EECS 16A Designing Information Devices and Systems I
Spring 2023 Exam Prep 3B

1. Hungry for Linearity (Fall 2022 Midterm 1 Question 6)

(a) Determine whether the following functions are linear. If so, show they are linear with the properties of linearity. If the function is not linear, clearly demonstrate at least one property of linearity is violated.

i. $g(x_1, x_2, x_3) = -\pi x_1 + e^6 x_2 - \sqrt{2} x_3$

ii. $f(x) = 3\sqrt{x^2}$

(b) Now consider an arbitrary matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ and vector $\vec{x} \in \mathbb{R}^{n \times 1}$. Determine whether the following functions are linear. If so, show they are linear with the properties of linearity. If the function is not linear, clearly demonstrate at least one property of linearity is violated.

i. $f(\vec{x}) = \mathbf{A}^2 \vec{x}$

ii. $f(\vec{x}) = \mathbf{A}\vec{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ where $\mathbf{A} \in \mathbb{R}^{2\times 2}$ and vector $\vec{x} \in \mathbb{R}^{2\times 1}$.

2. Oski on Sproul (Fall 2022 Midterm 1 Question 3)

Oski is studying how long it takes students to get through Sproul plaza. In particular, he is interested in how long each interaction takes in seconds: talking to a club *(t)*, accepting a flyer *(a)*, rejecting a flyer *(r)*, and taking a picture *(p)*.

- Ayah talked to 2 clubs, accepted 10 flyers, rejected 4 flyers, and took 1 picture. Her trip took a total of 96 seconds.
- Mira talked to 0 clubs, accepted 6 flyers, rejected 15 flyers, and took 2 pictures. Her trip took a total of 171 seconds.
- Rishi talked to 1 club, accepted 2 flyers, rejected 14 flyers, and took 2 pictures. Her trip took a total of 58 seconds.

(a) Formulate a matrix-vector representation for this set of linear equations in the form of $\mathbf{A}\vec{x} = \vec{b}$ where *t, a, r, p* are the variables in $\vec{x}$.

$$\mathbf{A} = \begin{bmatrix} & \\ & \\ & \\ & \\ & \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} t \\ a \\ r \\ p \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

(b) Oski's collaborator, Tree, gathered some more data and arrived at matrix $\mathbf{A}$ and vector $\vec{b}$ below. However, Tree wrote in cursive and Oski cannot read the value represented by $\mathbf{u}$ in matrix $\mathbf{A}$. What values of $\mathbf{u}$ would guarantee *no solution* to $\mathbf{A}\vec{x} = \vec{b}$? Justify your answer.

$$\mathbf{A} = \begin{bmatrix} 2 & 17 & 0 & 3 \\ 8 & 2 & 0 & 1 \\ 16 & 4 & \mathbf{u} & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 70 \\ 30 \\ 60 \\ 90 \end{bmatrix}$$

(c) Finally, Oski asks you to help out with one more augmented matrix problem. Help solve it using Gaussian elimination.

$$\mathbf{M} = \left[\begin{array}{ccc|c} 4 & 4 & 0 & 24 \\ 1 & 3 & 0 & 14 \\ 2 & 2 & 6 & 18 \end{array}\right]$$

3. From Independence to Dependence (Fall 2022 Midterm 1 Question 8)

Let $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & -6 \\ -2 & 4 \end{bmatrix}$ and $\vec{v}_1$ and $\vec{v}_2$ be two vectors in $\mathbb{R}^2$. Prove that the set of vectors $\{\mathbf{A}\vec{v}_1, \mathbf{A}\vec{v}_2\}$ must be linearly dependent.