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4. Autocorrelation (10 points)

Let's define the *autocorrelation* of a vector $\vec{x} \in \mathbb{R}^N$. Recall a zero-padded discrete-time signal is

$$x[n] = \begin{cases} x_n, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

The autocorrelation of $\vec{x}$ is then defined as the correlation of $\vec{x}$ with itself, or

$$\text{autocorr}(\vec{x})[k] = \text{corr}_{\vec{x}}(\vec{x})[k] = \sum_{n=-\infty}^{\infty} x[n]x[n-k]$$

(a) (4 points) For the following problems, **select the statements that are *always true*** given the provided assumptions.

i. Assumption: All entries of $\vec{x}$ are positive, i.e., $x_n \geq 0$ for all indices n. (**Select all that apply.**)

☐ $\text{autocorr}(\vec{x})[0] = \|\vec{x}\|^2$

☐ $\text{autocorr}(\vec{x})[k] \geq 0$ for all k

☐ There exists some k where $\text{autocorr}(\vec{x})[k] < 0$

☐ $\text{autocorr}(\vec{x})[k] = \text{autocorr}(-\vec{x})[k]$ for all k

ii. Assumption: In addition to $x_n \geq 0$, let $\vec{y} = \alpha\vec{x}$ for $\alpha \in \mathbb{R}$. (**Select all that apply.**)

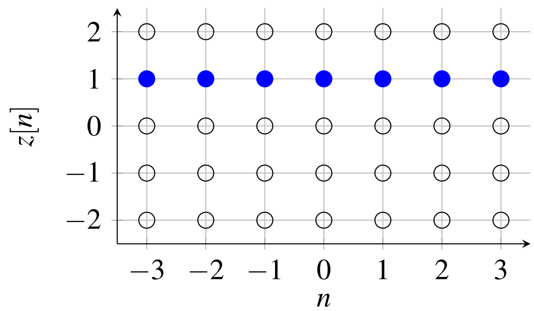
☐ $\text{autocorr}(\vec{y})[\alpha] = \text{autocorr}(\vec{x})[-\alpha]$

☐ $\text{autocorr}(\vec{y})[k] = \text{autocorr}(\vec{x})[k]$ for all k

☐ $\text{autocorr}(\vec{y})[k] = \alpha^2 \cdot \text{autocorr}(\vec{x})[k]$ for all k

☐ $\text{corr}_{\vec{y}}(\vec{x})[k] = \alpha \cdot \text{autocorr}(\vec{x})[k]$ for all k

(b) (2 points) In this question, you will be plotting a signal by filling in bubbles on the graph. The example below shows you how to plot $z[n] = 1$.

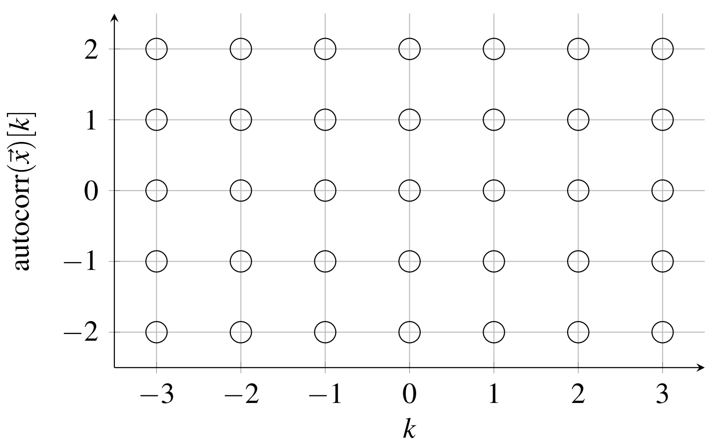


Now, suppose we have an arbitrary vector $\vec{x}$ with the following two properties:

i. $\|\vec{x}\|^2 = 1$

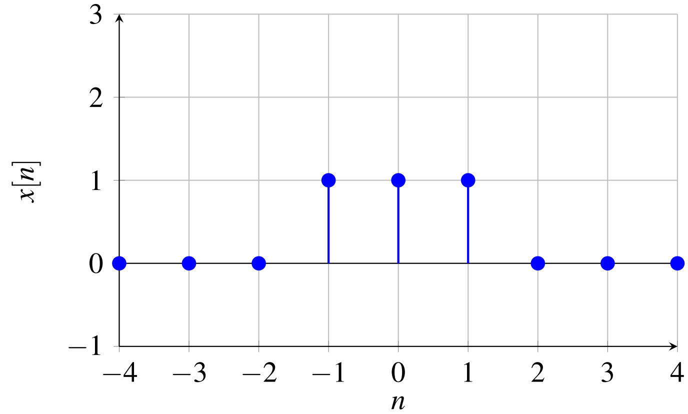
ii. $\vec{x}$ is orthogonal to any shifted zero-padded version of itself.

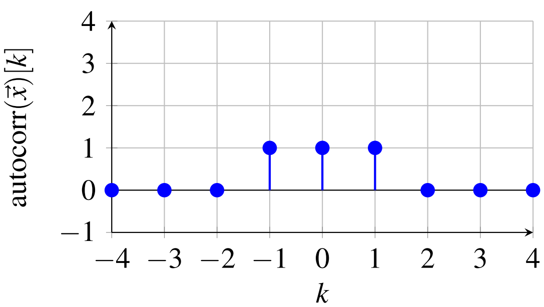
Plot $\text{autocorr}(\vec{x})[k]$ as a function of k. To do so, **fill in the values** of $\text{autocorr}(\vec{x})[k]$ for $k = -3, \ldots, 3$.

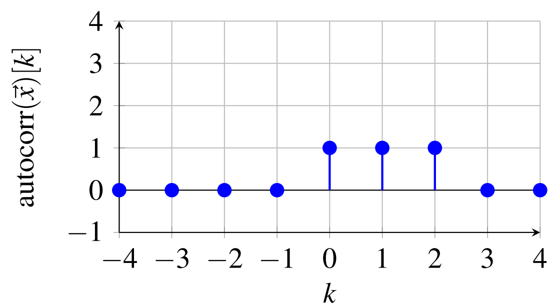


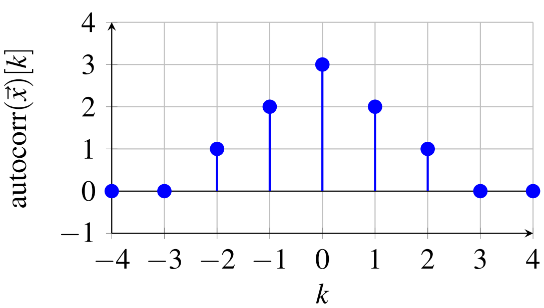
(c) (4 points) For each of the following signals, **select its autocorrelation plot.**

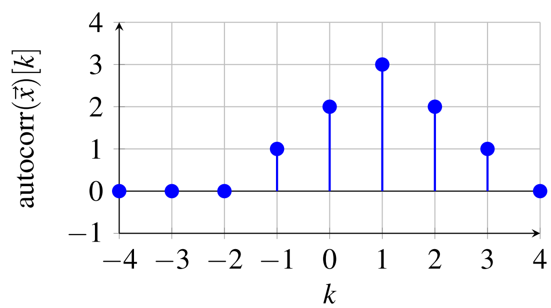
i. (2 points) $\vec{x}$ plotted below

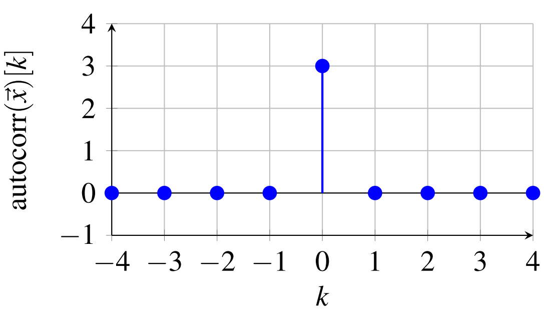


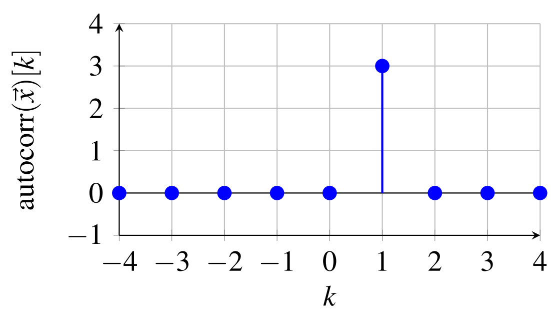




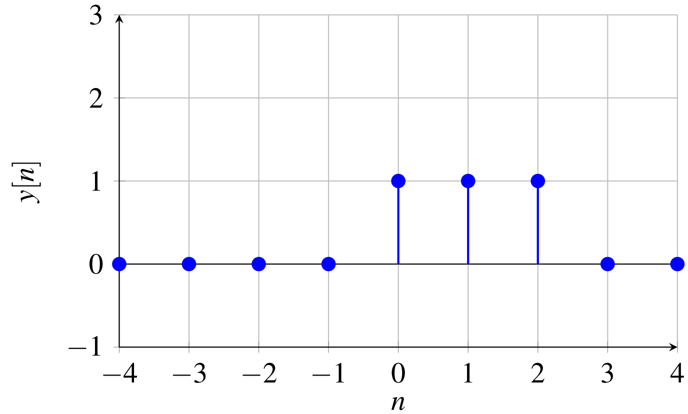








ii. (2 points) $\vec{y}$ is a shifted version of $\vec{x}$ such that $y[n] = x[n-1]$, as shown below.



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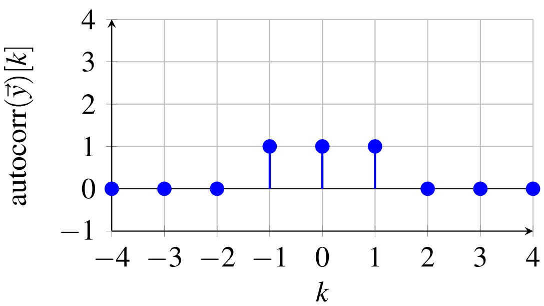

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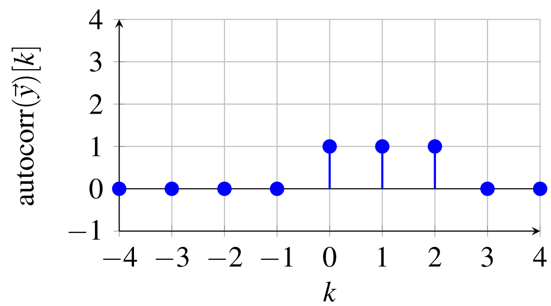

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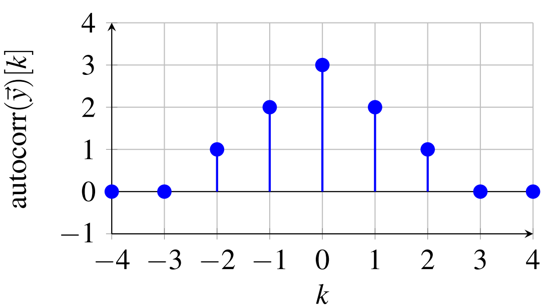

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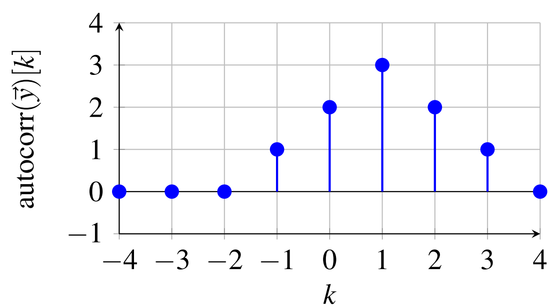

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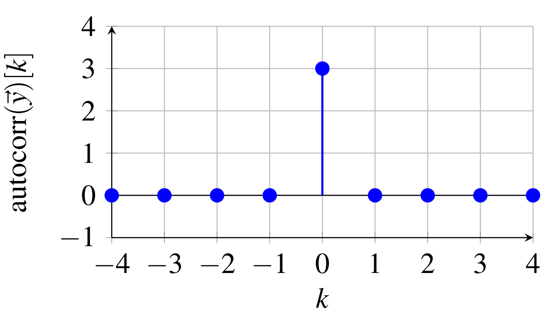

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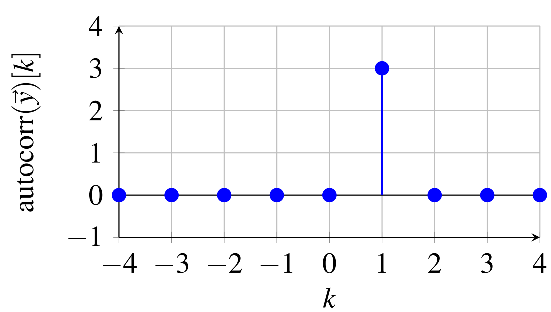

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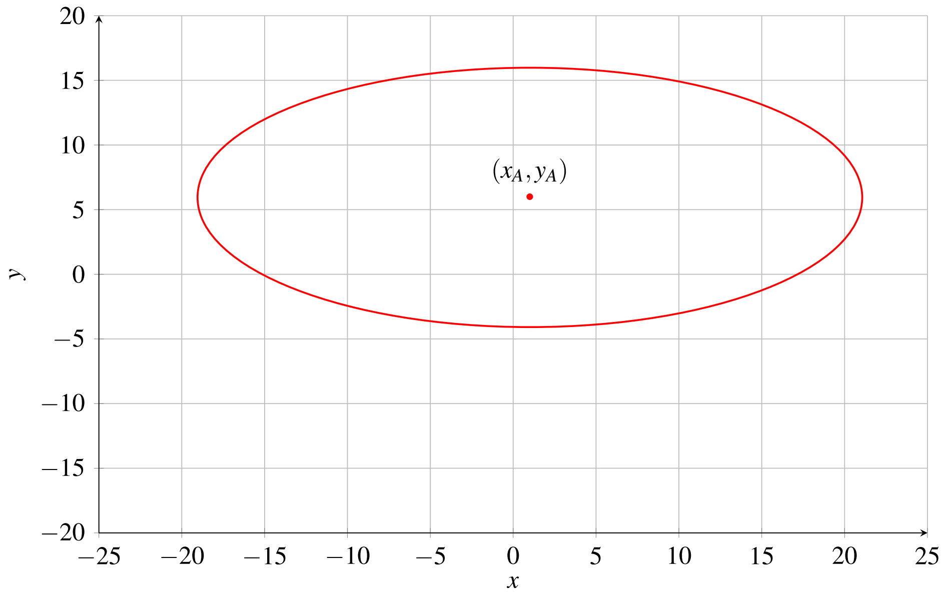
9. Non-Isotropic World (7 points)

You are an astronaut living in a colony on a distant planet. After some exploration you have gotten lost and are now trying to trilaterate your location (x_m, y_m) using received signals from beacons with known locations. However, on this particular planet radio waves propagate two times faster in the x-direction (latitude) than the y-direction (longitude).

(a) (1 point) The first distance reading is received from Beacon A, located at $(x_A, y_A) = (1, 6)$, showing `'Distance from Beacon A = 10'`. Thus, the elliptical equation governing the 1st beacon is

$$\frac{(x-1)^2}{4} + \frac{(y-6)^2}{1} = 10^2$$



From the information provided by Beacon A, **how many possibilities exist for your location**?

- ◯ Infinitely many possibilities
- ◯ Two possible locations
- ◯ One possible location

(b) (1 point) You then receive a second reading from Beacon B, located at $(x_B, y_B) = (1, -6)$, showing '`Distance from Beacon B = 10`'. Thus, the elliptical equation governing the 2nd beacon is

$$\frac{(x-1)^2}{4} + \frac{(y+6)^2}{1} = 10^2$$

From the information provided by Beacon A and Beacon B, how many possibilities exist for your location?

- ◯ Infinitely many possibilities
- ◯ Two possible locations
- ◯ One possible location

(c) (1 point) You receive a third reading from Beacon C, located at $(x_C, y_C) = (-1, 0)$, showing '`Distance from Beacon C = 9`'. Thus, the elliptical equation governing the 3rd beacon is

$$\frac{(x+1)^2}{4} + \frac{y^2}{1} = 9^2$$

From the information provided by all three beacons, how many possibilities exist for the your location?

- ◯ Infinitely many possibilities
- ◯ Two possible locations
- ◯ One possible location

(d) (4 points) Use the elliptical equations from Beacons A, B, and C. For your convenience, here they are again:

$$\frac{(x-1)^2}{4} + \frac{(y-6)^2}{1} = 10^2$$

$$\frac{(x-1)^2}{4} + \frac{(y+6)^2}{1} = 10^2$$

$$\frac{(x+1)^2}{4} + \frac{y^2}{1} = 9^2$$

i. How many *unique* linear equations are necessary to determine your location (x_m, y_m)?

◯ One linear equation

◯ Two linear equations

◯ Three linear equations

ii. Derive a possible set of linear equations that can be used to solve for your location.
(There are multiple correct answers, you only need to select as many equations as you think are necessary to successfully calculate your location)

☐ $y=0$	☐ $x+y=24$	☐ $-2x+4y=9$
☐ $x=0$	☐ $x-y=24$	☐ $x+12y=17$
☐ $x+48y=0$	☐ $2x+4y=9$	☐ $x-12y=17$