EECS 16A   Designing Information Devices and Systems I

Fall 2021   Final

1. A Quirky Quantum Question

(a) In quantum mechanics, states of particles are represented by vectors in a vector space. In this problem, we'll say that all states exist in $\mathbb{R}^2$.

A particular matrix, $\hat{\mathbf{H}}$ (called the Hamiltonian operator), has the unique property that its eigenvalues represent a particle's allowed energy values. Quantum mechanics tells us that if the values of $\hat{\mathbf{H}}$ are real, it must be symmetric – that is, it can be written as

$$\hat{\mathbf{H}} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Assume that we know $a > 0$ and $b > 0$. What further condition on a and b forces the allowed energy values (the eigenvalues) to always be nonnegative?

(b) Miki experimentally determines that particles associated with the $\hat{\mathbf{H}}$ matrix from Question 1 have allowed energy values $\lambda_1 = \frac{5}{2}$ and $\lambda_2 = \frac{9}{2}$. Find a and b.

(c) Now, given a new matrix $\hat{\mathbf{H}} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, let the eigenvalues be $\lambda_1 < \lambda_2$ and the normalized eigenvectors be $\vec{v}_{\lambda_1}$ and $\vec{v}_{\lambda_2}$ (corresponding to eigenvalues λ_1 and λ_2, respectively, and scaled to a magnitude of 1), span $\mathbb{R}^2$. If a particle is in some state $\vec{v}_s \in \mathbb{R}^2$, then it can be expressed as $\vec{v}_s = \alpha \vec{v}_{\lambda_1} + \beta \vec{v}_{\lambda_2}$, where α and β are real constants.

If $v_s = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, what are possible magnitudes of α? (In quantum mechanics, α^2 represents the probability that measuring the particle's energy will yield λ_1.)

2. Inner Products

(a) For the following inner product defined on $\mathbb{R}^2$, which inner product properties hold?

$$< \vec{x}, \vec{y} > = \begin{bmatrix} 2 \\ 1 \end{bmatrix}^T (3\vec{x} + 3\vec{y})$$

    i. Symmetry (True/False)

    ii. Linearity (True/False)

    iii. Positive-Definiteness (True/False)

(b) Given the following valid inner product over the vector space of 2x2 real matrices $\mathbb{R}^{2x2}$, defined as
$$<A,B>=<\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}>= a_{11}b_{11}+a_{12}b_{12}+a_{21}b_{21}+a_{22}b_{22}$$ for any $A, B \in \mathbb{R}^{2x2}$. Given this inner product definition, what is $\left\| \begin{bmatrix} 2 & 5 \\ 6 & 2 \end{bmatrix} \right\|^2$ (or alternate exam values $\left\| \begin{bmatrix} 7 & 1 \\ 0 & 3 \end{bmatrix} \right\|^2$?

3. Least Squares with Shazam

(a) The application Shazam is able to detect what song is playing by means of an *acoustic footprint*. This is a small set of information that identifies the song. Shazam then checks that footprint in its database, to check for another song that has that footprint.

Here is the footprint we obtained via sampling: (we are representing the footprint as a vector)

$$\vec{x}_{sample} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \tag{1}$$

Say Shazam has narrowed it down to the following three songs with the corresponding footprints:

"Electric Love - Børns": $\vec{x_1} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

"She's Electric - Oasis": $\vec{x_2} = \begin{bmatrix} 2 \\ -2 \\ -8 \\ 7 \end{bmatrix}$

"Electric Feel - MGMT": $\vec{x_3} = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 2 \end{bmatrix}$

Shazam is going to determine which song it is by projecting the footprint of our sample onto each of the song candidates, and ranking the songs based on the normalized inner product of $\vec{x}_{sample}$ onto the footprints.
Based on this information, which song is playing?

(b) Shazam has a feature where it displays lyrics in time with the song that is playing - to do this, it needs to figure out where in the song we are. To do this, it takes the cross-correlation of a snippet of the song with the full song. Assume that the song is sampled at 30 samples a second. Here is the cross correlation:
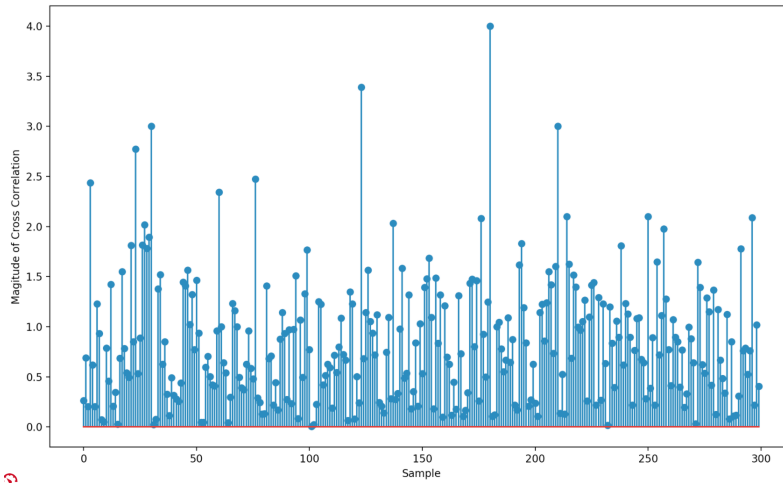
Make the best estimate as to when the sample was taken in seconds.

(c) Shazam wants to partner with Spotify. For its Discover Weekly algorithms, Spotify would like to know what characteristics of a song make it attractive to a first-time listener. Shazam provides the number of Shazams for a set of new songs to Spotify, who combines it with their data on the songs. Here is the table of data that Spotify assembles:

Shazam Popularity	Tempo	Danceability	Acoustic-ness
1109	100	0.8	0.6
5501	90	0.5	0.2
2031	68	0.4	0.7
13045	120	0.9	0.2

Spotify would now like to use the data it has to predict what the number of shazams for a new song, whose characteristics (Tempo, Danceability, Acoustic-ness) are represented as $\vec{a}_n$.

Which is the correct formula for how Spotify would use Least Squares to calculate this? Let M be the matrix of Tempo, Danceabilty and Acoustic-ness, and $\vec{b}_n$ be the number of shazams they get:

$$M = \begin{bmatrix} 100 & 90 & 68 & 120 \\ 0.8 & 0.5 & 0.4 & 0.9 \\ 0.6 & 0.2 & 0.7 & 0.2 \end{bmatrix}, \vec{b}_n = \begin{bmatrix} 1109 \\ 5501 \\ 2031 \\ 13045 \end{bmatrix}$$

Options:

   i. $(MM^T)^{-1}M\vec{b}_n$

  ii. $\vec{a}_n^T(MM^T)^{-1}M\vec{b}_n$

 iii. $\vec{a}_n^T(\vec{b}_n^T M^T)^T$

 iv. $\vec{b}_n^T M\vec{a}_n$

v. $\vec{b_n}^T M \vec{a_n}$

[0.1in]

(d) Say Spotify gets some new data to incorporate into its data set, the energy of the song. Here is the table with the added data:

Shazam Popularity	Tempo	Danceability	Acoustic-ness	Energy
1109	100	0.8	0.6	0.70
5501	90	0.5	0.2	0.35
2031	68	0.4	0.7	0.55
13045	120	0.9	0.2	0.55

Will it still be possible to run least squares with all of this data?

(e) What is the maximum number of features we could have per song, assuming we keep the number of songs the same?

Options:

   i. 4

   ii. 3

   iii. 2

   iv. 5

   v. As many as we want

4. **Towers of Sixteen**

The kingdom of Sixteen is easy to get lost in. Luckily, there are three beacons that send out audio signals to help lost travelers. You have a special device that records one sample every second and plots the cross correlation of the known signals emitted from each of these beacons with the signal it has received. After you begin recording the signals, you see the following three plots, associated with the three beacons named Gaussian, NVA, and Charge:
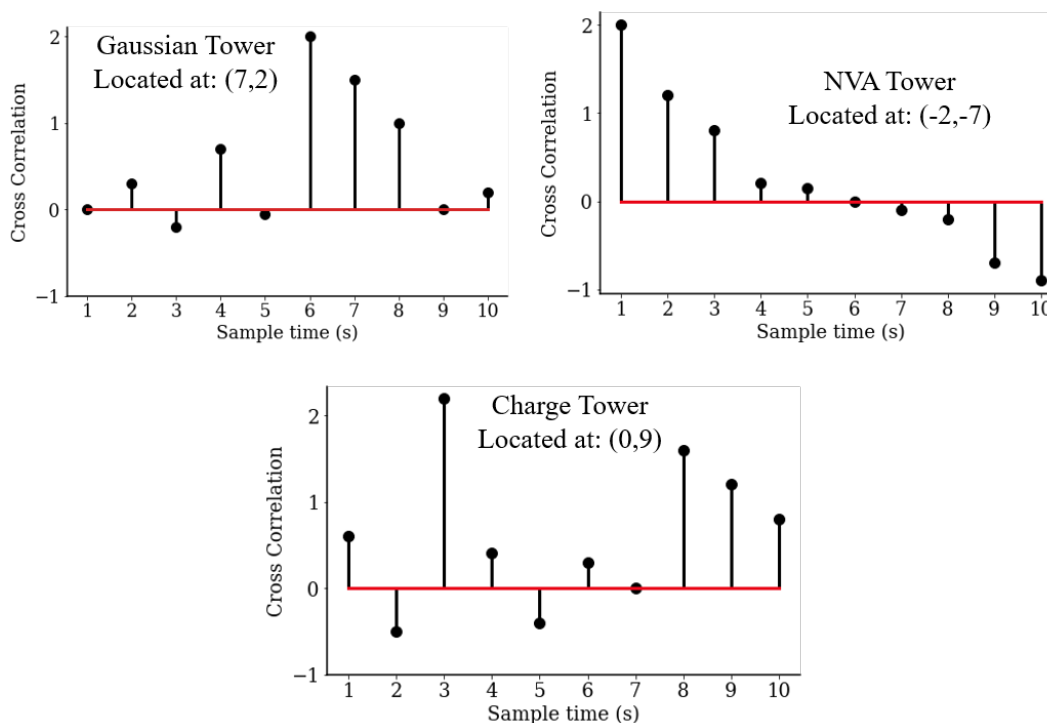
(a) How far away are you from each of the beacons? Provide a distance in meters. Hint: Approximate the speed of sound to 300 meters per second and consider what information the cross-correlation plots can provide.

   i. Distance from the Gaussian Tower:[ ]meters
   ii. Distance from the NVA Tower:[ ]meters
   iii. Distance from the Charge Tower:[ ]meters

You write a system of linear equations based on the data you have collected to help you find your location in the 2D space of the kingdom. To solve this system, you decide to use Gaussian Elimination and set up a matrix-vector equation in the form: $\mathbf{A}\vec{x} = \vec{b}$.

Which matrix or vector corresponds the most closely to the distances you found above correspond with? Answer with A, $\vec{x}$, or $\vec{b}$

(b) **Version 1:** You continue to explore another area of the kingdom and now want to share your location with your friends so they can also join you in your travels! Regardless of what you got from the previous part, assume that you have already calculated the new distances between yourself and each of the beacons:

Distance from the Gaussian Tower: $\sqrt{18}$ meters

Distance from the NVA Tower: LaTeX: $\sqrt{72}$ meters

Distance from the Charge Tower: $\sqrt{116}$ meters

Then, your location is at ([x], [y]).

Version 2: You continue to explore another area of the kingdom and now want to share your location with your friends so they can also join you in your travels! Regardless of what you got from the previous part, assume that you have already calculated the new distances between yourself and each of the beacons:

Distance from the Gaussian Tower: $\sqrt{101}$ meters

Distance from the NVA Tower: LaTeX: $\sqrt{65}$ meters

Distance from the Charge Tower: $\sqrt{73}$ meters

Then, your location is at ([x, y]).
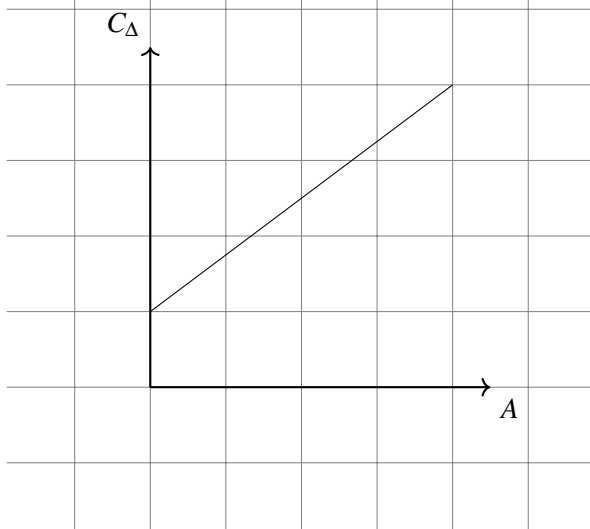
5. **Capacitive Touch Pixel**

In lab, we worked on a capacitive touch pixel that can detect whether a touch is present or not. Your TA Raghav wants to develop a capacitive touch pixel that can differentiate between no touch, weak touch, and strong touch. But he needs your help implementing this design.

Just like lab, we model the surface of a finger as a parallel plate. Thus, our finger forms a set of parallel plate capacitors, with total capacitance C_Δ. The difference between a weak touch and a strong touch is the area of the finger surface (stronger touch $\rightarrow$ greater area).

(a) (1 point) Which of the following curves depicts the correct relationship between C_Δ and the area of the finger surface A?
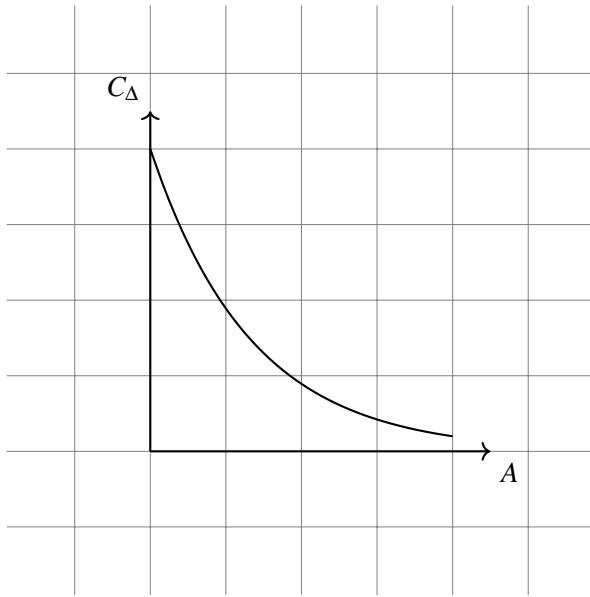

i.

ii.

C_Δ

A

iii.

C_Δ

A
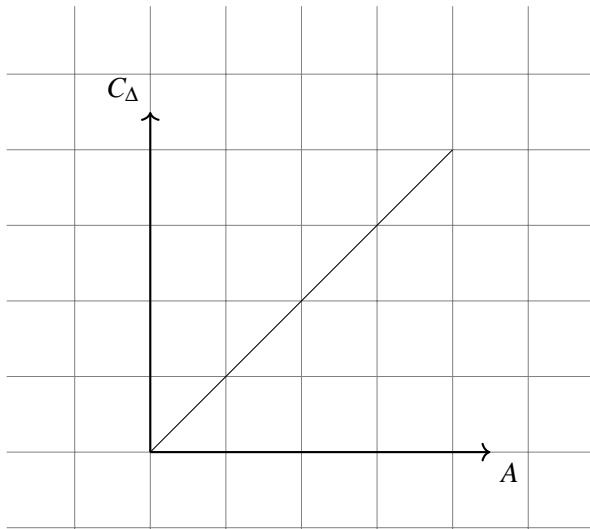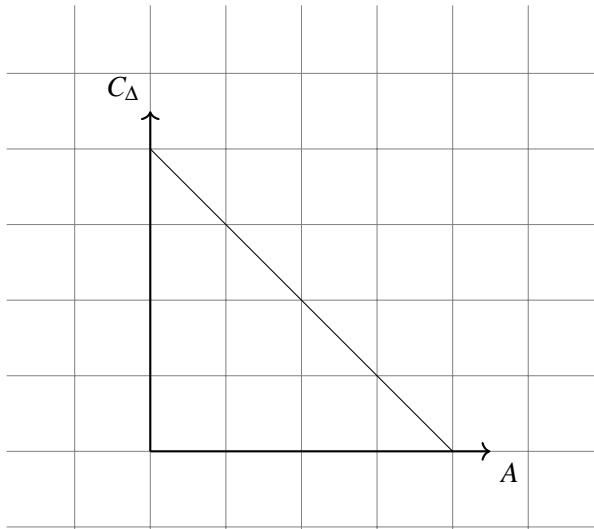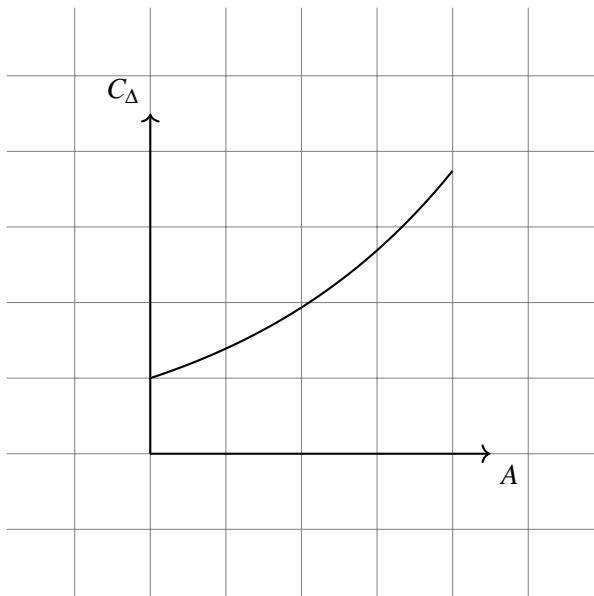
iv.

v.

(b) (1 point) Let's say you're given $C_{\Delta,\text{weak touch}} = 5$ nF (3.5 nF for alternate version) and $A_{\text{strong touch}} = 1.4 \cdot A_{\text{weak touch}}$ ($3 \cdot A_{\text{weak touch}}$ for alternate version). What is $C_{\Delta,\text{strong touch}}$ in nF?

(c) (4 points) Let C_{pixel} be the capacitance at the touch pixel. Now we know that $C_{pixel} = C_0 + C_\Delta$ where C_0 is the capacitance of the pixel itself and C_Δ is the capacitance that comes from the finger. These capacitances will influence the node voltage V_+ after charge-sharing.

Raghav solves the charge-sharing problem for you and gives you the following information for the subsequent problem:

$V_{+,\text{no touch}} < V_{ref,1} < V_{+,\text{weak touch}} < V_{ref,2} < V_{+,\text{strong touch}}$

Choose the right voltages for the following nodes such that the given circuit takes V_+ as input and lights up LED L_1 when there is a weak touch and both LEDs L_1 and L_2 when there is a strong touch.

Assume the positive supply rail of the comparator is set at a large V_{DD} and the negative supply rail is set to ground ($0V$).
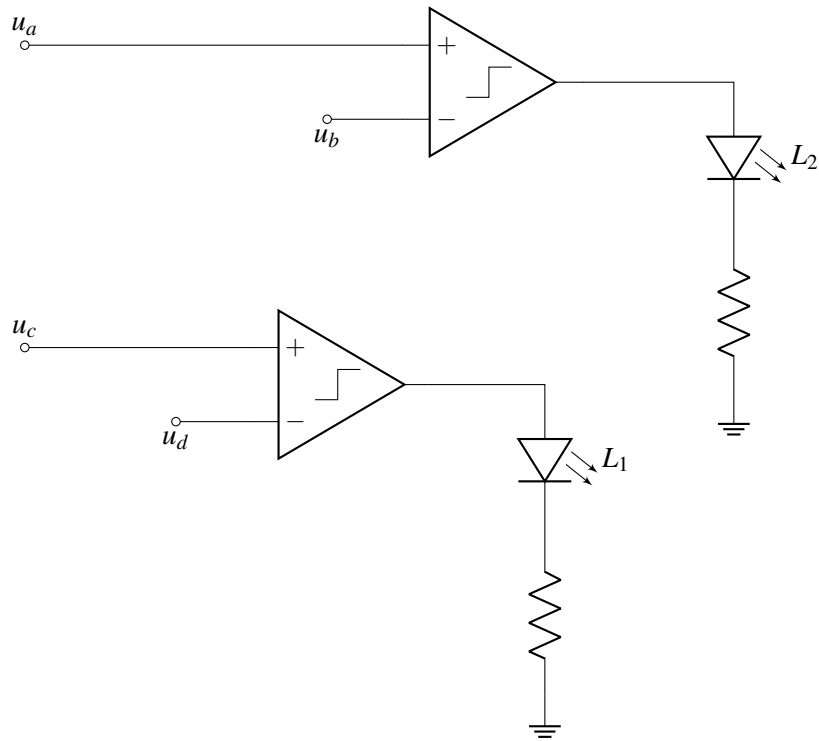
Find:

    i. u_a

    ii. u_b

    iii. u_c

    iv. u_d

Potential options:

    i. V_+

    ii. V_{DD}

    iii. $V_{ref,1}$

    iv. $0V$

    v. $V_{ref,2}$

   vi. $\frac{V_{DD}}{2}$

6. **Falcon Incubation [8 points]**

The Campanile, old as it is, needs to be cleaned sometimes. Unfortunately, this means our resident falcons will be displaced briefly. Being a conservation minded Berkeley Engineer, you decide to help them out by designing an artificial incubator. It has to hold the temperature at $37°$ C, and must turn on only once 3 eggs have been laid.

You decide to start by implementing a circuit that detects the number of eggs laid. For this, you use a resistive plate placed under the entire nest as shown in the following diagram:
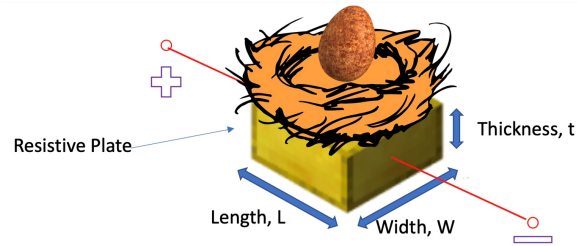
We know that the plate resistance increases as more eggs are laid, essentially working as a variable resistor.

(a) **(1 point)** Your TA, Aniruddh, provides you with this circuit diagram which is designed to output 5V when 3 or more eggs have been lain, and 0 V otherwise.
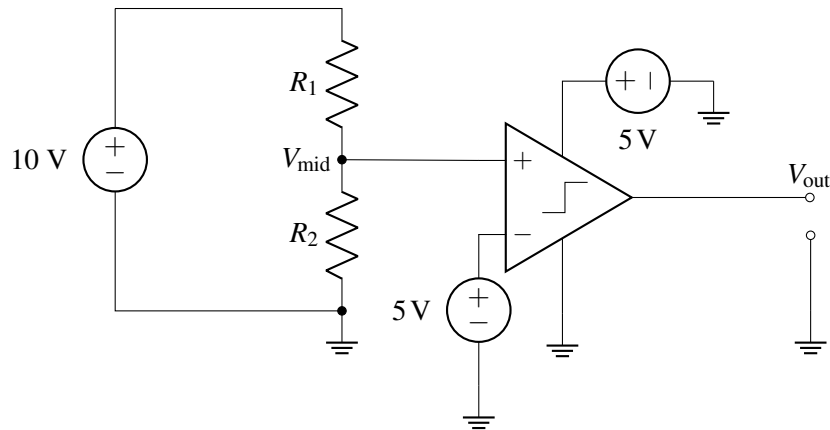
If you have a single fixed resistor, R_{fixed}, and the variable resistance plate under the nest has a range such that $0 < R_{var} < 2R_{fixed}$, where should you put the resistive plate so that the circuit behaves as expected (with the fixed resistor in the other resistor location)?

  i. The resistive plate must be placed where R_1 is.
 ii. The resistive plate must be placed where R_2 is.
iii. The resistive plate can be placed in either location.

(b) **(1 point)** Regardless of your previous answers, assume we use a fixed resistor for $R_2 = 150\Omega$ (or 100Ω for the alternate question). What is the maximum value of the other resistor such that the circuit outputs +5V (or operates on the threshold of the output voltage switch)?

(c) **(4 points)** You now want to work on the temperature control of the incubation unit. You are provided with a *thermistor*, a temperature dependent resistor, which follows the following resistance (kΩ) vs. temperature (°C) relationship:

$$R(T) = 2.5 + \frac{1}{2}(T - 37) \qquad [k\Omega]$$

For the alternate question:

$$R(T) = 2 + \frac{1}{2}(T - 37) \qquad [k\Omega]$$

The thermistor can thus be used as a measurement of the current temperature. At 37°C, you want to output 0V to the temp controller–in other words, the below circuit should output 0V at this temperature.
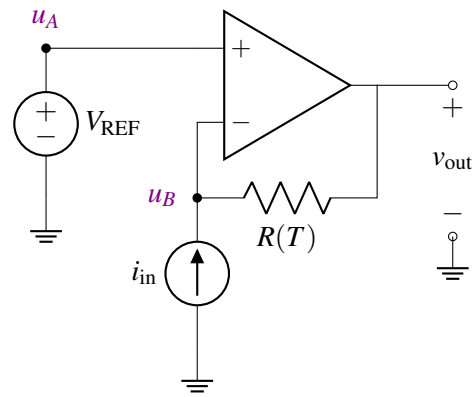
If i_{in} = 4 mA (2 mA for the alternate question), what should V_{REF} be set to, in Volts (V)?

(d) **(2 points)** You are given the following op-amp pin diagram. Match the nodes from the previous circuit diagram (shown below for your convenience) to their corresponding pin numbers on the op-amp.
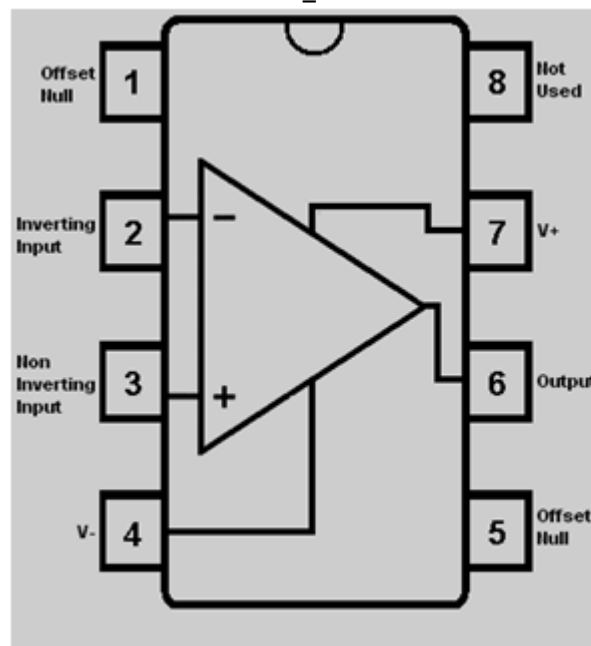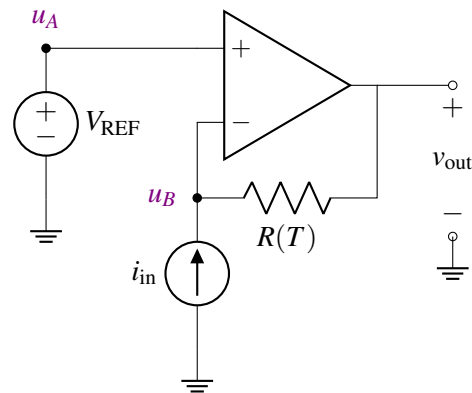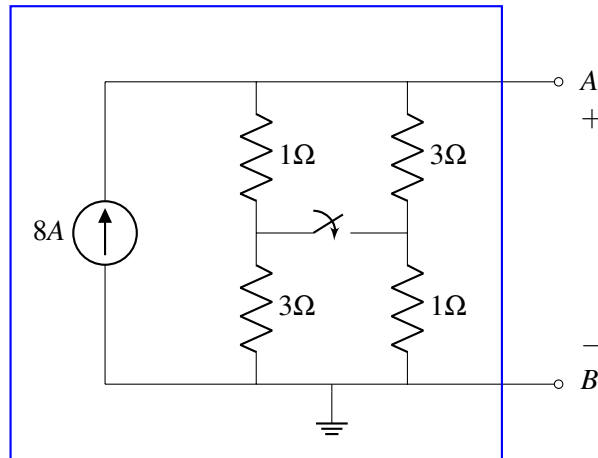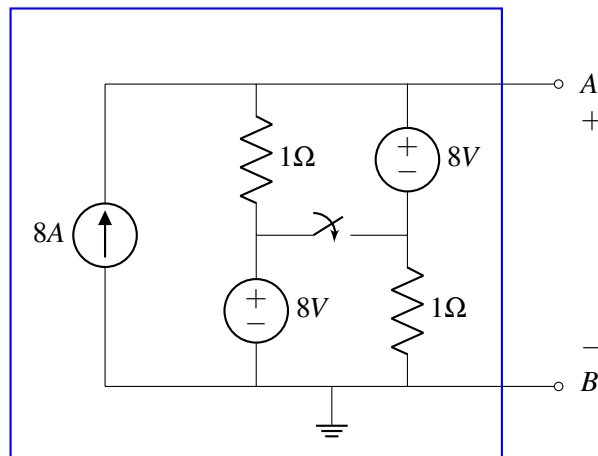

- u_B
- u_A

- 5V Supply
- -5V Supply
- V_{out}

7. A Thevenin Paradox

(a) Calculate the Thevenin equivalent voltage across terminals A and B first when the switch is open and again when the switch is closed.
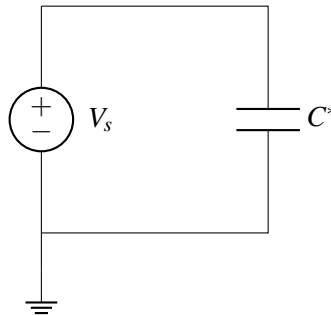

(b) In this new circuit, what is the Thevenin equivalent voltage across terminals A and B when the switch is open and when the switch is closed?

8. Charge Sharing Choices

(a) C^* is attached to voltage source V_s as shown below and allowed to reach steady state (assume there is zero charge on C^* before it is attached to the voltage source).
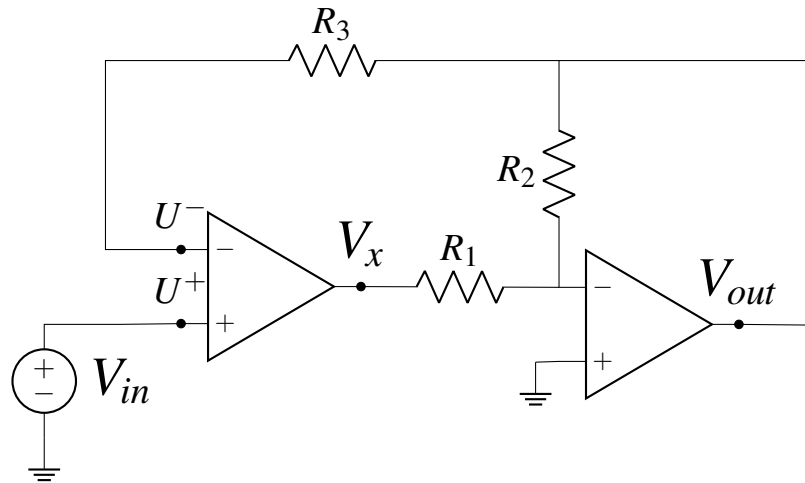
Next, C^* is attached to the circuit shown below. Initially, all the switches are open and the voltage across C^* is still V_s. Which set of switches (ϕ_1, ϕ_2, or ϕ_3), when closed, will cause the potential at node U_x to equal $\frac{1}{2}V_s$ once steady state is reached? Assume that $C^* = C$.
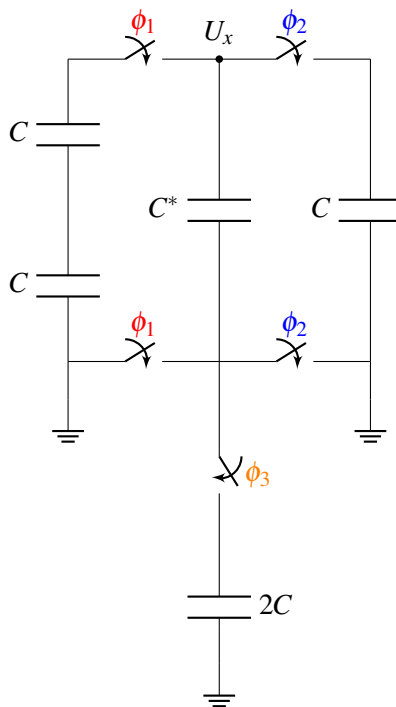
9. OpAmps1: Checking for feedback type (6 points)

Version 1:

We are trying to determine if the circuit in the above schematics is in negative or positive feedback. In order to do that, we zero out all independent sources and wiggle the output V_{out} by increasing it. How would the following node voltages change? Note that $V_{error} = U^+ - U^-$

U^-=Increases/Decreases/Stays the same?

U^+=Increases/Decreases/Stays the same?

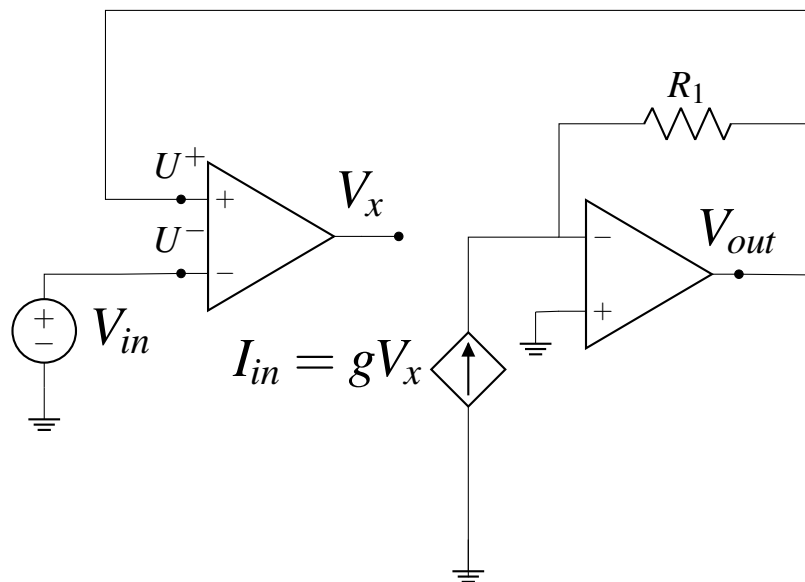V_{error}=Increases/Decreases/Stays the same?

V_x=Increases/Decreases/Stays the same?

V_{out}=Increases/Decreases/Stays the same?

Based on the change in node voltages, the circuit is in negative/positive feedback.

Version 2:

We are trying to check if the circuit in the above schematics is in negative or positive feedback. In order to do that, we zero out all independent sources and wiggle the output V_{out} by decreasing it. How would the

following node voltages and currents change? Note that $V_{error} = U^+ - U^-$ and $I_{in} = gV_x$ where g is a given constant and is equal to $g = 5\text{A/V} = 5/\Omega$

U^-=Increases/Decreases/Stays the same?

U^+=Increases/Decreases/Stays the same?
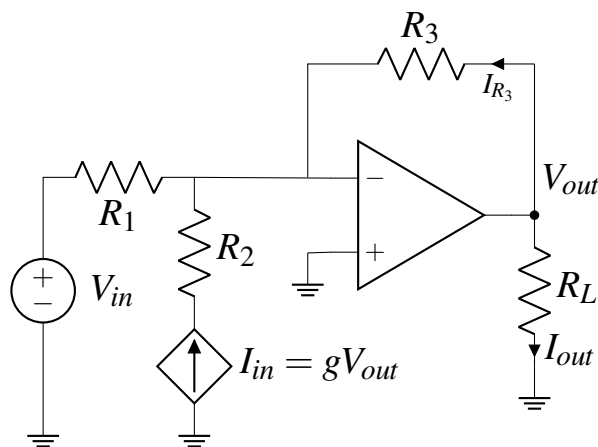
V_{error}=Increases/Decreases/Stays the same?

V_x=Increases/Decreases/Stays the same?

I_{in}=Increases/Decreases/Stays the same?

V_{out}=Increases/Decreases/Stays the same?

Based on the change in node voltages and currents, the circuit is in negative/positive feedback.

10. **Op Amps 2**

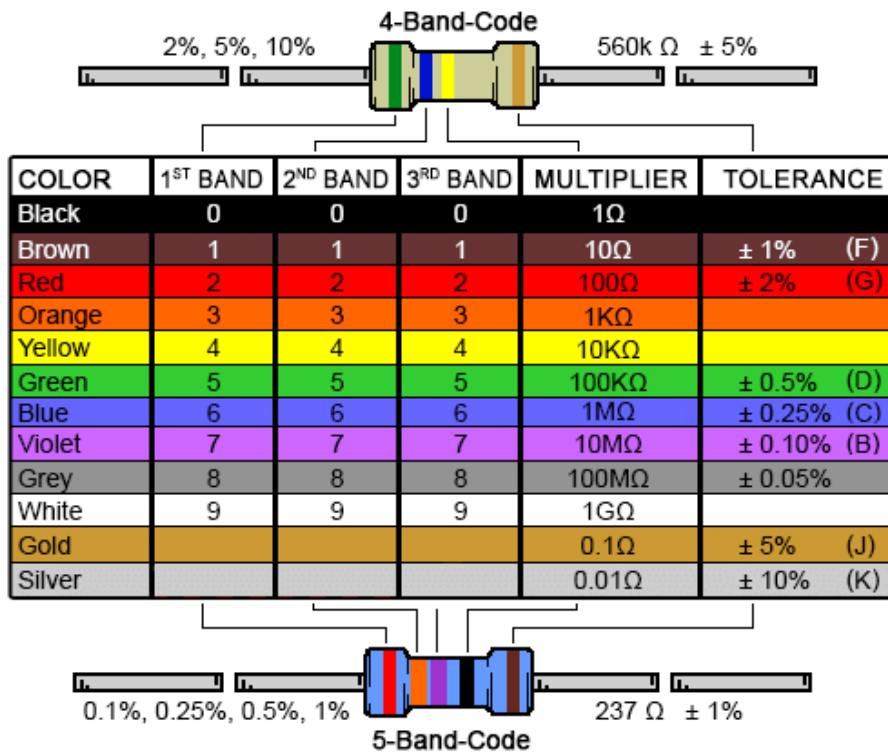

Based on the above schematics and for $V_{in} = -4\text{V}$, $I_{in} = gV_{out}$, $R_1 = 1\Omega$, $R_2 = 10\text{k}\Omega$, $R_3 = 1\Omega$ and $R_L = 1\text{k}\Omega$, determine if the circuit is in negative feedback, an expression for I_{R_3}, and the value of I_{out}. Note that g is the transconductance is equal to $g = 1\text{A/V} = 1/\Omega$.

(a) This circuit is negative feedback. True/False?

(b) What is the correct expression for I_{R_3}?

(c) What is I_{out} (mA)?

11. **Resistor Band**

(a) Suppose you need a 51Ω resistor your your circuit with a tolerance of $\pm 5\%$. Referring to the diagram above, which four-band color code would this correspond to?

4-Band-Code

2%, 5%, 10%   560k Ω ± 5%

COLOR	1ST BAND	2ND BAND	3RD BAND	MULTIPLIER	TOLERANCE	
Black	0	0	0	1Ω		
Brown	1	1	1	10Ω	± 1%	(F)
Red	2	2	2	100Ω	± 2%	(G)
Orange	3	3	3	1KΩ		
Yellow	4	4	4	10KΩ		
Green	5	5	5	100KΩ	± 0.5%	(D)
Blue	6	6	6	1MΩ	± 0.25%	(C)
Violet	7	7	7	10MΩ	± 0.10%	(B)
Grey	8	8	8	100MΩ	± 0.05%	
White	9	9	9	1GΩ		
Gold				0.1Ω	± 5%	(J)
Silver				0.01Ω	± 10%	(K)

0.1%, 0.25%, 0.5%, 1%   237 Ω ± 1%

5-Band-Code

12. Orthogonality

(a) $f(x)$ and $g(x)$ are both polynomials with degree at most 2. $f(x) = x^2 - 2x + 1$ ($f(x) = -x^2 + 2x - 1$ for the alternate version). We define inner product between two polynomials as $\langle f(x), g(x) \rangle = f(0)g(0) + f(1)g(1) + f(2)g(2)$.
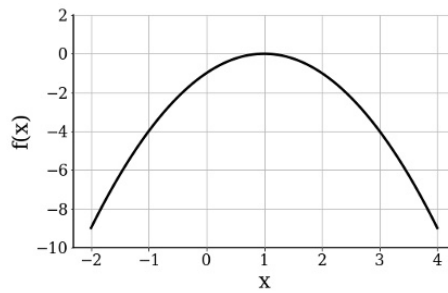
If $g(x)$ is <u>orthogonal</u> to $f(x)$, which is possible equation for $g(x)$?
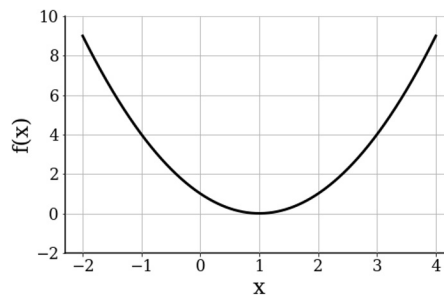
i. $-x^2 + 2x - 1$

ii. $x^2 + x - 1$

iii. $x - 1$

iv. x

Alternate version:

    i. $2x^2 - 5x - 2$

    ii. $5x + 2$

    iii. $-x^2 + 2x - 1$

    iv. $-x + 1$
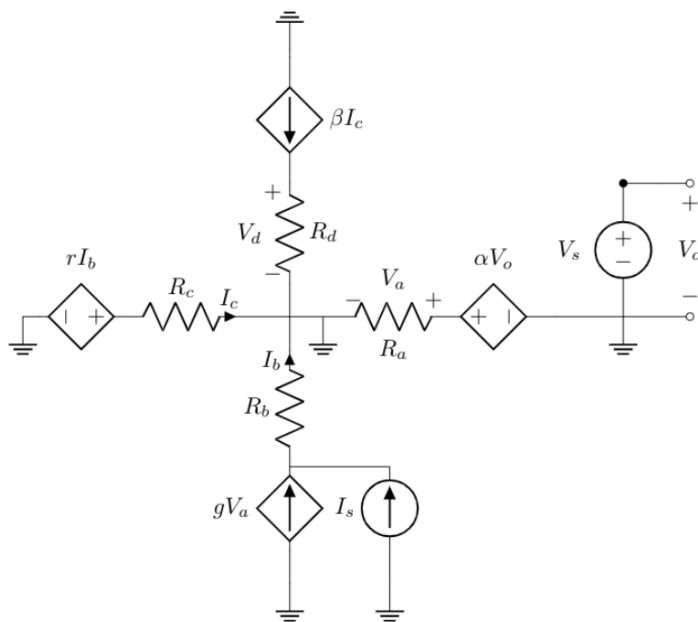
13. Satellite Codes

(a) Which of the following vectors is best suited as a satellite code?
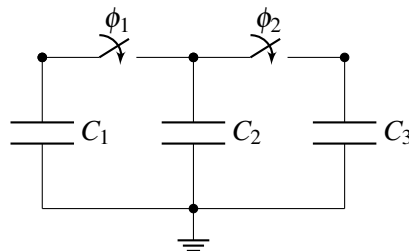
$$\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

14. Superposition Fun

(a) Suppose we want to solve this circuit using superposition. If we want to start solving by leaving V_s on, how many other sources do we need to null out?

15. Charge Sharing Cycles

The circuit begins in phase 1 (all switches open). During phase 1, the ϕ_1 switch is closed and the ϕ_2 switch is open. During phase 2, the ϕ_2 switch is closed and the ϕ_1 switch is open. The circuit moves from phase 0 to phase 1 to phase 2 and then back to phase 0 (defined as a cycle). Let t indicate the number of cycles completed and $\vec{x}[t] = \begin{bmatrix} Q_{C_1}^{(t)} & Q_{C_2}^{(t)} & Q_{C_3}^{(t)} \end{bmatrix}^T$ indicate the distribution of charge across the three capacitors after t cycles have occurred. Let $\vec{x}[0] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$. You are given $C_1 = C_2 = C_3$.

(a) Find $\vec{x}[1]$, the distribution of charges after phase 1 and phase 2 have been completed once.

(b) Find a matrix T such that $T\vec{x}[t] = \vec{x}[t+1]$.

(c) Find $\lim_{t\to\infty} \vec{x}[t]$.

16. Linear Algebra Circuit (22 points)

(a) Consider the following circuit (4 points):

u_1

I_s V_s R u_2 R
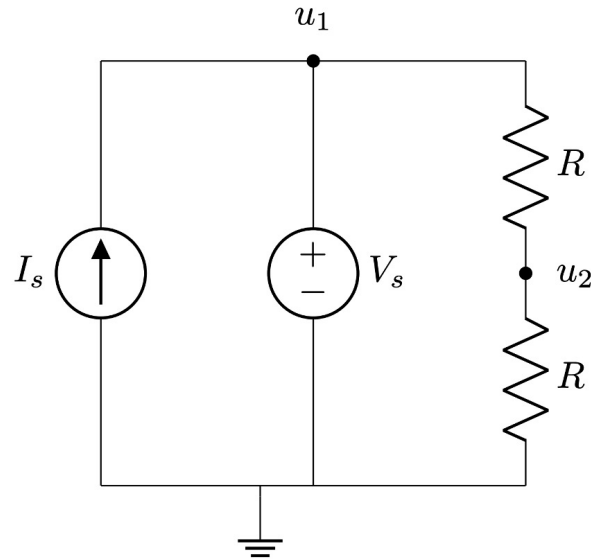
Let $\vec{x} = [V_s \ I_s]^T$, and let $\vec{u} = [u_1 \ u_2]^T$. Construct A such that $A\vec{x} = \vec{u}$, choose the most correct statement: For a given $\vec{u} = \vec{u}_0$, the matrix equation $A\vec{x} = \vec{u}_0$:

Options:

1) can have a single solution or no solution;

2) always has a single solution;

3) always has no solution;

4) always has infinite solutions;

5) can have infinite solutions or no solutions;

(b) Consider the following circuit (2 points):

u_1

V_{s1} V_{s2} R u_2 R
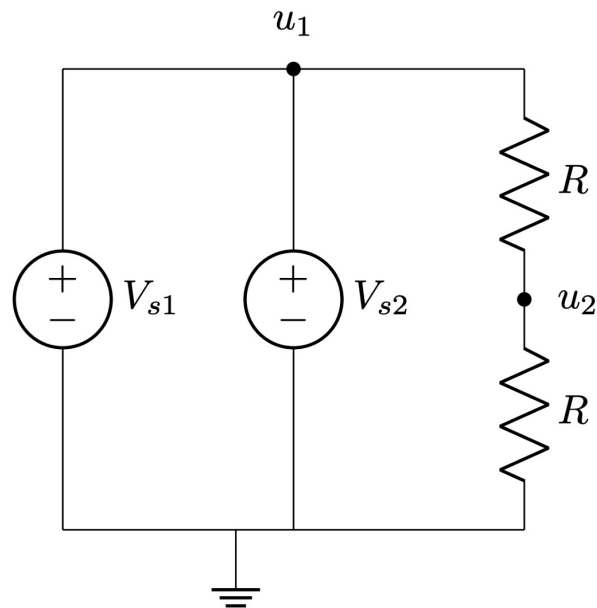
Let $\vec{x} = [V_{s1} \ V_{s2}]^T$, and let $\vec{u} = [u_1 \ u_2]^T$. Construct A such that $A\vec{x} = \vec{u}$. For a given $\vec{u} = \vec{u}_0$, the matrix equation $A\vec{x} = \vec{u}_0$ has a solution for the vector $\vec{x}$, then the solution is in the span of $[a \ 0.5]$, where a = ?.

(c) Consider the following circuit (6 points):
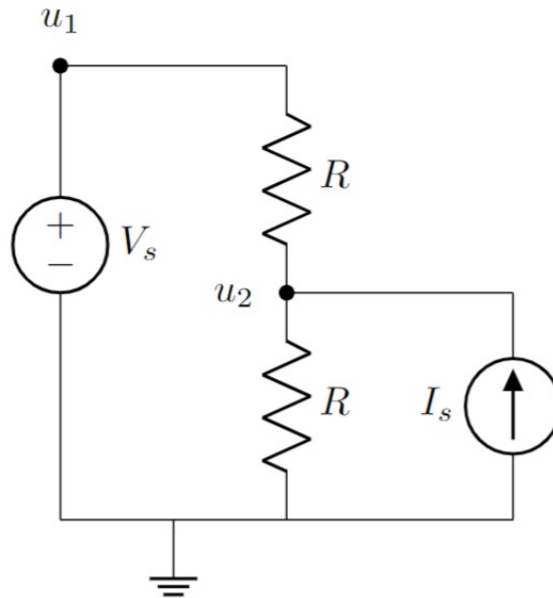
Let $\vec{x} = [V_s \ I_s]^T$, and let $\vec{u} = [u_1 \ u_2]^T$. Let $\vec{u} = \vec{u}_0$ and construct A such that $A\vec{x} = \vec{u}_0$. Choose the correct statements about:

1) A:

    i. Matrix A is invertible;

    ii. Matrix A is not invertible;
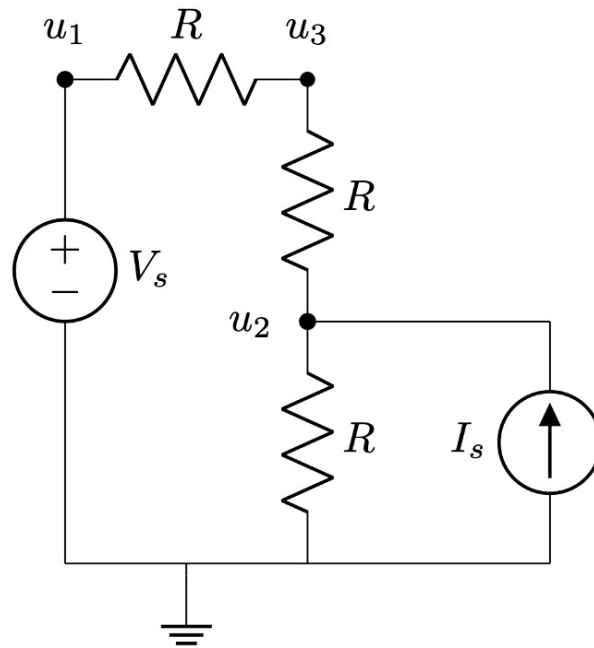
    iii. Not possible to determine if A is invertible.

2) u_1:

    i. u_1 depends only on V_s;

    ii. u_1 depends only on I_s;

    iii. Neither of the other choices;

3) u_2:

    i. u_2 depends only on V_s;

    ii. u_2 depends only on I_s;

    iii. Neither of the other choices;

(d) Consider the following circuit (10 points):

Let $\vec{x} = [V_s \; I_s]^T$, and let $\vec{u} = [u_1 \; u_2 \; u_3]^T$. Assume that all measurements of the node voltages are noisy with some small error. Given $R = 1\Omega$, $\vec{u} = \vec{u}_0 = [3 \; 2 \; 2]^T$. We can construct $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ such that

$A\vec{x} = \vec{u}_0$.

What is the best approximation for V_s and I_s in volts and amperes, respectively?