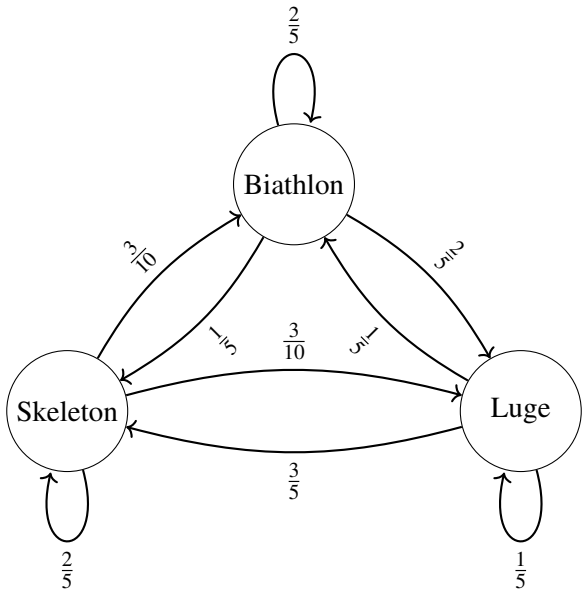
EECS 16A Designing Information Devices and Systems I
Spring 2023 Exam Prep 6B

1. Cooling Off at The Olympics (Spring 2022 Midterm 1 Question 9)

A number of Berkeley students tuned into the Winter Olympics. You want to analyze the dynamics of the viewer traffic.

(a) You are able to construct the following transition diagram.



The current number of students watching each sport at time-step t is given by the state vector $\vec{x}[t]$ defined as:

$$\vec{x}[t] = \begin{bmatrix} x_B[t] \\ x_S[t] \\ x_L[t] \end{bmatrix} = \begin{bmatrix} \text{number of students watching Biathlon at time } t \\ \text{number of students watching Skeleton at time } t \\ \text{number of students watching Luge at time } t \end{bmatrix}$$

Explicitly write out the transition matrix $\mathbf{T}$ from the provided diagram such that $\vec{x}[t+1] = \mathbf{T}\,\vec{x}[t]$. Is the system conservative? Justify your answer.

(b) Suppose we have a different transistion matrix given by:

$$\mathbf{T}_2 = \begin{bmatrix} \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

Does a steady state vector exist for this system? If so, identify a steady state vector $\vec{x}_{steady}$ such that $\mathbf{T}_2\vec{x}_{steady} = \vec{x}_{steady}$.

(c) Say you know the state at time t, $\vec{x}[t]$, for the system described in part (b) with $\mathbf{T}_2$. For *any* given $\vec{x}[t]$, is it possible to find the previous state, $\vec{x}[t-1]$? **Justify**.

2. Proof (Spring 2021 Midterm 1 Question 8)

You are told that a $\mathbf{A} \in \mathbb{R}^{2\times 2}$ is a conservative transition matrix. **Prove that it has an eigenvalue of λ = 1.**

