EECS 16A Designing Information Devices and Systems I
Spring 2019 Midterm 2

1. How is your semester so far? (1 point)

2. Do you have any summer plans? (1 point)

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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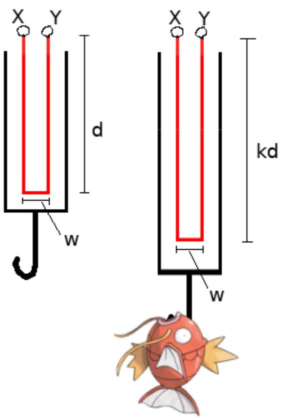
3. Fisherman Friend (10 points)



My fisherman friend wants to make an automatic fishing rod. First, he wants to create a circuit that can sense when a fish is biting. He has already made a special stretchy fishing hook, shown on the left.

The entire apparatus is stretchy, and the interior string is a resistor with terminals X and Y and resistivity ρ. Without a fish on it, the string has cross-sectional area A and length $2d$ (You can assume that the width w is negligible, or $w \ll d$).

When a fish bites, the length of the string stretches by a factor of $k > 1$, but the volume of the string remains constant. The resistivity ρ also remains constant.

My fisherman friend wants to know what kind of circuit he should attach to X and Y. Please help him.

(a) (4 points) If the resistor has resistance R without a fish on it, find the resistance when a fish bites.

Solution: First, volume is given by $V = 2dA$, so if $d \mapsto kd$, then $A \mapsto A/k$. Resistance is given by $R = \rho 2d/A$. So after a bite, $R' = \rho 2k^2 d/A = k^2 R$.

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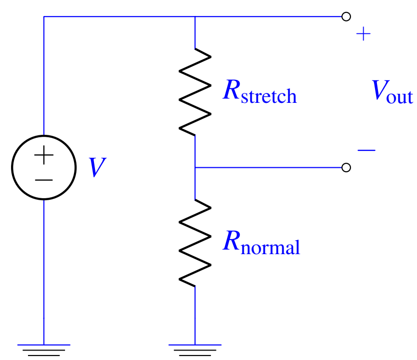
(b) (6 points) My fisherman friend has a voltmeter. He also has a voltage source and an extra non-stretchy resistor. He has an idea of what kind of circuit to use, which is shown below, where the two nodes across which he measures the voltage is labeled by V_{out}. But he's not sure which of the two resistors should be stretchy. Can you label the normal resistor by R_{normal} and the stretchy resistor by R_{stretch} such that V_{out} **increases** when a fish bites? Derive an expression for V_{out} to justify your answer.

Solution:



The voltage measured is $V_{out} = \frac{R_{\text{stretch}}}{R_{\text{stretch}}+R_{\text{normal}}}V$, so the voltage changes to $V_{out} = \frac{k^2 R_{\text{stretch}}}{k^2 R_{\text{stretch}}+R_{\text{normal}}}V$ when a fish bites. This change is an increase because $\frac{k^2 R_{\text{stretch}}}{k^2 R_{\text{stretch}}+R_{\text{normal}}}$ gets closer to 1.

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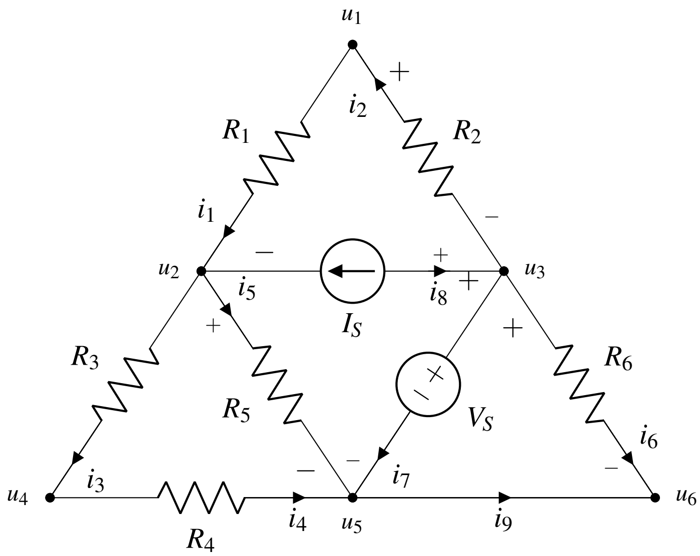
4. It's a Triforce! (10 points)



(a) (3 points) Which of the elements listed below have current-voltage labeling that violates Passive Sign Convention? Fill in the circle on the left of all the correct answer(s).

◯ I_s ◯ V_s ◯ R_2 ◯ R_5 ◯ R_6

Solution: The correct answers are I_S and R_2, since the current directions disagree with the signs. Swap the signs for both components to fix them.

(b) (3 points) There is a subset of nodes in the given circuit that are redundant. Fill in the circles on the left of all the nodes that can be merged into a single node.

◯ u_1 ◯ u_2 ◯ u_4 ◯ u_5 ◯ u_6

Solution: The correct answer is u_5 and u_6 since there is a short connecting them.

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(c) (4 points) Write the KCL equation for node u_2 in terms of the node potentials and other circuit elements.

Solution:

KCL at u_2 gives us:

$$-i_1 + i_8 + i_3 + i_5 = 0$$

Note that in this solution, we have assumed that the current i_8 is flowing out of node u_2. You could have also assumed the current flows into the node if you corrected it from the previous part. So long as you were consistent with your polarity and current direction, you should have gotten the correct answer.

The equations for the current through each of the branches are:

$$i_1 = \frac{u_1 - u_2}{R_1}$$
$$i_3 = \frac{u_2 - u_4}{R_3}$$
$$i_8 = -I_S$$
$$i_5 = \frac{u_2 - u_5}{R_5}$$

The final expression is then:

$$-\frac{u_1 - u_2}{R_1} - I_S + \frac{u_2 - u_5}{R_5} + \frac{u_2 - u_4}{R_3} = 0$$

Or any equivalent equation.

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5. You Can't Just Beam Yourself Up (20 points)

You got stranded on Bohemian Junkheap while on your way to planet Risa for your spring break vacation. Now you need to scavenge for equipment parts to contact the rest of your friends on Risa.

You found a signal encoder, which can be modeled as a 2-terminal network. Then you programmed it to send out the following bits as a train of voltage pulses: $\{1\ 1\ 0\ 1\ 0\}$. You did the following measurements to test its output, as shown in Figures (5.1) and (5.2):

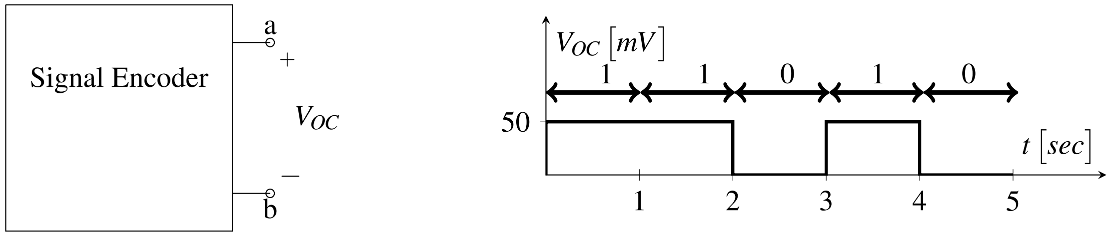


Figure 5.1: Measurement in open-circuit condition

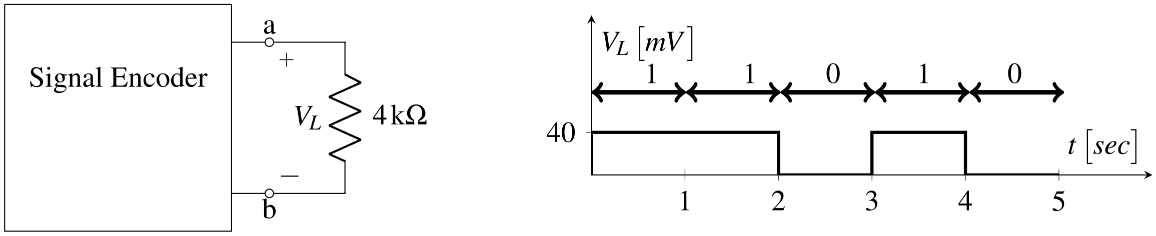


Figure 5.2: Measurement with resistive load

(a) (10 points) Find the Thévenin equivalent circuit of the signal encoder, i.e. **plot V_{TH} below as a function of time and find the value of R_{TH}.**

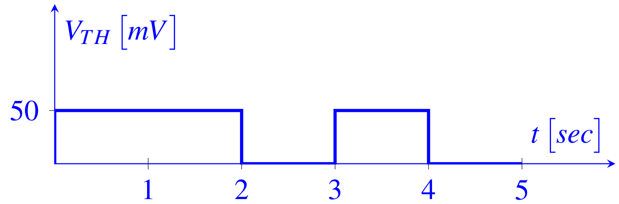
Solution: The open circuit voltage, V_{OC} is equal to the Thévenin voltage of the signal encoder. Hence,

$$V_L = \frac{4k}{4k + R_{th}} V_{OC}. \tag{1}$$

For $V_{OC} = 50mV$, $V_L = 40mV$. So

$$40m = \frac{4k}{4k + R_{th}} \times 50m \tag{2}$$

$$R_{TH} = 1\,\text{k}\Omega \tag{3}$$



(b) (10 points) Since the signal encoder output is not very strong, you need an amplifier to boost the output voltage. Luckily your friend Urmita arrives through the Astral Gate with an amplifier, which can be modeled and connected as shown in Figure 5.3.

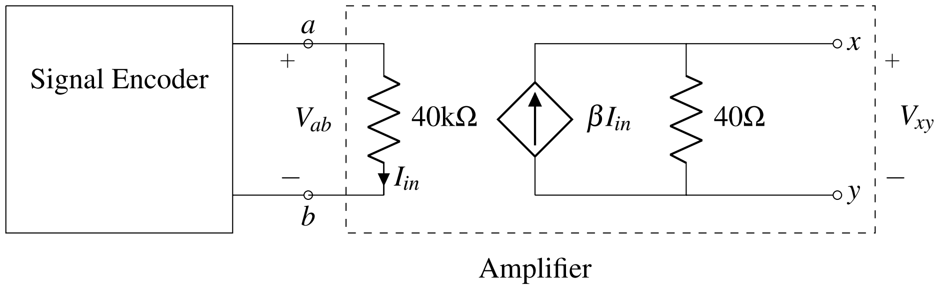


Figure 5.3: Encoder connected to an amplifier

You need to adjust the value of β for the dependent source, so that the amplifier has a gain of 100, i.e. $V_{xy} = 100V_{ab}$. **Find the value of β.**

Solution: The gain of the amplifier can be found by:

$$
\begin{aligned}
I_{in} =& \frac{V_{ab}}{40\text{k}\Omega} \\
V_{xy} =& \beta I_{in} \times 40\Omega \\
=& \beta \frac{V_{ab}}{40\text{k}\Omega} \times 40\Omega \\
=& 0.001\beta V_{ab}
\end{aligned}
$$

Hence the gain is

$$
\begin{aligned}
\frac{V_{xy}}{V_{ab}} =& 0.001\beta \\
\implies \beta =& \frac{100}{0.001} \\
\implies \beta =& 100,000.
\end{aligned}
$$

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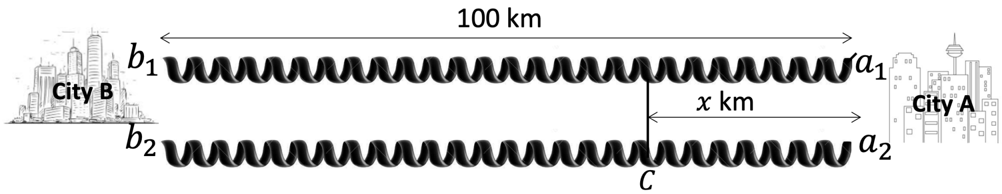
6. Telephone cable (15 points)

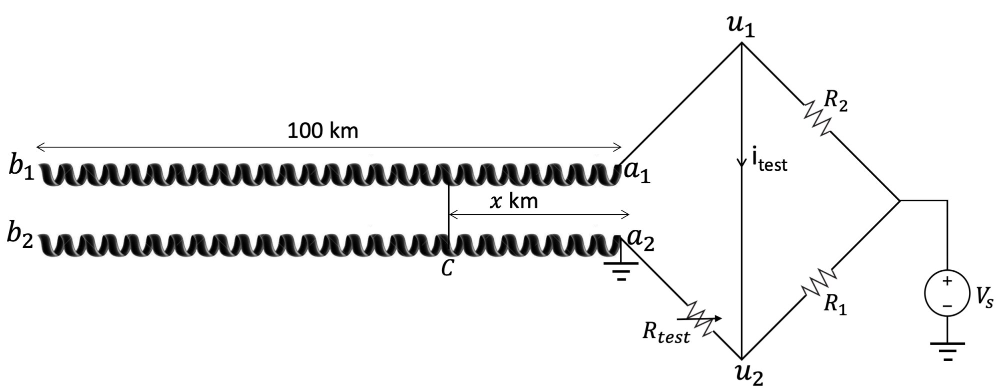


The telephone cable between city **A** and city **B** is broken at an unknown location **C**, which means the broken part of the cable is shorted at the location **C**. City **A** and city **B** are 100 km apart. The telephone cable resistance is $6\,\Omega/\text{km}$ for a single cable. Let's help the technician to find out where the location **C** is.

(a) (5 points) To build a model of the telephone wire, we can regard the telephone cable from a_2 to **C** as a single resistor R_{a2C}. The distance between a_2 and **C** is x km. What's the resistance of R_{a2C}?

Solution: $R_{a2C} = 6x\ \Omega$

(b) (10 points) To figure out where the location **C** is, the technician in city **A** attaches a testing circuit between a_1 and a_2 shown in the figure below, where $R_1 = R_2 = 1000\,\Omega$. R_{test} is an adjustable resistor and its resistance can be adjusted between $100\,\Omega \sim 1000\,\Omega$. The technician measures the current i_{test} while changing R_{test}.



When the technician measures $i_{test} = 0$, the adjustable resistor $R_{test} = 480\,\Omega$. Use this information to help the technician find out where the location **C** is (solve for x)?

Solution: Since nodes u_1 and u_2 are connected with a wire, from the definition of a wire as an element, the voltage across the wire is zero. Hence the two potentials on its ends are always the same, $u_1 = u_2$. When $i_{test} = 0$, the circuit is equivalent to the one where we replace the wire with an open circuit (with $u_1 = u_2$ still) and solve for two voltage dividers separately.

$$\text{since} \quad u_1 = u_2 \tag{4}$$

$$\frac{2R_{a2C}}{2R_{a2C}+R_2} = \frac{R_{test}}{R_{test}+R_1} \tag{5}$$

$$\text{from } R_1 = R_2 \tag{6}$$

$$\text{we can get } 2R_{a2C} = R_{test} \tag{7}$$

$$2*6x = 480 \tag{8}$$

$$x = 40km \tag{9}$$

$$\tag{10}$$

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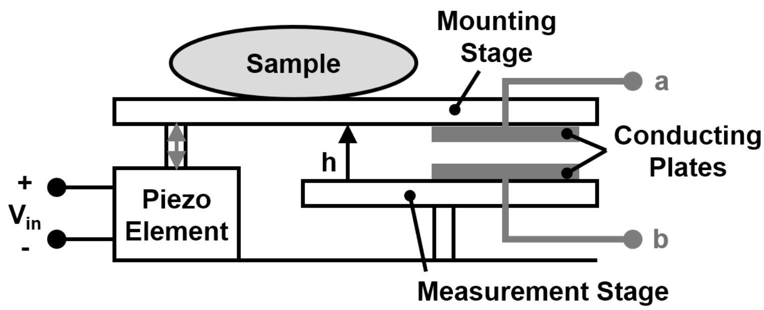
7. Piezo positioner (15 Points)

A Piezo positioner is a device often used to precisely control the position of mounting stages - for example in microscopes. The overall structure of a Piezo positioner is shown below.



Using this instrument, you can adjust the height of the mounting stage h by changing the input voltage (V_{in}), utilizing a special component called "piezoelectric element". The resulting relationship between V_{in} and h is given as follows:

$$h = \alpha_0 V_{\text{in}} + h_0.$$

In other words, h is at some default position h_0 when there's no input, and the stage rises as we increase V_{in}.

In addition, the Piezo positioner has other parts to make it possible for you to **monitor** the height of the mounting stage. This monitoring system also has two conductive plates of the area A, each attached to the mounting stage and the measurement stage. You can connect any circuit you may have to these plates through the electrode **a** and **b**. Note that two plates form a capacitor. Assume that the space between two plates is filled with the air, whose permittivity is given as ε_0. Ignore the thickness of the stages as well as the conducting plates.

(a) (5 points) The initial V_{in} is 0 V. Let's say now you want to **raise** the location of the sample by Δh. What is the new V_{in}? Also, express the capacitance between a and b after the movement as a function of given parameters.

Solution:

$$V_{\text{in}} = \Delta h / \alpha_0$$

$$C_{\text{ab}} = \frac{\varepsilon_0 A}{h} = \frac{\varepsilon_0 A}{\Delta h + h_0}$$

(b) (10 points) Now you notice that it is possible to monitor the location of the mounting stage by measuring C_{ab}. To measure this capacitance, your friend proposed the following method:

- Phase 0: short the C_{ab} to make the initial charge 0.
- Phase 1: connect a current source I_S to charge C_{ab} exactly for T seconds
- Phase 2: disconnect the current source, and then measure the voltage between a and b.

Calculate the voltage betwen a and b at the end of Phase 2, in terms of C_{ab}.

Solution:

$$
\begin{aligned}
I_{\mathrm{S}} &= C_{\mathrm{ab}} \frac{dV_{\mathrm{ab}}}{dt} \\
dV_{\mathrm{ab}} &= \frac{I_{\mathrm{S}}}{C_{\mathrm{ab}}} dt \\
V_{\mathrm{ab}} &= \int_0^T \frac{I_{\mathrm{S}}}{C_{\mathrm{ab}}} dt = \frac{I_{\mathrm{S}}}{C_{\mathrm{ab}}} T
\end{aligned}
$$

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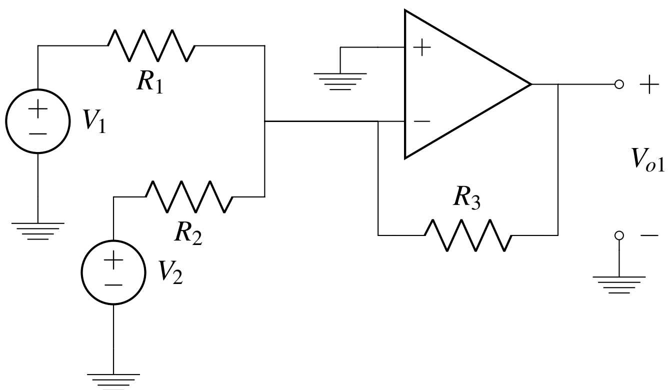
8. Swimming Pool Pressure Sensors (20 points)

A swimming pool has two pressure sensors that output voltage measurements that scale linearly with the amounts of pressure measured by the sensors. One sensor measures the pressure at the top of the swimming pool, which is the air pressure of the environment, and outputs V_1. The second sensor measures only the pressure exerted by the water in the swimming pool, and outputs V_2.



The pool manager would like to have the above op amp circuit output V_{o1} to continuously monitor the pressure the bottom of the pool would sense (i.e. the circuit that sums the outputs of both sensors).

(a) (5 points) In terms of V_1, V_2, R_1, R_2, and R_3, what is V_{o1}? Mathematically justify your answer, i.e. perform circuit analysis to find V_{o1}.

Solution: Applying the Golden Rules of an op amp in negative feedback as well as the KCL at the inverting input of the op amp, we obtain

$$-\frac{V_{o1}}{R_3} - \frac{V_1}{R_1} - \frac{V_2}{R_2} = 0 \Rightarrow V_{o1} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2.$$

(b) (3 points) The pool manager gives you an inverting amplifier that negates V_{o1}, the output of the above circuit (i.e. the output of the inverting amplifier is $-V_{o1}$). What condition(s) do you need on R_1, R_2, and R_3 (i.e. what equation(s) involving R_1, R_2, and R_3 must be satisfied) so that the output of the inverting amplifier is $-V_{o1} = V_1 + V_2$?

Solution: We need $R_1 = R_2 = R_3$. Then, $-V_{o1} = V_1 + V_2$.

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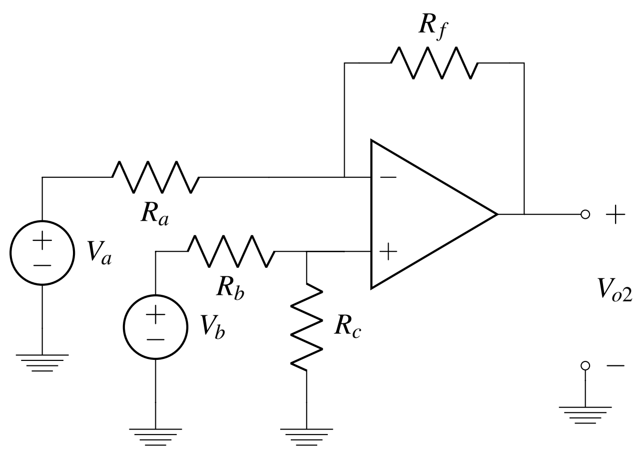
One day, a janitor accidentally breaks the screen of the second sensor while cleaning, and the value of V_2 cannot be read from the sensor anymore. Luckily, the circuit above still functions correctly and $-V_{o1}$, the output of the inverting amplifier, still measures the sum of the two sensors (i.e. $V_1 + V_2$). The pool manager asks you to use the below circuit to retrieve V_2 from V_{o1} and V_1.



(c) (7 points) What is V_{o2} in terms of V_a, V_b, R_a, R_b, R_c, and R_f? Mathematically justify your answer, i.e. perform circuit analysis to find V_{o2}.

Solution: Applying the KCL at the inverting input of the op amp and the fact that the current going into the inverting input of the op amp is zero from the Golden Rules, we obtain

$$\frac{U_- - V_a}{R_a} + \frac{U_- - V_{o2}}{R_f} = 0.$$

The voltage at the non-inverting input of the op amp is

$$U_+ = \frac{R_c}{R_b + R_c} V_b,$$

from the voltage divider equation. We know from the Golden Rules of an op amp in negative feedback that $U_+ = U_-$. Therefore, we have

$$\frac{V_{o2}}{R_f} = \frac{U_-}{R_a} - \frac{V_a}{R_a} + \frac{U_-}{R_f} \Rightarrow V_{o2} = \frac{R_f + R_a}{R_a} U_- - \frac{R_f}{R_a} V_a.$$

Using the fact that $U_+ = U_-$, we obtain

$$V_{o2} = \frac{(R_f + R_a) R_c}{R_a (R_b + R_c)} V_b - \frac{R_f}{R_a} V_a.$$

This problem can also be solved using superposition. With V_a turned on, the circuit is an inverting amplifier. With V_b turned on, the circuit is a voltage divider followed by a non-inverting amplifier.

(d) (3 points) What condition(s) do you need on R_a, R_b, R_c, and R_f (i.e. what equation(s) involving R_a, R_b, R_c, and R_f must be satisfied) such that the above circuit performs a simple subtraction operation, i.e. $V_{o2} = V_b - V_a$?

Solution: If $R_a = R_f$, we have

$$V_{o2} = \frac{2R_aR_c}{R_a(R_b+R_c)}V_b - V_a \;\Rightarrow\; V_{o2} = \frac{2R_c}{R_b+R_c}V_b - V_a.$$

If $R_b = R_c$, then

$$V_{o2} = V_b - V_a.$$

Therefore, the conditions we need are $R_a = R_f$ and $R_b = R_c$.

(e) (2 points) The manager would now like to recover V_2 (the output of the second sensor) from $-V_{o1}$ (the output of the inverting amplifier) **and** V_1 (the output of the first sensor). What must V_a and V_b be in the above circuit to have $V_{o2} = V_b - V_a = V_2$, the output of the second sensor? You do not have the tools to multiply any sensor output, but you have $-V_{o1}$ and V_1 available for use.

Solution: If $V_b = -V_{o1}$ and $V_a = V_1$, then $V_{o2} = V_2$.

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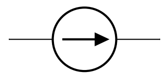
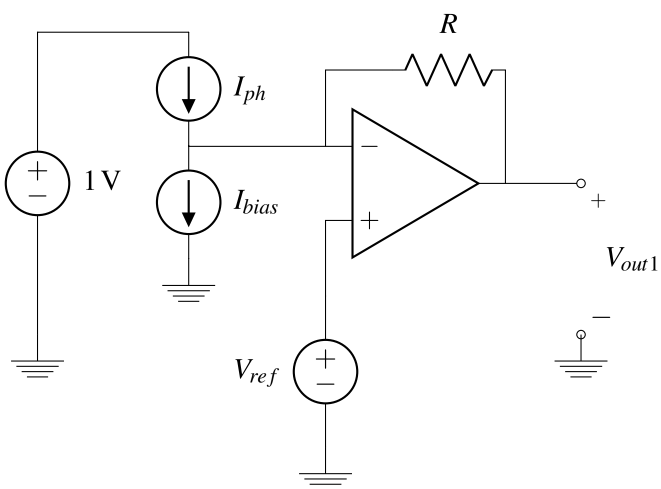
9. Communicating with Light (15 points)

After finishing EE16A, you got the opportunity to work with Professor Vladimir. His group designs computer chips that communicate with light, and your job is to design the reciever. Your receiver connects to a photodetector, that can be modeled as current source.



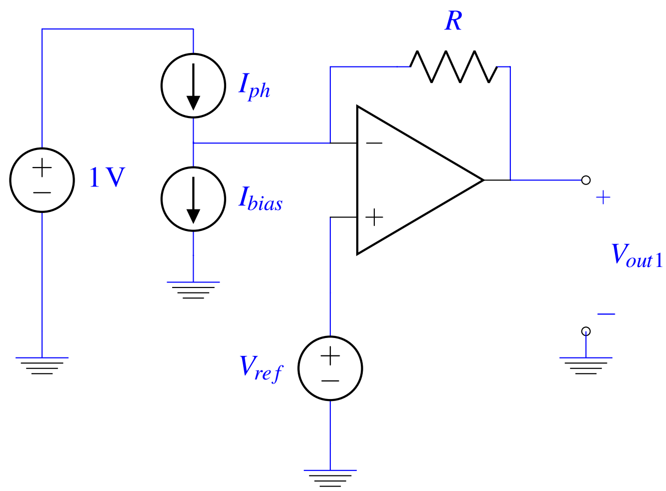
When light is shining on the reciever, the current becomes $I_{ph} = 1\,\mu A$. Otherwise, with no light shining on the receiver the current is $I_{ph} = 0\,A$. The circuit should output 0 V to the processor when there is light shining on the receiver, and output 1 V to the processor when there is no light shining on the receiver.

(a) (10 Points) Your friend suggests the following circuit to turn the current into a voltage. Find the output voltage, V_{out} as a function of I_{ph}, and find a value of R such that the circuit outputs 0 V when $I_{ph} = 1\,\mu A$. $I_{bias} = 0.5\,\mu A$ and $V_{ref} = 0.5\,V$. Show your work.



Solution:

We begin by labeling a few nodes in the diagram shown below:



Let's check if the Op Amp is in negative feedback. Moving the ouput voltage up a little moves u^- up, which decreases $v_c = u^+ - u^-$ making the output voltage want to go back down. Since moving the output voltage up made the voltage want to come back down, the circuit is in negative feedback.

Since the circuit is in negative feedback, $V_{ref} = u^- = 0.5\,\text{V}$. Next we perform nodal analysis on the node u^-:

$$
\begin{aligned}
-I_{ph} + I_{bias} + \frac{u^- - u_{out}}{R} \\
u_{out} &= (I_{bias} - I_{ph})R + u^- \\
u_{out} &= (I_{bias} - I_{ph})R + 0.5\,\text{V} \\
V_{out1} &= u_{out} - 0 = (I_{bias} - I_{ph})R + 0.5\,\text{V}
\end{aligned}
$$

From the above relationship, we see the circuit functions as required when $R = 1\,\text{M}\Omega$.

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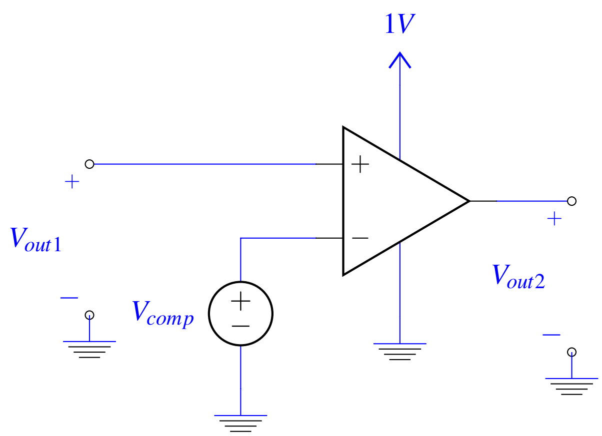
(b) (5 Points) After building the circuit and testing it, you find that I_{ph} varies as you get closer or further from light. This causes the output voltage of your circuit to not be precisely 0 V or 1 V.

However, the processor needs exactly 0 V when light is shining, and $1V$ when no light is shining. Vlad advises you add a comparator to your design. Finish the circuit below by calculating the value of V_{comp} for the comparator. Also select the polarity of the comparator by drawing a $+$ or $-$ in corresponding boxes in the diagram below.

Solution:



We need when we have $I_{ph,light}$:

$$(I_{bias} - I_{ph,light})R + 0.5\,\text{V} < V_{comp} < 1\,\text{V}$$

Then, the ideal V_{comp} is inbetween the range:

$$V_{comp} = \frac{1.5\,\text{V} + (I_{bias} - I_{ph,light})R}{2}$$