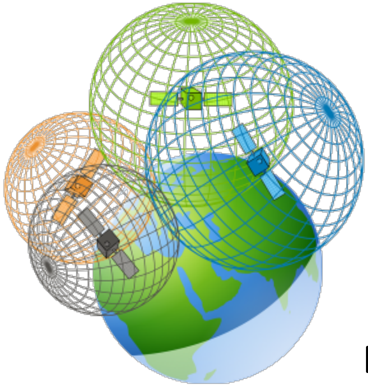
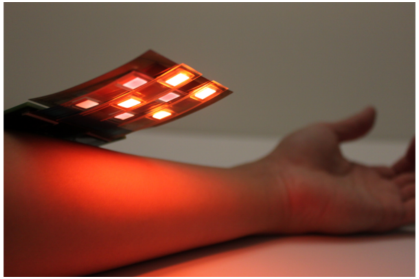




Welcome to EECS 16A!

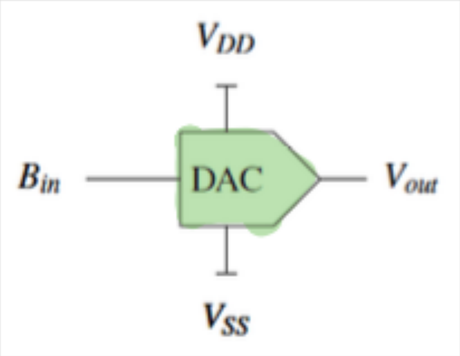
Designing Information Devices and Systems I

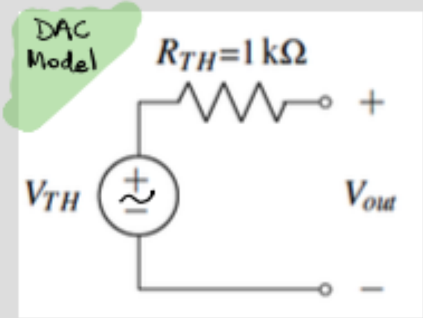
Ana Claudia Arias and Miki Lustig

Fall 2021

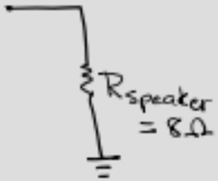
Module 2
Lecture 10
Negative Feedback
(Note 18)

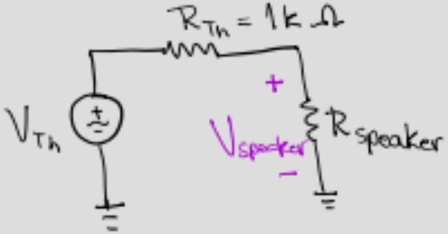
Last Class…





Speaker





Voltage Divider

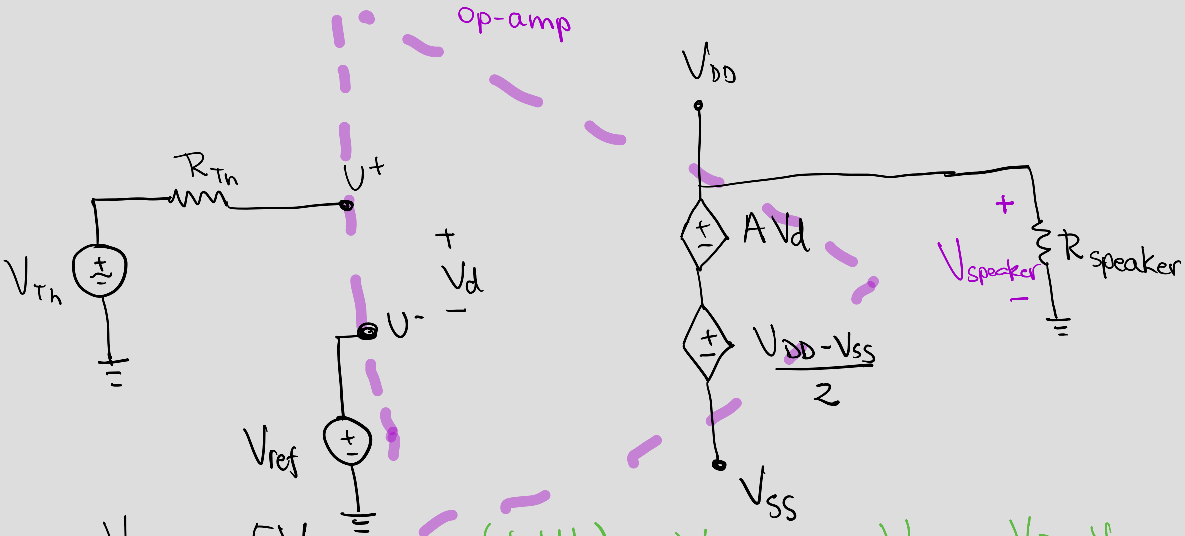
$$V_{speaker} = \frac{\overbrace{R_{speaker}}^{8\,\Omega}}{\underbrace{R_{Th}}_{1000\,\Omega} + \overbrace{R_{speaker}}^{8\,\Omega}} \cdot V_{Th}$$

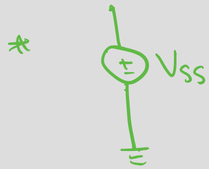
$$V_{speaker} = \frac{V_{th}}{126}$$

Not loud!
Too quiet!

Need to isolate DAC.

Digital to Analog Converter - DAC





$V_{DD} = -V_{SS} = 5V$

10V output

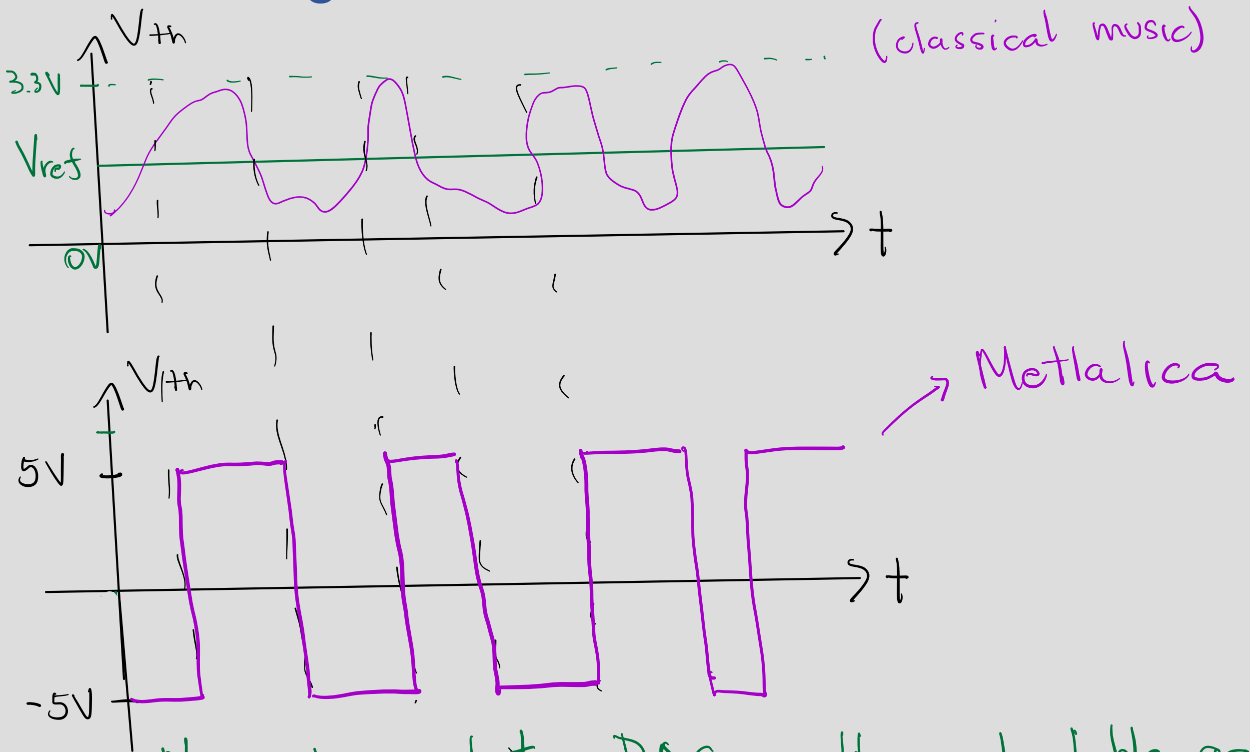
(Input)

$$V_d = U^+ - U^- = V_{th} - V_{ref}$$

(KVL) $V_{speaker} = \underbrace{V_{SS} + \frac{V_{DD}-V_{SS}}{2}}_{\frac{V_{DD}+V_{SS}}{2} \to 0} + AV_d = AV_d$

when: $V_{SS} < AV_d < V_{DD}$

Digital to Analog Converter - DAC



Need to isolate DAC with controlable gain!

e.g. 3x

Negative Feedback

$$S_{err} = S_{in} - S_{fb}$$

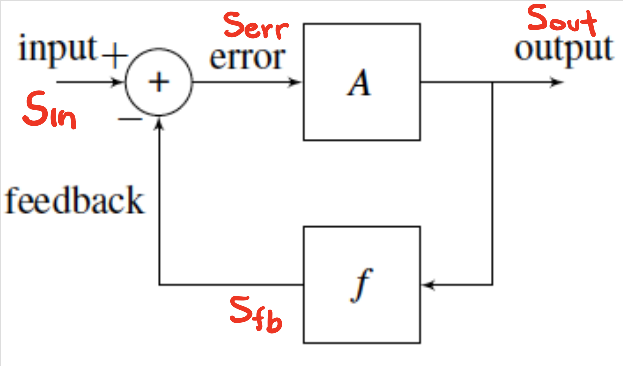
$$S_{out} = A \cdot S_{err}$$

$$S_{fb} = f \cdot S_{out}$$

$$\frac{S_{out}}{A} = S_{in} - S_{fb}$$

$$S_{out}\left(\frac{1}{A} + f\right) = S_{in}$$

$$\frac{S_{out}}{S_{in}} = \frac{1}{\frac{1}{A} + f} = \frac{A}{1 + Af}$$



- Making small adjustments to correct output on the fly
- Basis of control theory
- Many examples in daily life:

- Biology
- Self-driving car
- Human driving car
- Hand-eye coordination
- . . . .

Negative Feedback

$$\boxed{\frac{S_{out}}{S_{in}} = \frac{A}{1 + Af}}$$

- Describes the behaviour of the system – transfer function.
- How Sout depends on Sin

$$\frac{S_{out}}{S_{in}} \underset{A \to \infty}{=} \frac{1}{f}$$

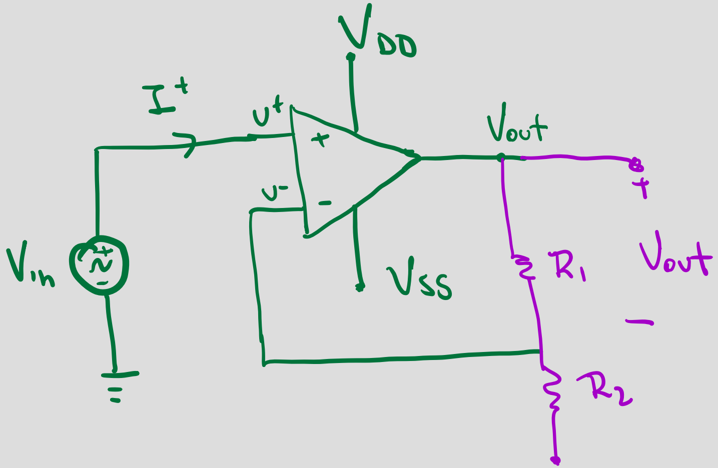
↳ We control the output via block $\boxed{f}$!

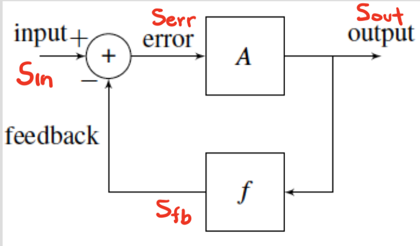
So $\boxed{V_{out} = \frac{1}{f} V_{in}}$ for very large gain.

↳ we can set f to get any output.

(Beautiful result). ☺

Need to isolate the DAC from speaker – OP-Amp with NFB





$U^+ = S_{in}$

$V_{out} = S_{out}$

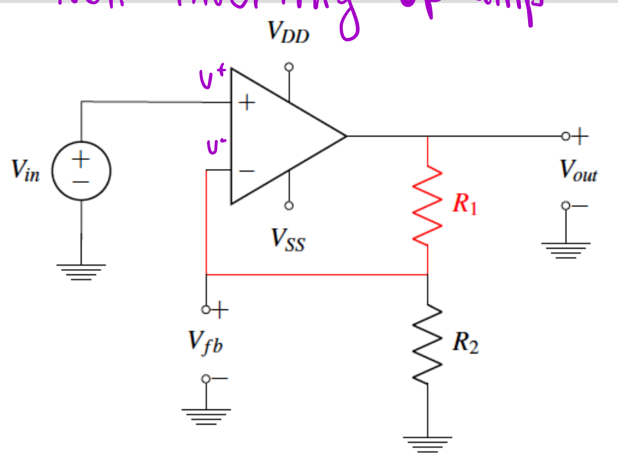
$U^- = S_{fb}$

$U^+ - U^- = S_{err}$

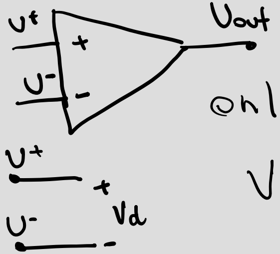
- We want to measure V_{out}, take a portion of the signal and feedback as U^-

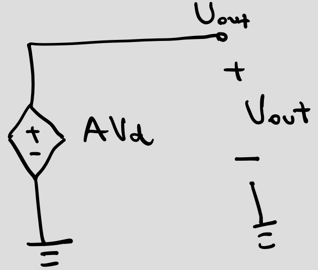
Op-Amp in negative feedback

Non-inverting op-amp



Model :





only for

$V_{SS} < V_{out} < V_{DD}$

Simpler model as the second source is not "needed".

(1) $V_d = U^+ - U^- = V_{in} - V_{fb}$

(2) $V_{out} = A V_d$

(3) $V_{fb} = \underbrace{\boxed{\dfrac{R_2}{R_1 + R_2}}}_{f} \cdot V_{out}$

"BUFFER circuit"

$$V_{out} = A\,(V_{in} - f \cdot V_{out})$$

$$V_{out}\,(1 + Af) = A\,V_{in}$$

$$A_v = \text{Gain} = \frac{V_{out}}{V_{in}} = \frac{A}{1 + Af}$$

$$A_v \underset{A \to \infty}{=} \frac{1}{f} \rightarrow \frac{R_1 + R_2}{R_2} = \boxed{1 + \frac{R_1}{R_2}}$$

Golden Rules of Op-Amps

For our design we want $A = 3$

$$V_d = \frac{V_{out}}{A} \quad \text{if } A \to \infty$$

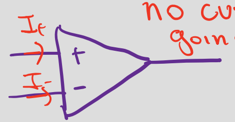
$$V_d = \frac{1}{A} \cdot \frac{A}{1+Af} V_{in} = \frac{V_{in}}{1+Af} = 0$$

In NFB: $U^+ = U^-$ and $A \to \infty$

Rules: (Golden Rules)

(1) $I^+ = I^- = 0$ (always true)

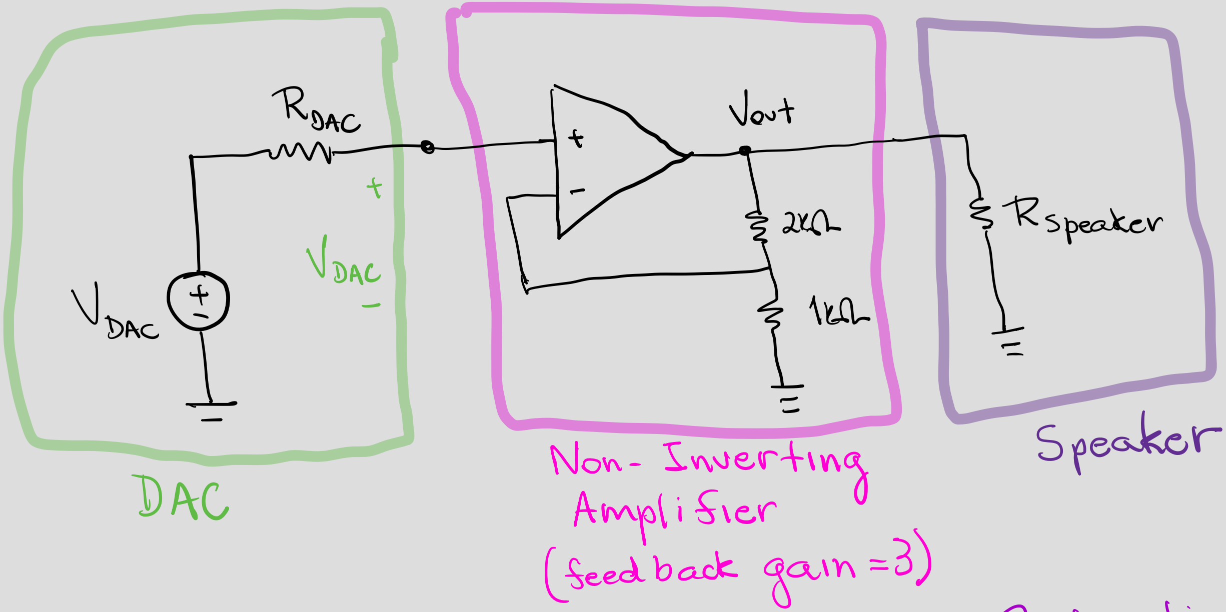
(2) $U^+ = U^-$ (only in NFB & $A \to \infty$)



no current going in

$I^+ = I^- = 0$

Let's go back to playing music

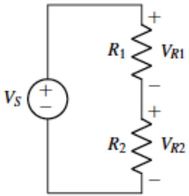


Party time!

Yay!

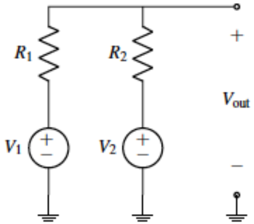
Today

Voltage Divider



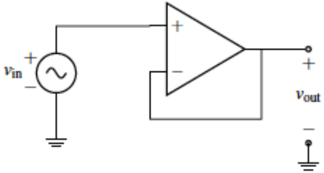
$$V_{R2} = V_S \left(\frac{R_2}{R_1+R_2}\right)$$

Voltage Summer



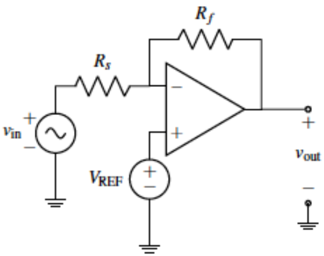
$$V_{out} = V_1 \left(\frac{R_2}{R_1+R_2}\right) + V_2 \left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



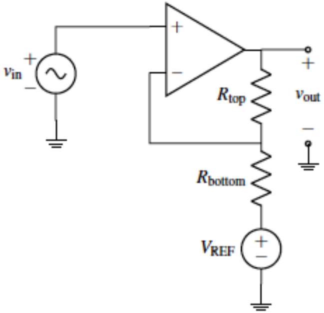
$$\frac{v_{out}}{v_{in}} = 1$$

Inverting Amplifier



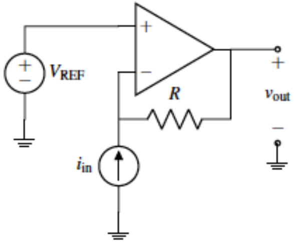
$$v_{out} = v_{in} \left(-\frac{R_f}{R_s}\right) + V_{REF} \left(\frac{R_f}{R_s} + 1\right)$$

Non-inverting Amplifier



$$v_{out} = v_{in} \left(1 + \frac{R_{top}}{R_{bottom}}\right) - V_{REF} \left(\frac{R_{top}}{R_{bottom}}\right)$$

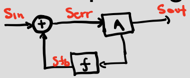
Transresistance Amplifier



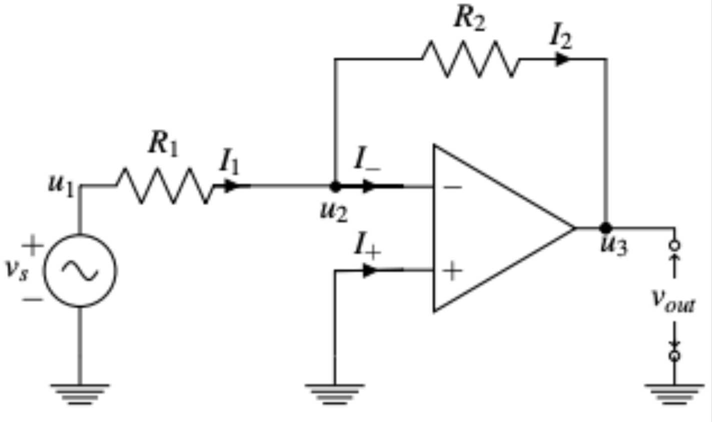
$$v_{out} = i_{in}(-R) + V_{REF}$$

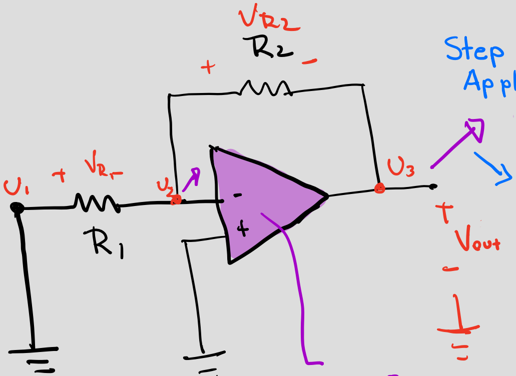
Checking for Negative Feedback (Determining the polarity of NFB)

Step 1 – Zero out all independent sources : replacing voltage sources with wires and current sources with open circuits as in superposition



Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.
- if the error signal decreases, the output must also decrease. The circuit is in negative feedback
- if the error signal increases, the output must also increase. The circuit is in positive feedback



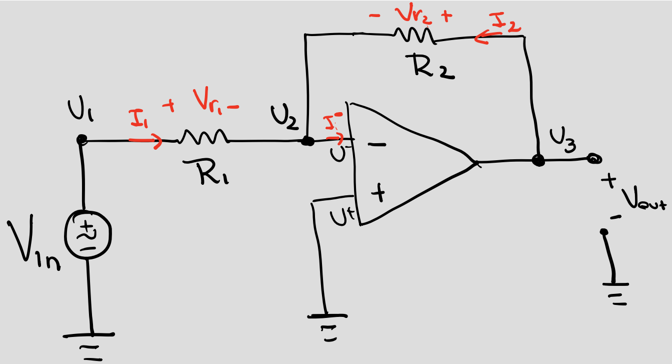


Step 2:
Apply disturbance
U_3 increases
U_2 increases (V^-)
U_3 decreases
(NFB) ✓

$A(U^+ - U^-)$

(when U^- increase
$A(U^+ - U^-)$ decreases)

Now lets solve it...



NFB ⇒ GR#2 applies
$U^+ = U^-$

① $U_1 = V_{in}$

$U_3 = V_{out}$

$U_2 = 0$ (circuit in NFB ⇒ GR#2 applies $U^+ = U^-$
↳ $U_2 = U^-$ we know $U^+ = 0$ ⇒ $U^- = 0$
$U^- = U_2$ ⇒ $U_2 = 0$)

② Element Definitions:

$V_{R_1} = I_1 R_1$

$V_{R_2} = I_2 R_2$

Voltage Def.

$V_{R_1} = U_1 - \cancel{U_2}^{0} = U_1 = V_{in}$

$V_{R_2} = U_3 - \cancel{U_2}^{0} = U_3 = V_{out}$

③ (KCL)

$I_1 + I_2 = \cancel{I^-}^{0}$ (GR#1)

$V_{in} = U_1 = I_1 R_1$

$V_{out} = U_3 = I_2 R_2$

$I_1 + I_2 = 0$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

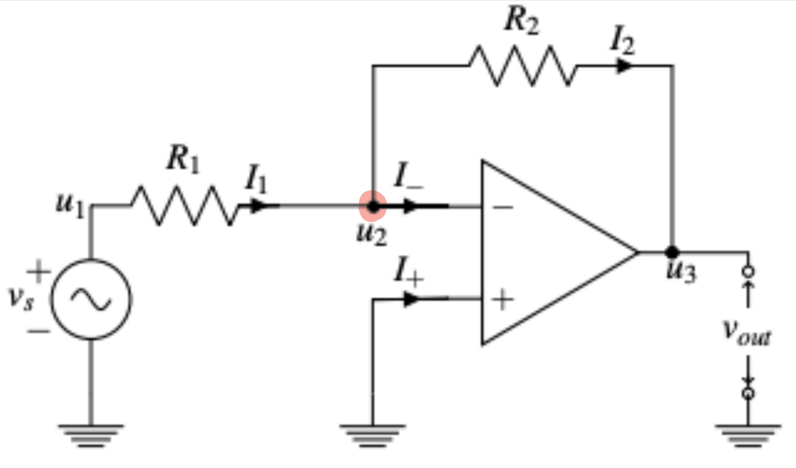
$$V_{out} = R_2 \cdot \left(-\frac{V_{in}}{R_1}\right)$$

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

Inverting Amplifier

$$\boxed{A_V = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

A faster way…



$\boxed{GR2}$: $U^+ = U^-$

$U_2 = U^-$
$U^+ = 0$ $\Rightarrow$ $\boxed{U_2 = 0}$

GR1 + KCL $\quad (I_1 = I_2 + I^-)$

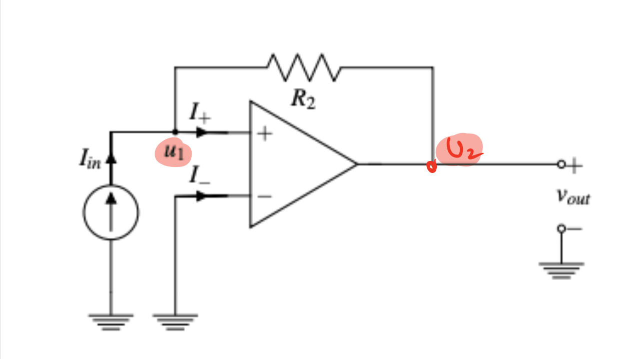
$$\frac{\cancel{U_2}^{\to 0} - U_1}{R_1} = \frac{U_3 - \cancel{U_2}^{\to 0}}{R_2} + \cancel{I^-}^{\to 0}$$

$$-\frac{\cancel{U_1}^{V_{in}}}{R_1} = \frac{\cancel{U_3}^{V_{out}}}{R_2}$$

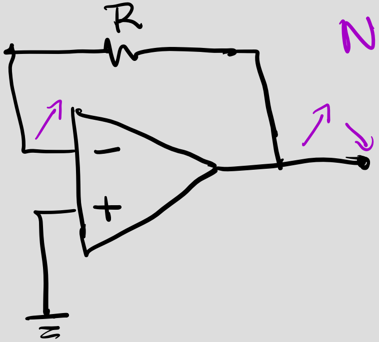
$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

Example circuit 2 (trans-resistance amplifier)

$I^+ = 0 \Rightarrow V_1 = V_2$

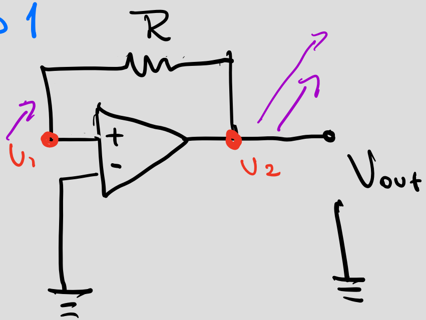


Invert polarity
$\Rightarrow$



NFB

Step 1



Step 2: Check for NFB

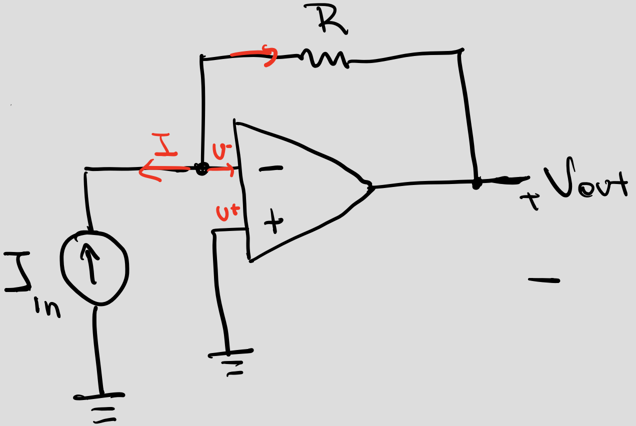
Increase output →

\+ moves up

output increases

by a lot

X Not in NFB



$$NFB: \quad U^+ = U^-$$

$$U^+ = 0 \rightarrow U^- = 0$$

GR # 2

$$\frac{\cancel{U^-}^{\,0} - V_{out}}{R} + (-I_{in}) + \cancel{I^-}^{\,0} = 0$$

GR2 GR1

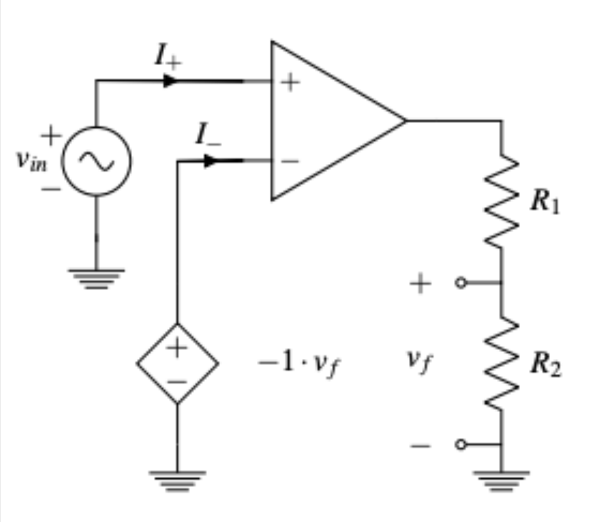
$$-\frac{V_{OUT}}{R} = I_{in}$$

$$V_{out} = -I_{in} R$$

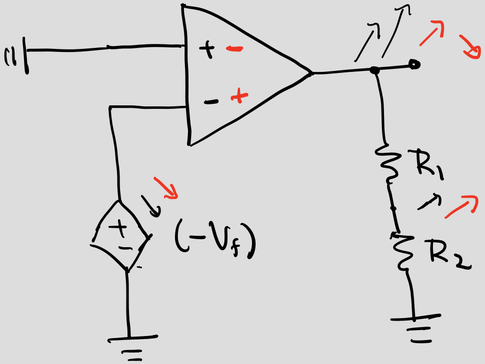
$$\boxed{\frac{V_{out}}{I_{in}} = -R}$$

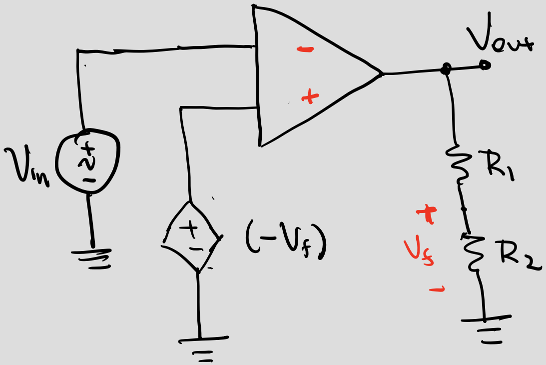
The input is current; output is Voltage: we use this model in the lab for photo sensors!

Example circuit 3 -



Check NFB:





Voltage Divider

$$V_f = \frac{R_2}{R_1+R_2} \cdot V_{out}$$

NFB (GR#2) $V^- = V^+$

$$V_{in} = -V_f$$

V^- V^+

$$V_{in} = -\frac{R_2}{R_1+R_2} V_{out} \Rightarrow \frac{V_{in}}{V_{out}} = -\frac{R_2}{R_1+R_2}$$

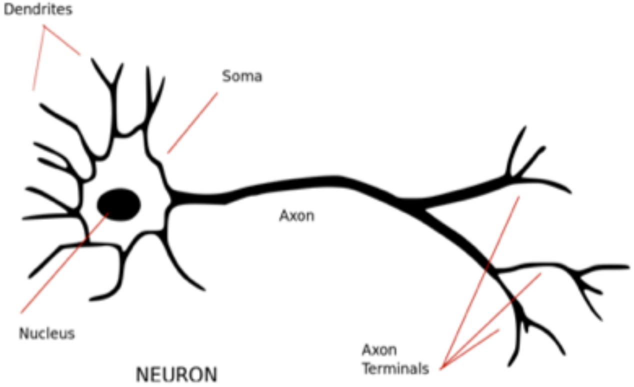
$$\boxed{A_V = \frac{V_{out}}{V_{in}}} = -\frac{R_1+R_2}{R_2} = \boxed{-\left(1+\frac{R_1}{R_2}\right)}$$

Artificial Neuron

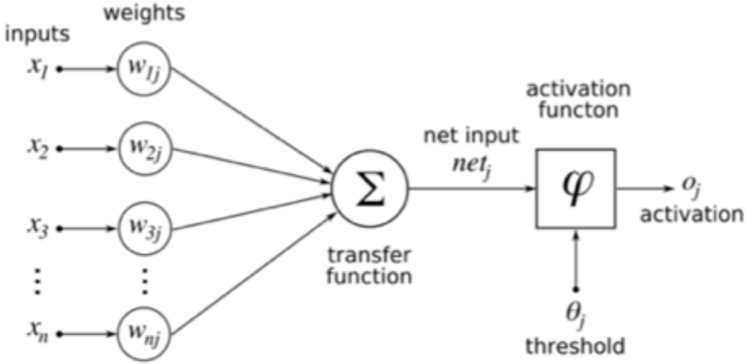
(Energy Efficient Neural Networks) — Yes we can!

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = a_1 V_1 + a_2 \cdot V_2$$



A biological Neuron



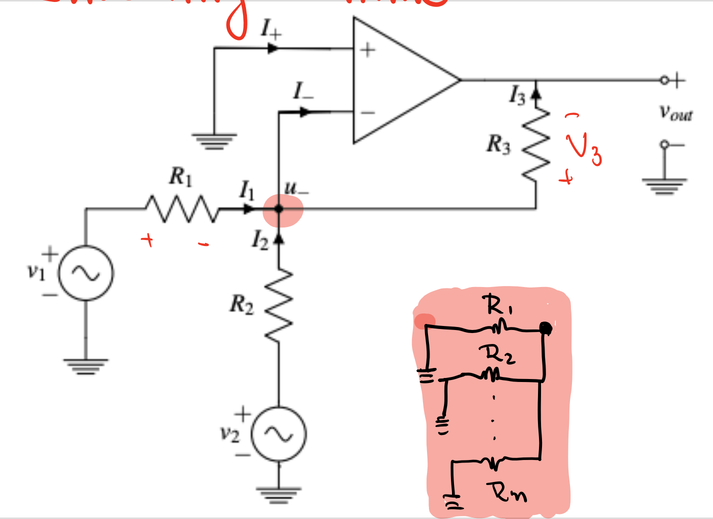
An Artificial Neuron

Artificial Neuron

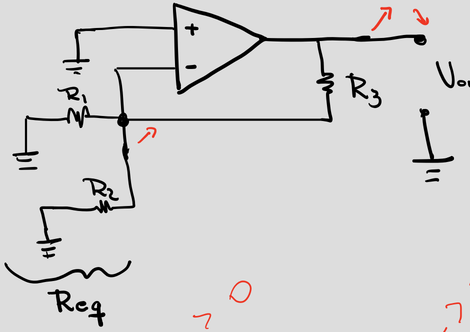
- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

$V_3 = V_{out} - V^-$

Inverting summer



Check for NFB:



$V^+ = V^-$: GR2

$V^+ = 0 \Rightarrow V^- = 0$

KCL: $\dfrac{\cancel{V^-}^{\,0} - V_1}{R_1} + \dfrac{\cancel{V^-}^{\,0} - V_2}{R_2} = \cancel{I^-}^{\,0} + \dfrac{V_{out} - \cancel{V^-}^{\,0}}{R_3}$

$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3}$$

$$\boxed{V_{out}} = \underbrace{-\frac{R_3}{R_1}}_{} \cdot V_1 + \left(\underbrace{-\frac{R_3}{R_2}}_{} V_2\right) + \cdots + \left(\underbrace{-\frac{R_3}{R_N}}_{} V_N\right)$$

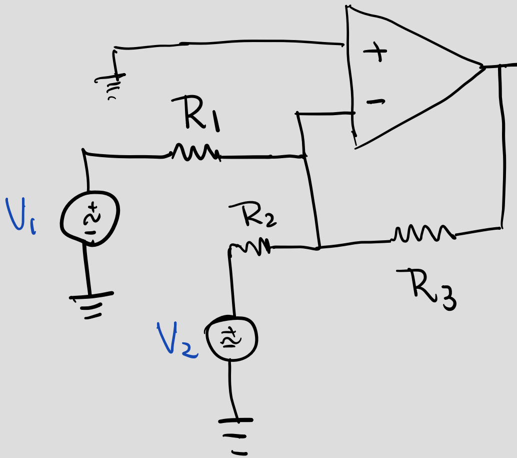
only negative coef.

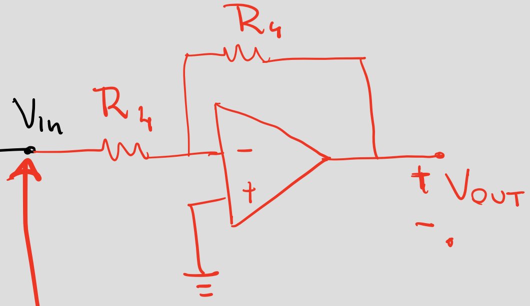
Weights

a_{11} v_1 a_{12} v_2 a_{1N} v_N

All weights are negative: How can we make a_1 and a_2 positive?

Add another inverting amplifier circuit.





$$V_{out_1} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} \cdot V_2$$

(V_{in})

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

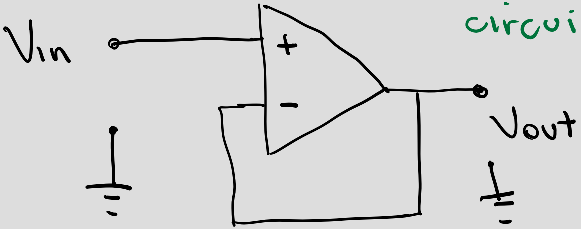
↳ result from Inverting amplifier

$$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$$

$V_{out} = -V_{in}$ (when R_1 and R_2 are the same)

Unity Gain Buffer

↳ Allows us to isolate circuits



$V^+ = V_{in}$

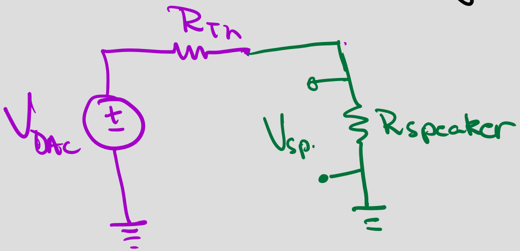
$V^- = V_{out}$

GR2

$V^+ = V^-$

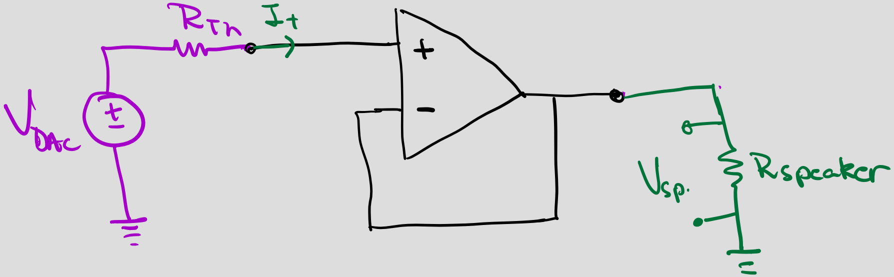
$\boxed{V_{in} = V_{OUT}}$

Speaker Design



$V_{speaker} = \frac{V_{DAC}}{126}$

loading



$I^+ = 0 \Rightarrow V^+ = V_{DAC}$

$V_{out} = V_{speaker} = V^- \Rightarrow V^+ = V^-$

$\boxed{V_{DAC} = V_{speaker}}$