EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 3B

1. **Mechanical Inverses**

   For each sub-part below, compute the inverse of $\mathbf{A}$ using the Gauss-Jordan method.

   (a) $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

   (b) Use the answer in part (a) to find the inverse of the rotation matrix $\mathbf{A} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$

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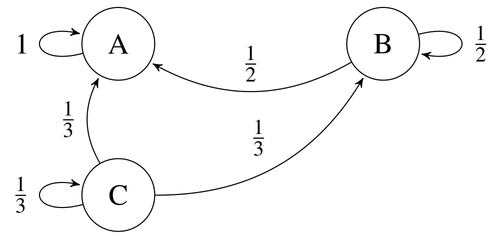
(c) $\mathbf{A} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

(d) (PRACTICE)

$$\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$$

2. Transition Matrix

Suppose we have a network of pumps as shown in the diagram below. Let us describe the state of A, B, and C using a state vector $\vec{x}[n] = \begin{bmatrix} x_A[n] \\ x_B[n] \\ x_C[n] \end{bmatrix}$ where $x_A[n]$, $x_B[n]$, and $x_C[n]$ are the states at time-step n.



(a) Find the state transition matrix S, such that $\vec{x}[n+1] \;=\; S\,\vec{x}[n]$.
Separately, find the sum of the terms for each column vector in S. Do you notice a pattern?

(b) Let us now find the matrix S^{-1} such that we can recover the previous state $\vec{x}[n-1]$ from $\vec{x}[n]$.
Specifically, solve for S^{-1} such that $\vec{x}[n-1] \;=\; S^{-1}\,\vec{x}[n]$.

(c) Now, draw the state transition diagram that corresponds to the S^{-1} that you just found. Again, find the sum of the terms for each column vector in S^{-1}. Do you notice a pattern?

(d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transition matrix of this "reversed" state transition diagram T. Does $T = S^{-1}$?

(e) Suppose we start in the state $\vec{x}[1] = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$. Compute the state vector after two time-steps, $\vec{x}[3]$.