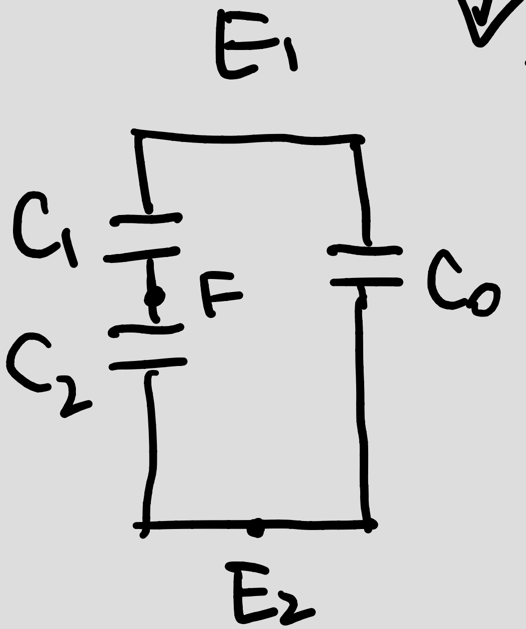
Circuit model

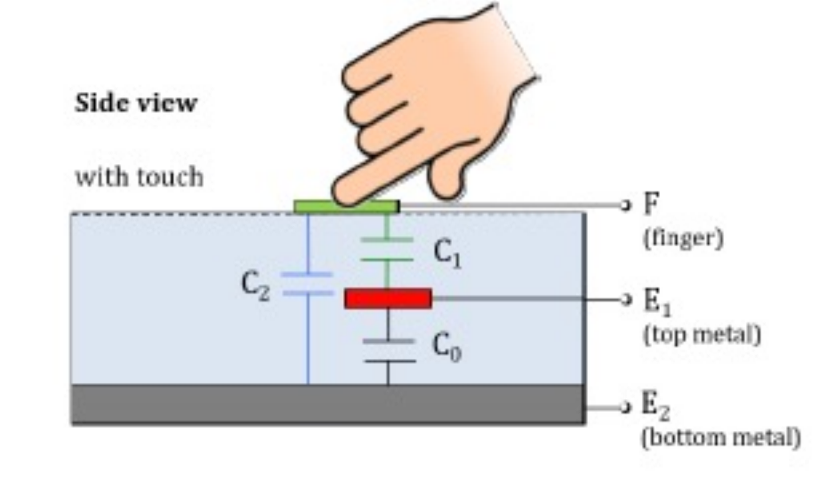


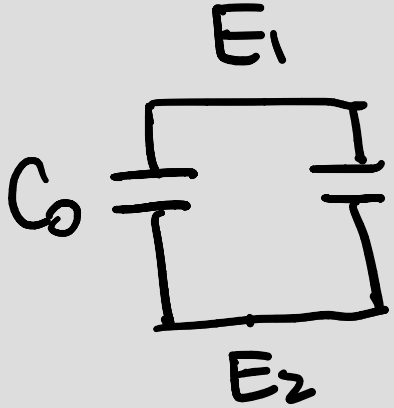
When no touch



Redraw to focus on terminals we can measure E_1 & E_2

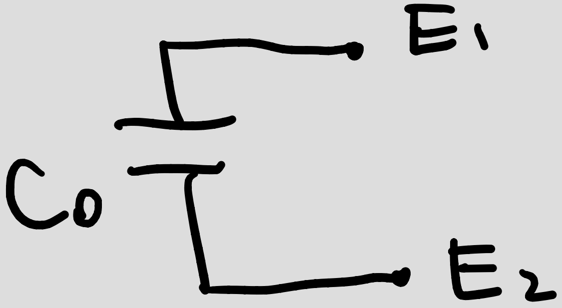






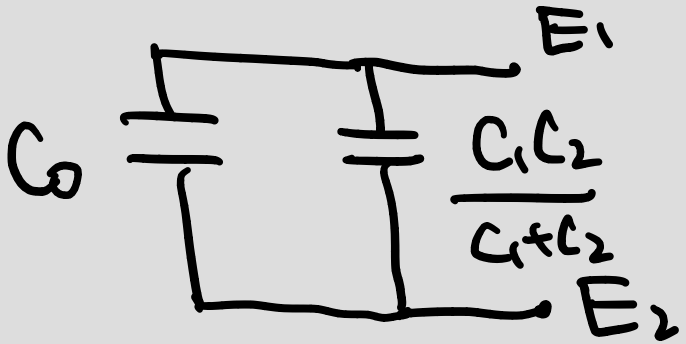
$C_1 // C_2 = \frac{C_1 C_2}{C_1 + C_2}$

No touch



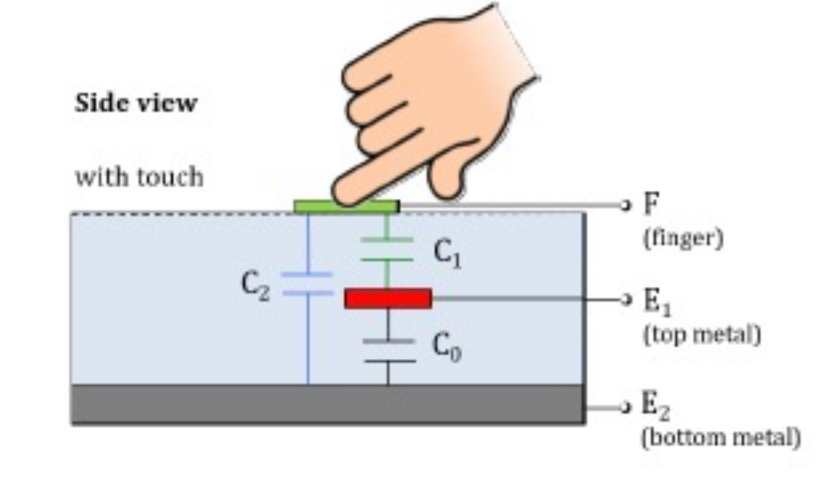
C_0

With Touch



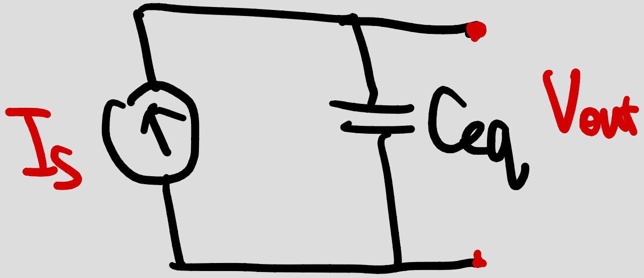
$\boxed{C_0 + \frac{C_1 C_2}{C_1 + C_2}}$

$\frac{C_1 C_2}{C_1 + C_2} = C_\Delta$ (change)



How do we measure change in capacitance?

Option 1:



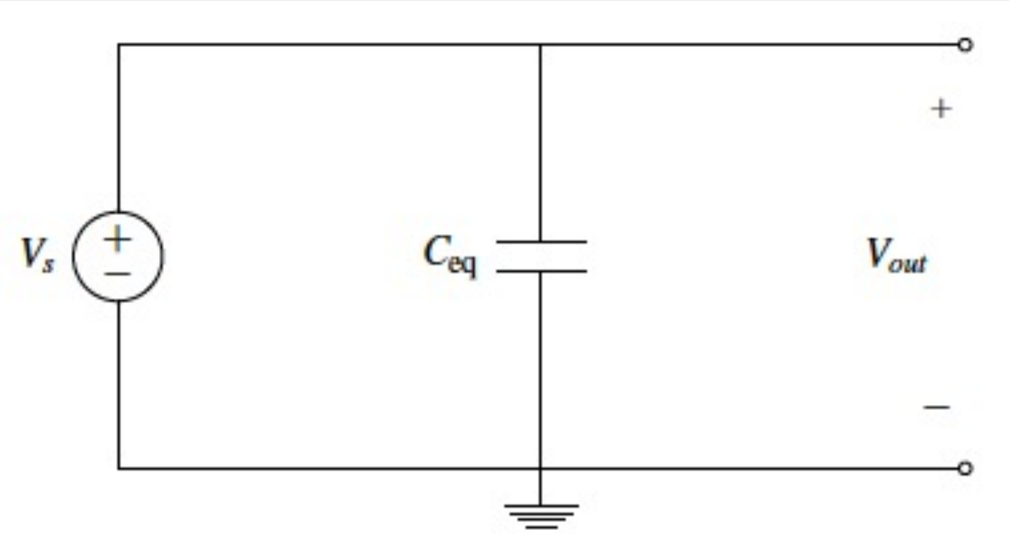
Assume $V_{out}(0) = 0$

$$I_s = C_{eq} \frac{dV_{out}(t)}{dt} \rightarrow V_{out}(t) = \int_0^t \frac{I_s}{C_{eq}} \, dt$$

$$V_{out} = \frac{I_s \cdot t}{C_{eq}} \Rightarrow C_{eq} = \frac{I_s \cdot t}{V_{out}}$$

Can't build current source easily

Measuring Capacitance Models – Attempt #1

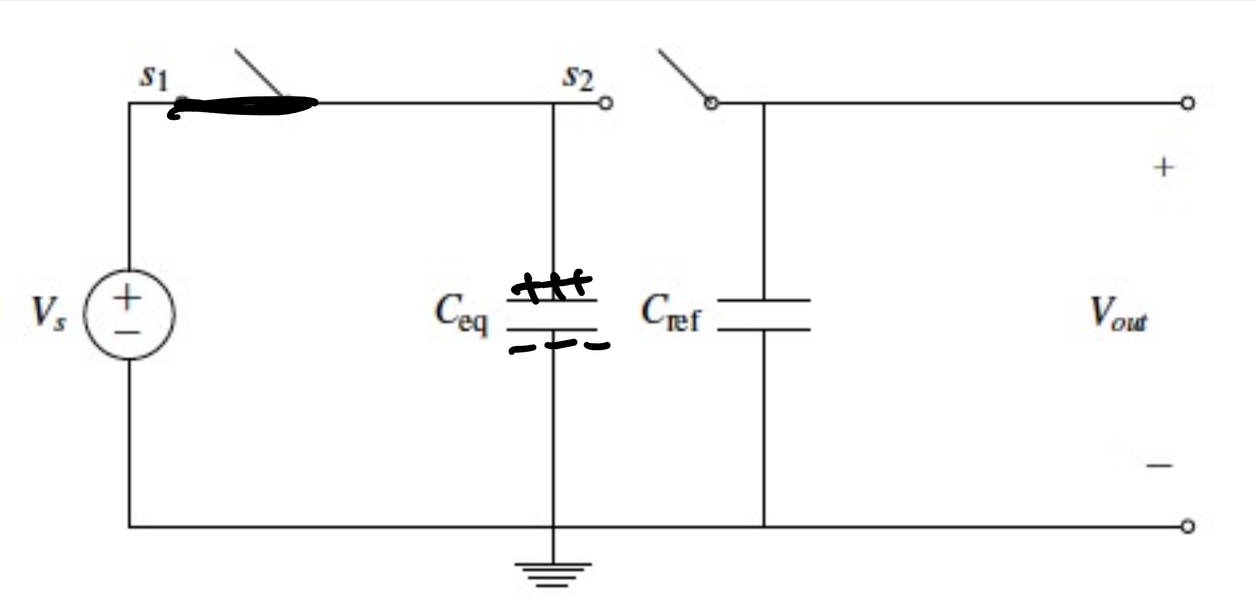


If there is touch : $V_{out} = V_s$

If there is no touch: $V_{out} = V_s$

Bad Idea!

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor



① Close both switches

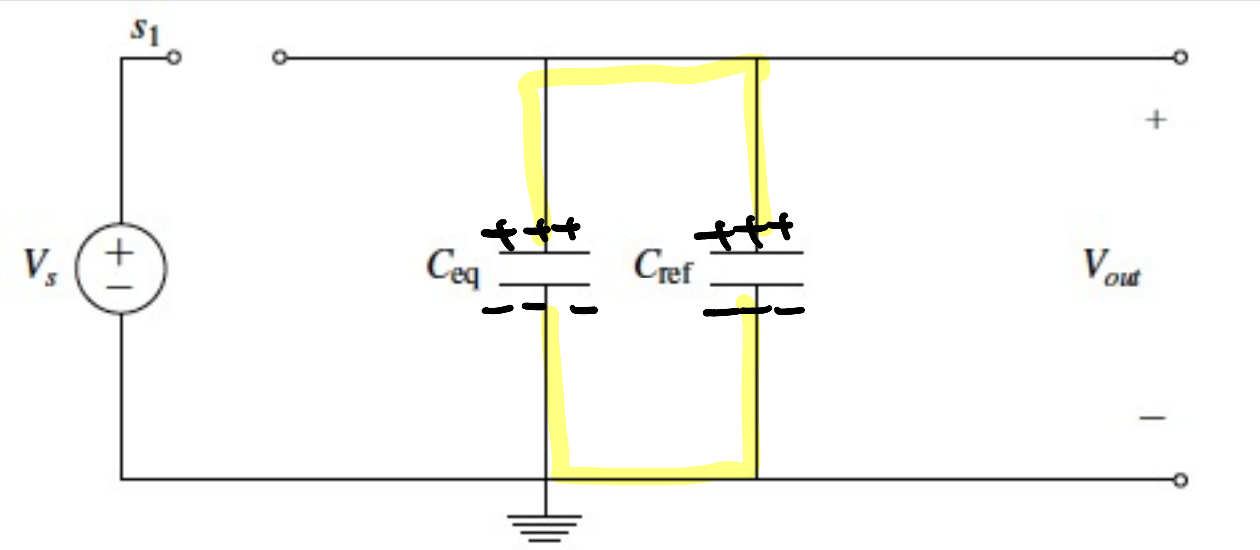
Same as before

② phase 1 : Close S1, open S2

$$Q = V_s \cdot C_{eq}$$

Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

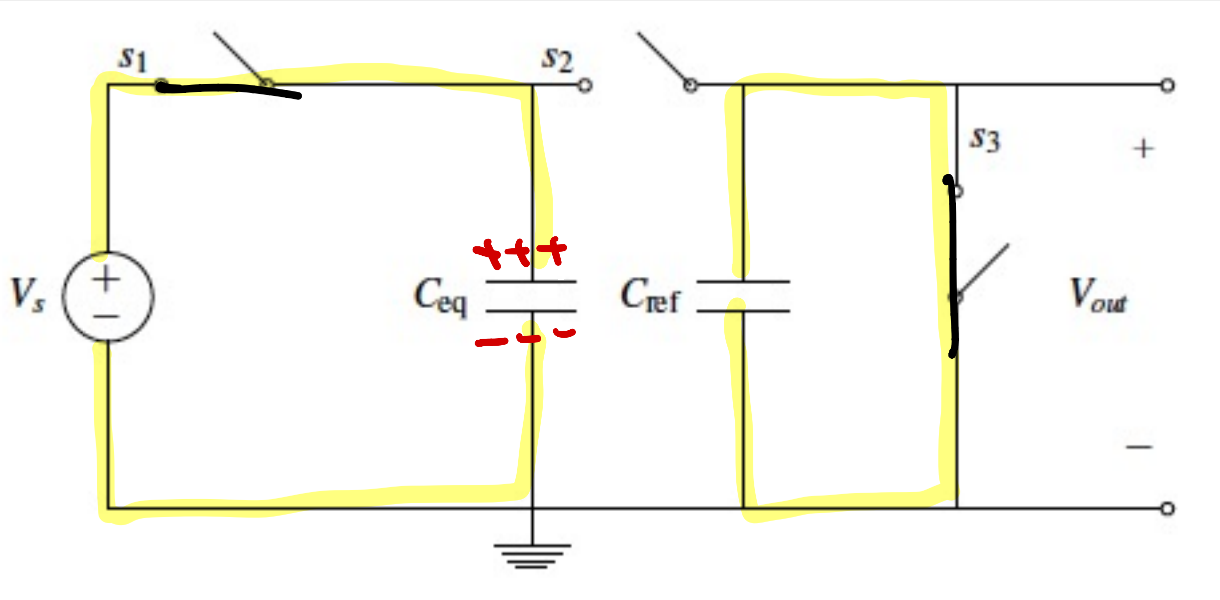
phase 2:
Close S2, open S1



Initial condition?

Charge sharing

Measuring Capacitance Models – Attempt #3 – known initial condition



Phase 1: S_1, S_3 closed, S_2 open

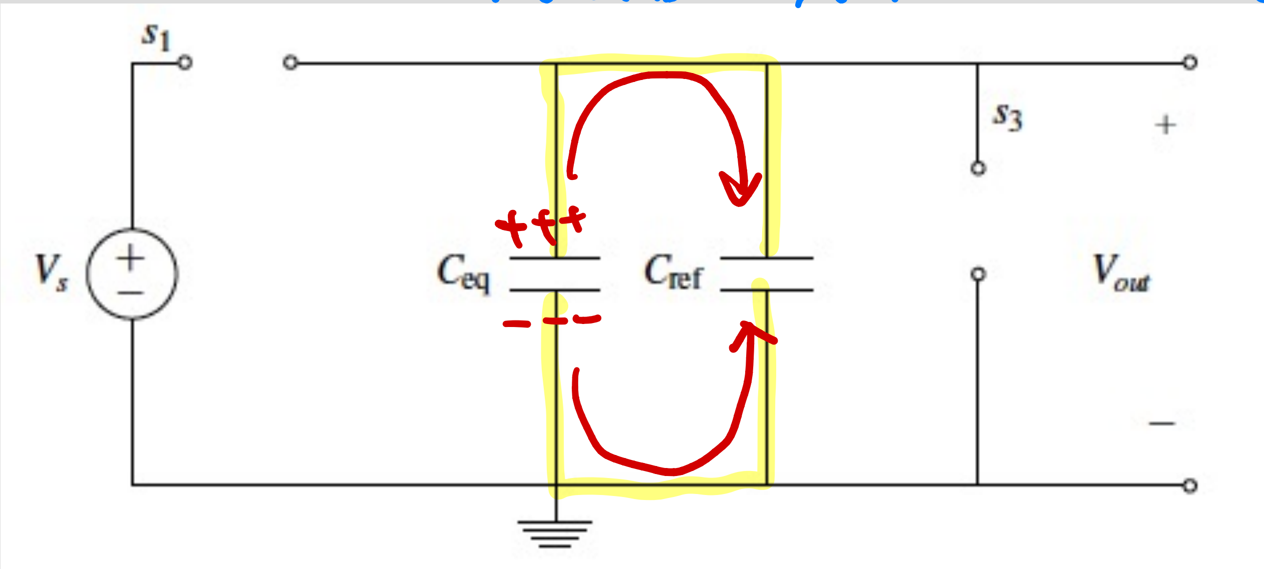
- Charge C_{eq}
- Discharge C_{ref}

$Q_{ref} = C_{ref} \cdot V_{out} = 0 \quad (V_{out} = 0)$

$Q_{eq} = C_{eq} \cdot V_s$

Measuring Capacitance Models – Attempt #3 – known initial condition

redistribute, until same voltage



Phase 2: S_1, S_3 open, S_2 closed

Voltage across C_{eq}: V_{out}

Voltage across $C_{ref} = V_{out}$

$$Q_{total_2} = C_{eq} \cdot V_{out} + C_{ref} \cdot V_{out}$$

Effect of touch on total capacitance

Total charge is conserved!!

$$Q_{total_1} = Q_{total_2}$$

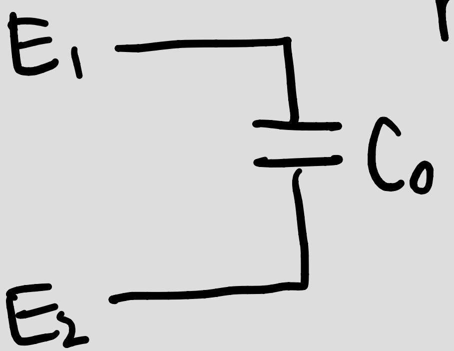
$$C_{eq} \cdot V_s = C_{eq} \cdot V_{out} + C_{ref} \cdot V_{out}$$

$$V_{out} = \boxed{\frac{C_{eq}}{C_{eq} + C_{ref}} \cdot V_s}$$

By touching, we change voltage

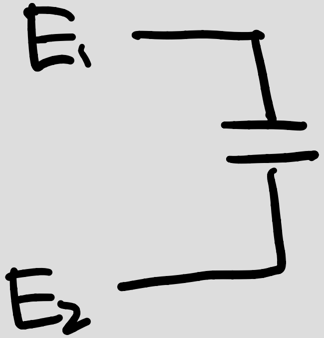
Effect of touch on total capacitance

no touch



$$V_{out} = \frac{C_0}{C_0 + C_{ref}} \cdot V_s$$

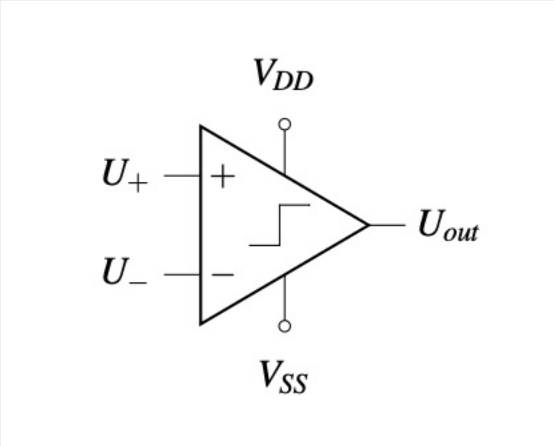
with touch



$$C_0 + \underbrace{\boxed{\frac{C_1 C_2}{C_1 + C_2}}}_{C_\Delta}$$

$$V_{out} = \frac{C_0 + C_\Delta}{C_0 + C_\Delta + C_{ref}} \cdot V_s$$

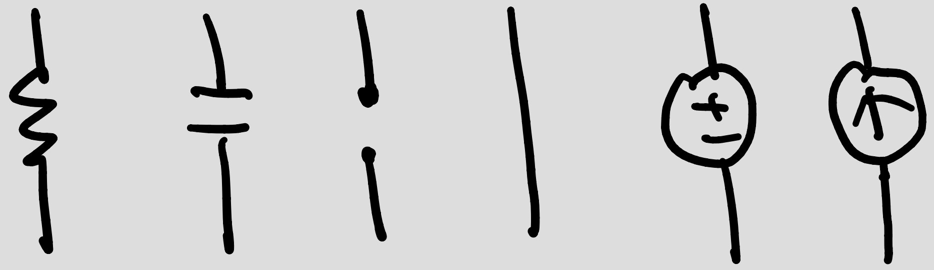
How can we go from voltage measurement to binary answer: touch or no touch?



- Threshold Voltage (V_{th}): between V_{out_touch} & $V_{out\_notouch}$
- Above V_{th} : 1 (touch)
- Below V_{th} : 0 (no touch)

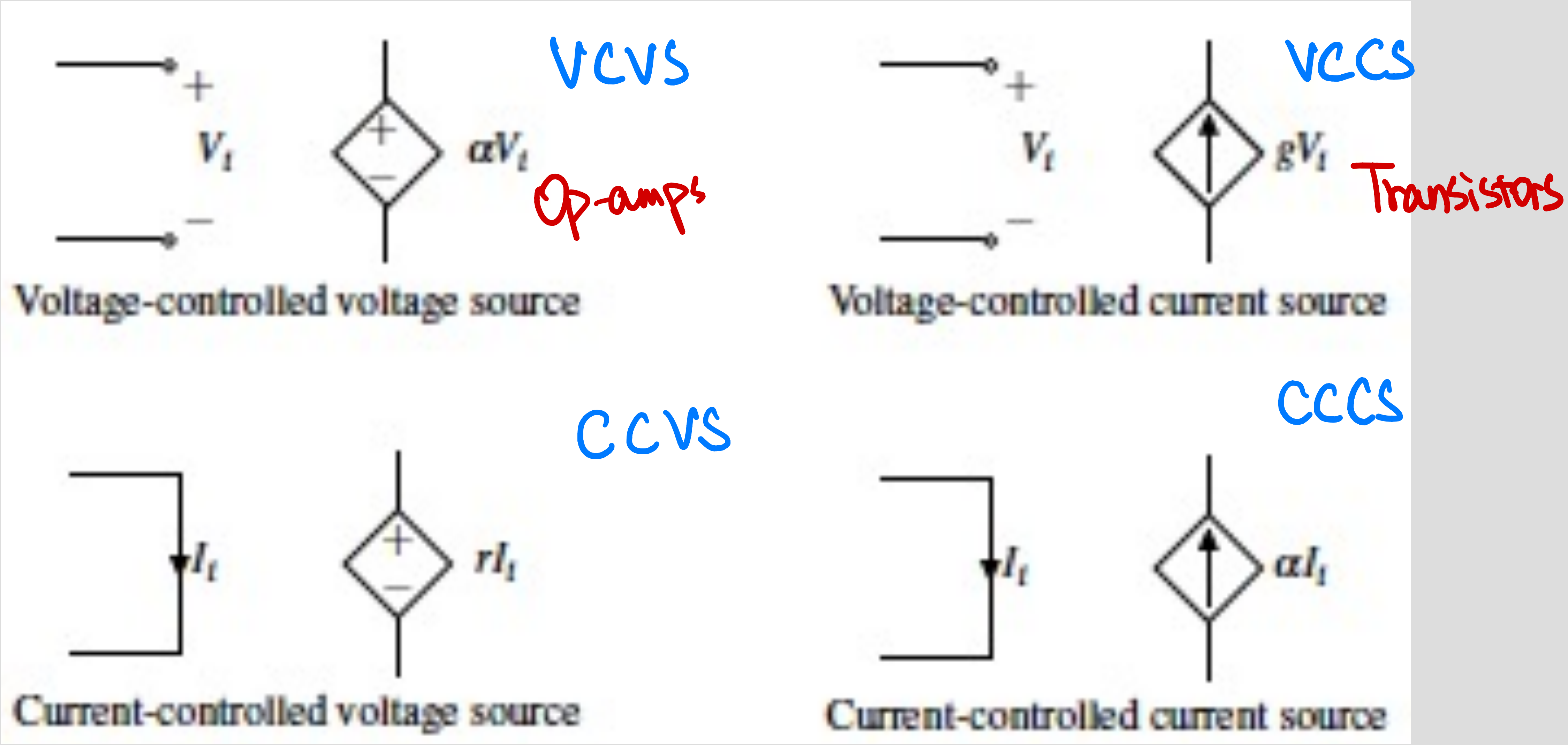
We need to compare V_{out} to V_{th}

So far:



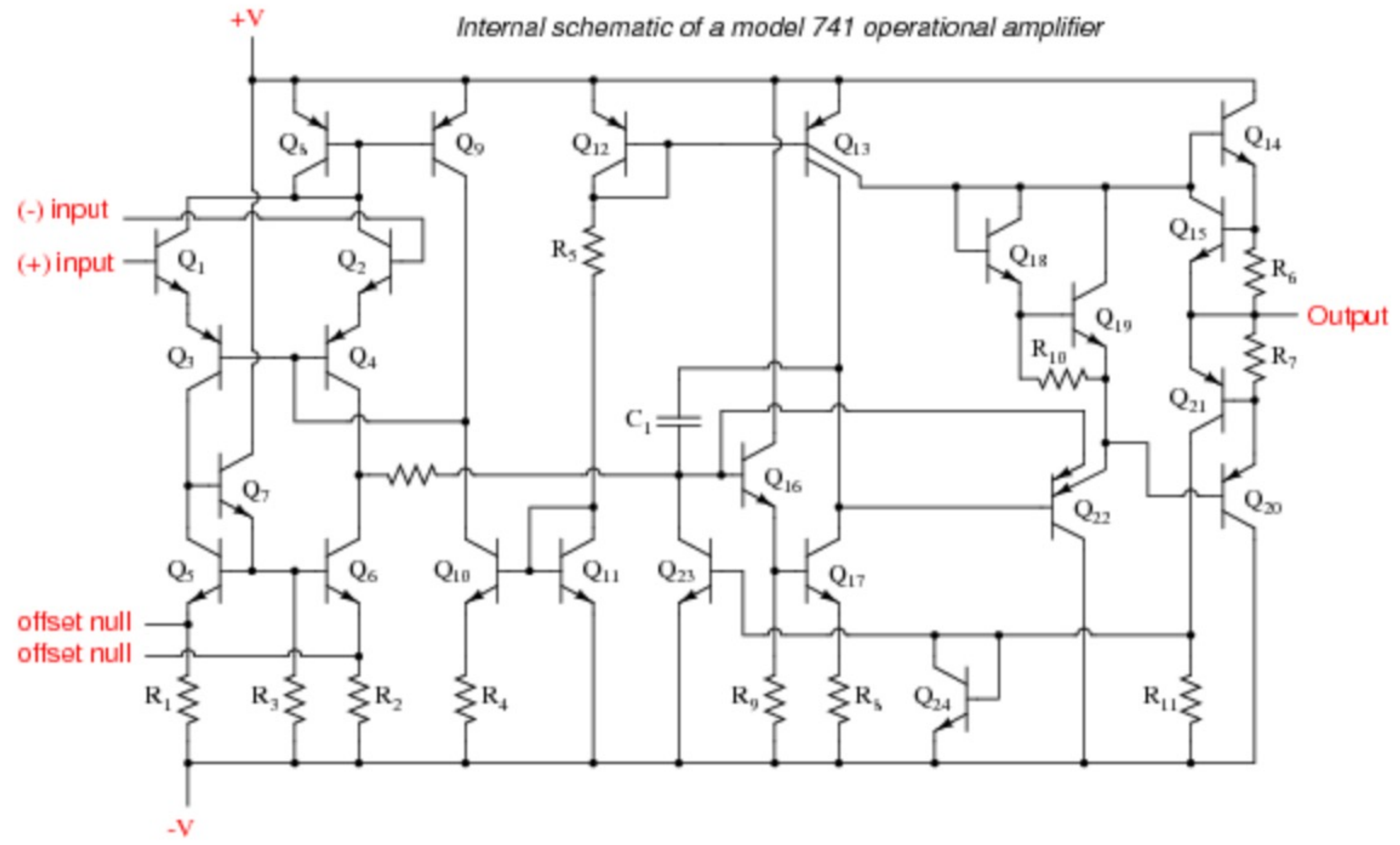
How can we go from voltage measurement to binary answer: touch or no touch?

- New tools are needed – new circuit elements



An example of an Op-amp circuit diagram

EE105
EE140



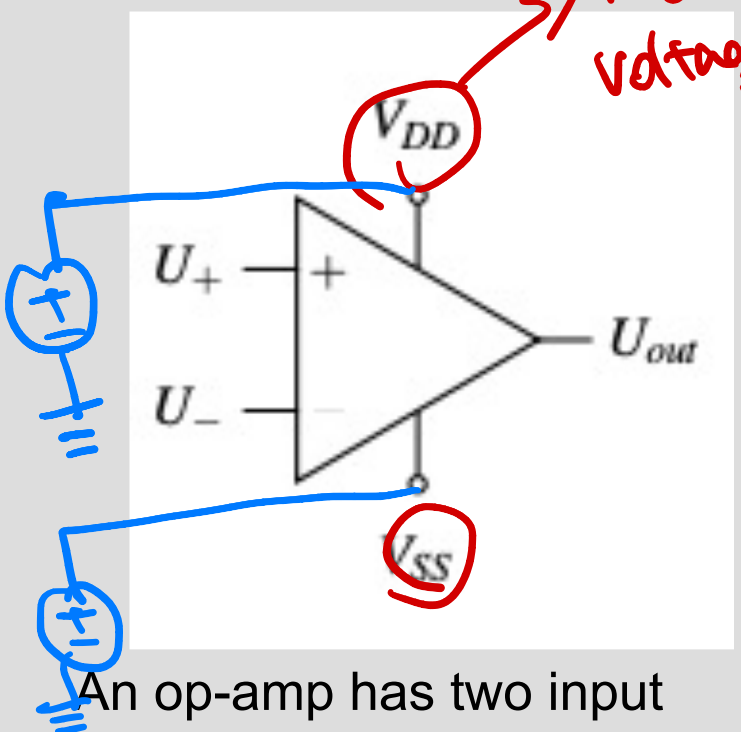
Schematic diagram of a model 741 op-amp.

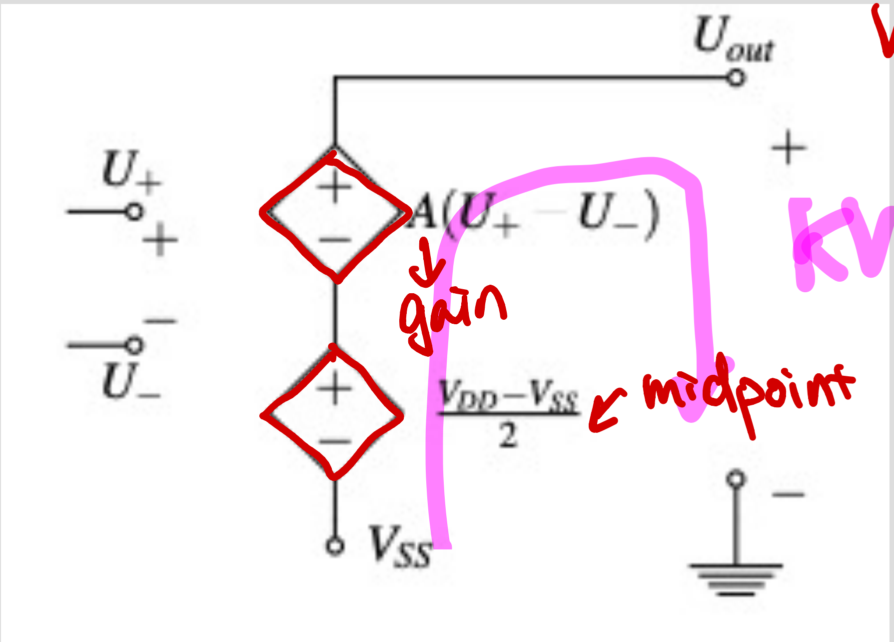
Operational Amplifier

An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.

Independent voltage source

Ideal → $A = \infty$





gain

midpoint

KVL

$V_{out} = V_{ss} + \frac{V_{DD}-V_{ss}}{2} + A(U^+ - U^-)$

$= \frac{V_{DD}+V_{ss}}{2} + A(U^+ - U^-)$

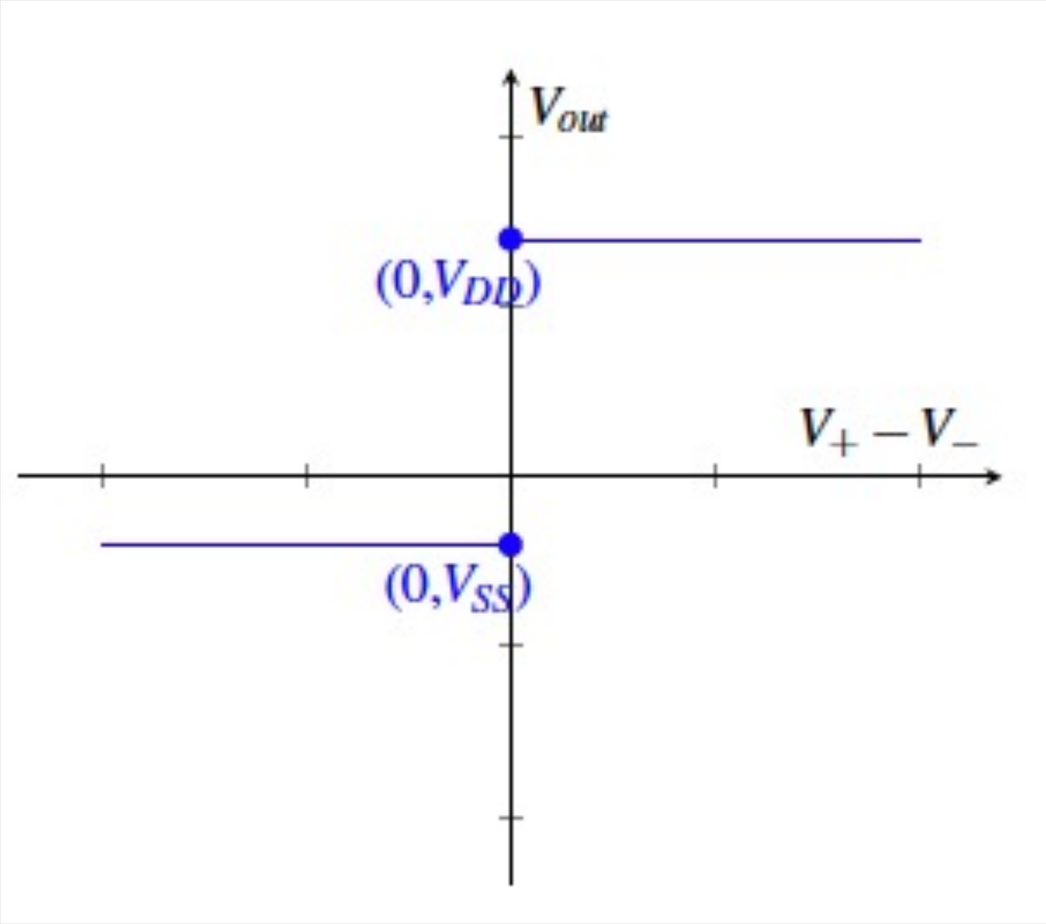
An op-amp has two input terminals marked (+) and (−) with potentials U+ and U−, two power supply terminals called VDD and VSS, and one output terminal with potential Uout.

U_+ connect to V_{out}

U_- connect to V_{th}

V_{DD}, V_{ss} limits upper & lower bounds

Comparator – optimized for binary output & speed

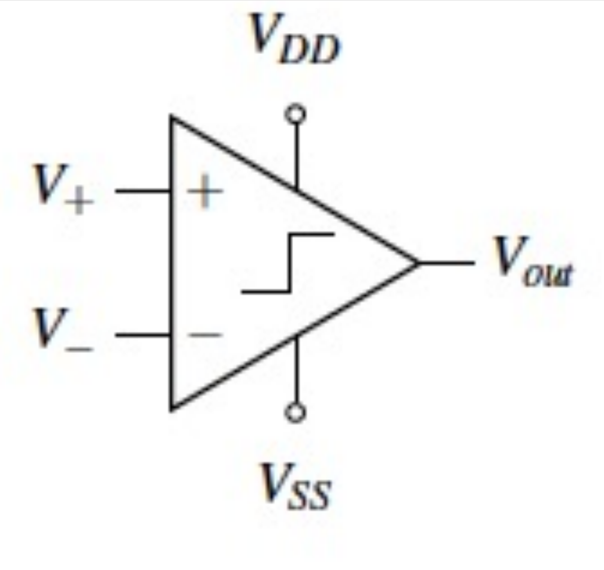


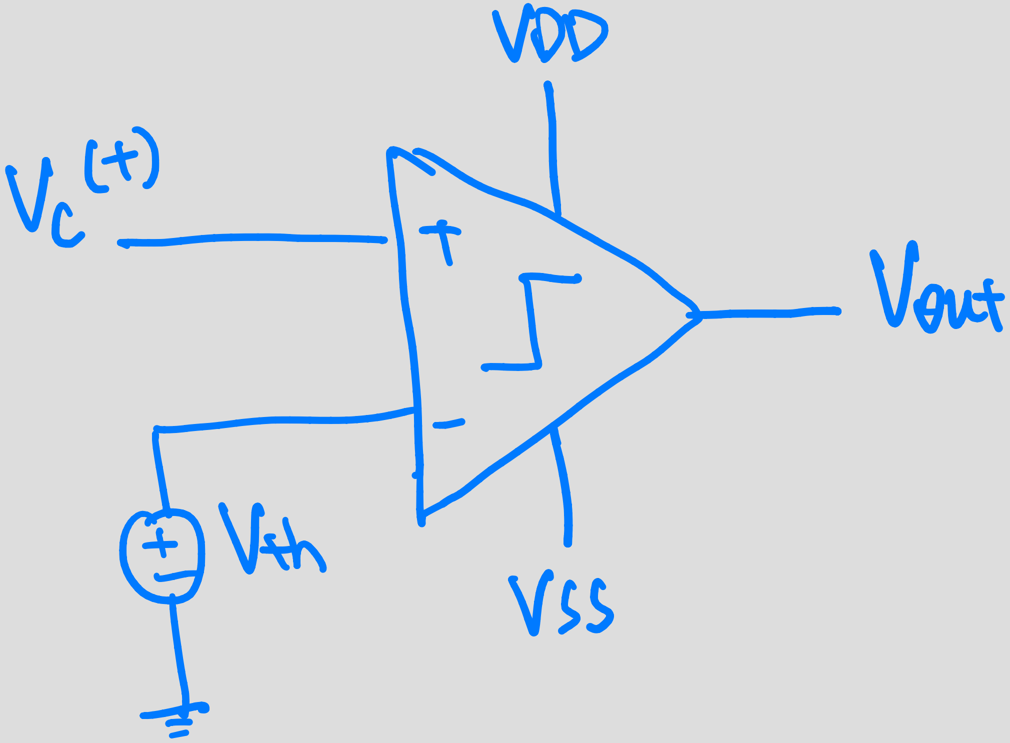
$V_{out} = V_{DD}$ if $V^* > V_{DD}$

$V_{out} = V_{SS}$ if $V^* < V_{SS}$

Assume $A = \infty$

Comparator – optimized for binary output

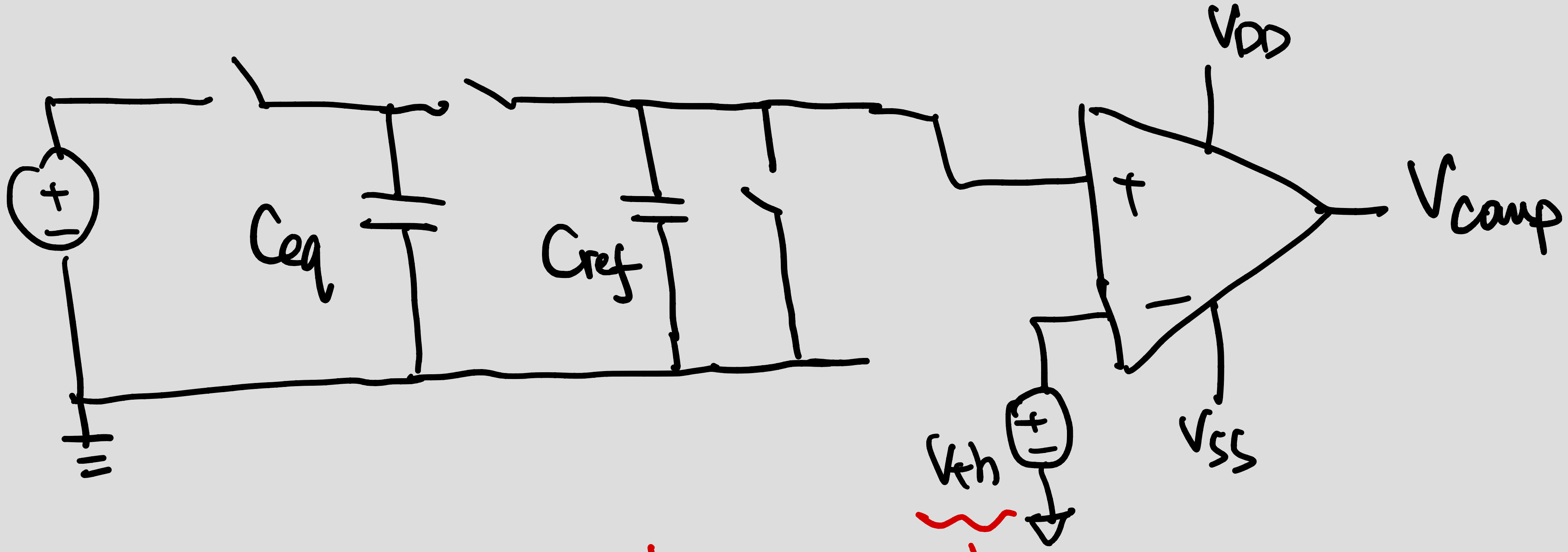


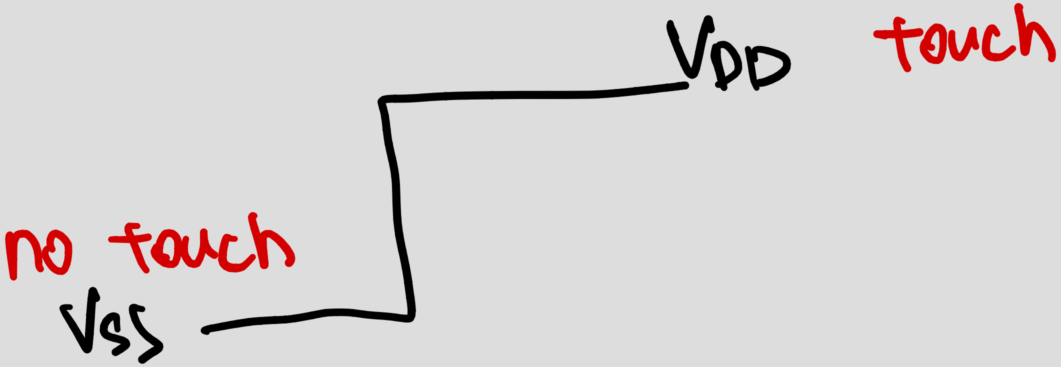


If $V_c(t) > V_{th}$, $V_{out} = VDD$

If $V_c(t) \leq V_{th}$, $V_{out} = VSS$

Back to our Capacitive Touchscreen





between V_{touch} & $V_{notouch}$

Enjoy Spring Break!