EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 4A

1. Exploring Column Spaces and Null Spaces

- The **column space** is the **span** of the column vectors of the matrix.
- The **null space** is the set of input vectors that when multiplied with the matrix result in the zero vector.

For the following matrices, answer the following questions:

i. What is the column space of $\mathbf{A}$? What is its dimension? (The **dimension** of a vector space is defined as the minimum number of vectors needed to span the space.)

ii. What is the null space of $\mathbf{A}$? What is its dimension?

iii. Do the columns of $\mathbf{A}$ span $\mathbb{R}^2$?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 1 & 1 & 3 & -3 \end{bmatrix}$

(f) What do you notice about the relationship between the dimension of the column space, the dimension of the null space, and their sum in all of these matrices?

2. Row Space

Consider:

$$\mathbf{V} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 0 & 4 \\ 6 & 4 & 10 \\ -2 & 4 & 2 \end{bmatrix}$$

Row reducing this matrix yields:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Show that the row spaces of **U** and **V** are the same. Argue that in general, Gaussian elimination preserves the row space.

(b) Show that the null spaces of **U** and **V** are the same. Argue that in general, Gaussian elimination preserves the null space.

(c) We define the row rank of a matrix as the dimension of the row space. We say that a matrix has full row rank if its row rank is equal to the number of rows in the matrix. What is the row rank of **U** and **V**, and do they have full row rank?