EECS 16A Designing Information Devices and Systems I
Spring 2023 Discussion 12B

1. Inner Product Properties

For this question, we will verify our definition of the Euclidean inner product in Cartesian coordinates

$$\langle \vec{x}, \vec{y} \rangle \; = \; x_1 y_1 + x_2 y_2 + \ldots + x_n y_n \; , \quad \text{for any} \;\; \vec{x}, \vec{y} \in \mathbb{R}^n$$

indeed satisfies the key properties required for all inner products for the 2-dimensional case. Suppose $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^2$ for the following parts (there is a slightly different definition for complex vectors):

(a) Show symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$.

(b) Show linearity: $\langle \vec{x}, c\vec{y} + d\vec{z} \rangle = c\langle \vec{x}, \vec{y} \rangle + d\langle \vec{x}, \vec{z} \rangle$, where $c, d \in \mathbb{R}$ are real numbers.

(c) Show non-negativity: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

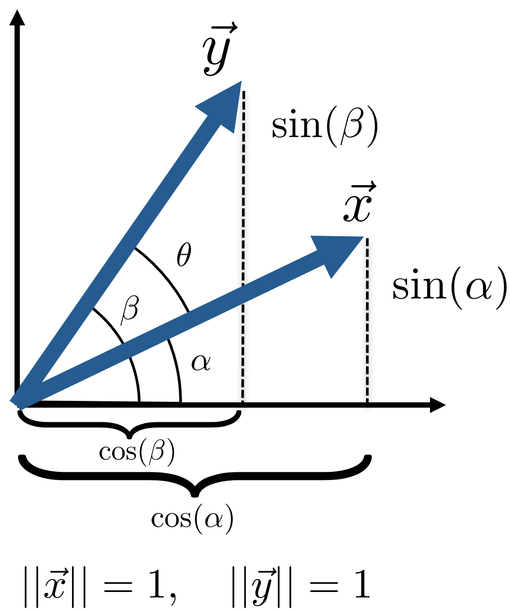
2. Geometric Interpretation of the Inner Product

In this problem, we explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in $\mathbb{R}^2$.

Remember that the formula for the inner product of two vectors can be expressed in terms of their magnitudes and the angle between them as follows:

$$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \|\vec{y}\| \cdot \cos\theta$$

The figure below may be helpful in illustrating this property:



For each subpart, give an example of any two (nonzero) vectors $\vec{x}, \vec{y} \in \mathbb{R}^2$ that satisfy the stated condition and compute their inner product.

(a) Give an example of a pair of parallel vectors (vectors that point in the same direction and have an angle of 0 degrees between them).

(b) Give an example of a pair of anti-parallel vectors (vectors that point in opposite directions).

(c) Give an example of a pair of perpendicular vectors (vectors that have an angle of 90 degrees between them).