EECS 16A Designing Information Devices and Systems I
Spring 2023 Homework 9

This homework is due Friday, March 24, 2023 at 23:59.
Self-grades are due Friday, April 7, 2023 at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw9.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

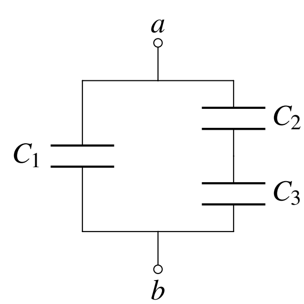
For this homework, please read Notes 16 and Sections 17.1 - 17.2 from Note 17. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them. Sections 17.1 - 17.2 in Note 17 will provide an overview of the capacitive touchscreen and how to measure capacitance.

(a) How do we calculate the equivalent capacitance of series and parallel capacitors? Compare this with how we calculate resistor equivalences.

(b) Consider the capacitive touchscreen. Briefly describe how it works: what quantity changes when your finger touches it? Compare and contrast it to the resistive touchscreens we have seen in previous lectures and homeworks.

2. **Equivalent Capacitance**

(a) Find the equivalent capacitance between terminals a and b of the following circuit in terms of the given capacitors C_1, C_2, and C_3. Leave your answer in terms of the addition, subtraction, multiplication, and division operators **only**.



(b) Find and draw a capacitive circuit using three capacitors, C_1, C_2, and C_3, that has equivalent capacitance of

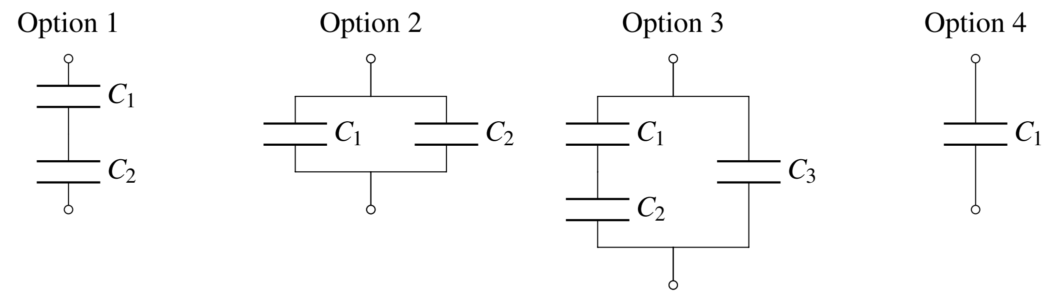
$$\frac{C_1(C_2+C_3)}{C_1+C_2+C_3}$$

3. Modeling Weird Capacitors

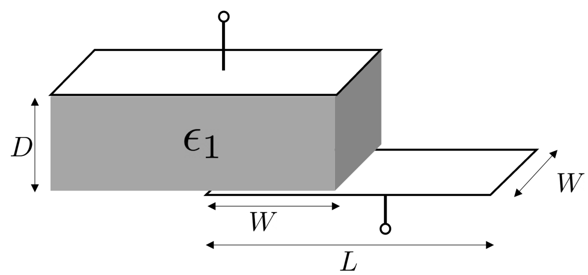
For each part of this problem,

i. Pick the circuit option from below that *best* models the given physical capacitor.

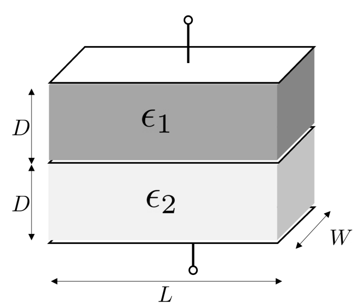
ii. Calculate the total equivalent capacitance of the circuit in terms of the given quantities (e.g. $\varepsilon_1, \varepsilon_2, \varepsilon_3, L, W, D$).



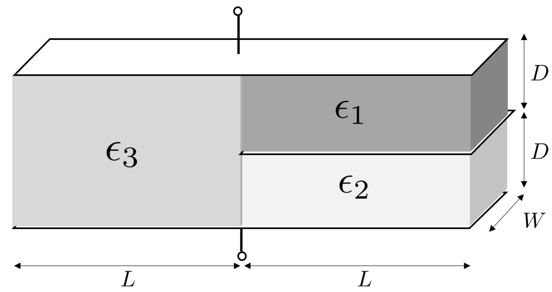
(a) A parallel plate capacitor with plate dimensions L and W, separated by a gap D, is filled with an insulator of permittivity ε_1, with the bottom plate displaced with overlap W as shown below. You can assume $W < L$ and $D << W$.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of L, W, D, ε_1.



(b) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with two insulators of permittivities ε_1 and ε_2 as shown below. You can assume there's a plate between the two dielectrics.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2$.

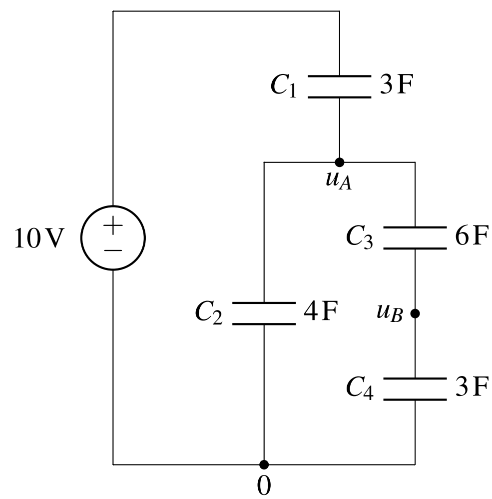


(c) A parallel plate capacitor with plate dimensions L and W, separated by a gap $2 \cdot D$, is filled with three different materials with permittivities ε_1, ε_2, and ε_3 as shown in the figure below. You can assume there's a plate between the two dielectrics on the right.
(i) Pick the circuit option from above that best models this physical capacitor, and (ii) calculate the total equivalent capacitance of the circuit in terms of $L, W, D, \varepsilon_1, \varepsilon_2, \varepsilon_3$.



4. Circuit with Capacitors

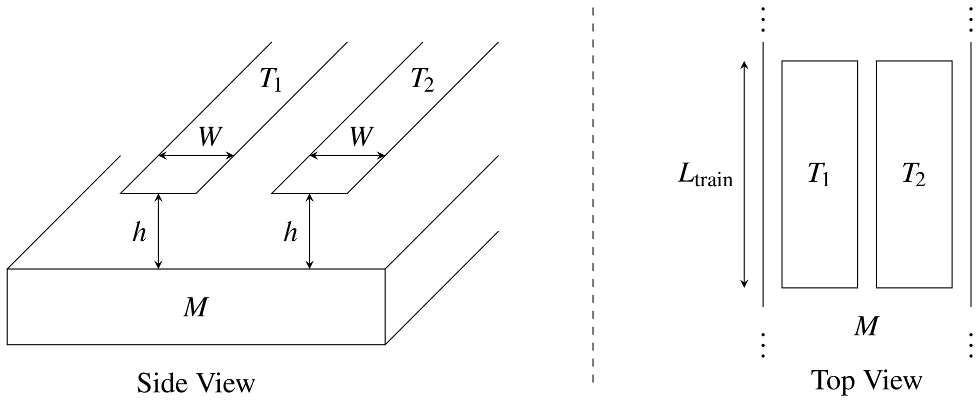
Find the voltages at nodes u_A and u_B, and currents flowing through all of the capacitors at steady state. Assume that before the voltage source is applied, the capacitors all initially have a charge of 0 Coulombs.



5. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from the ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how maglev trains use capacitors to stay elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, we put two parallel strips of metal (T_1, T_2) along the bottom of the train and we have one solid piece of metal (M) on the ground below the train (perhaps as part of the track).



Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., we can use the simple equations developed in lecture to model the capacitance), as a function of L_{train} (the length of the train), W (the width of T_1 and T_2), h (the height of the train off of the track), and ε (the permittivity of the air between the train and the track) what is the capacitance between T_1 and M? What is the capacitance between T_2 and M?

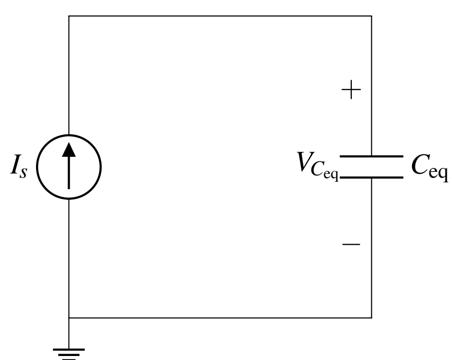
(b) Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other. *Hint: there should be three nodes in your circuit:* T_1, T_2, *and* M.

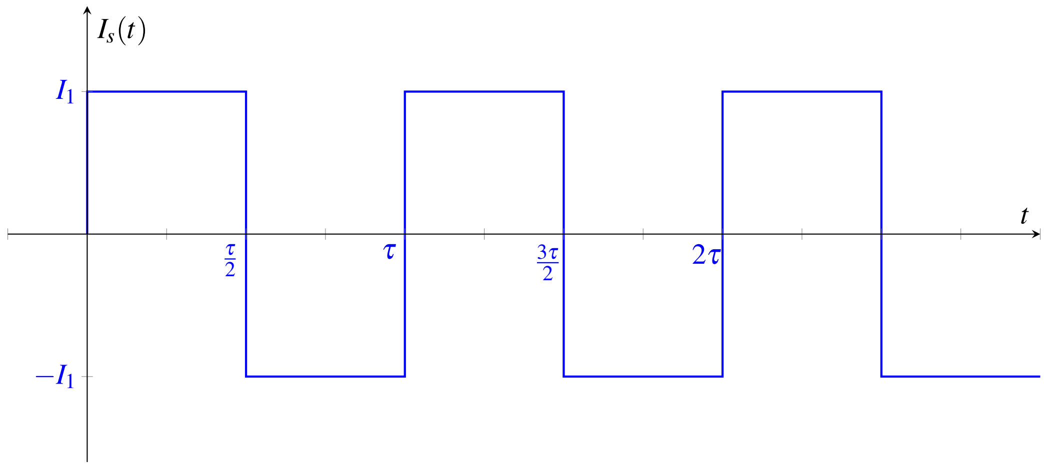
(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance between T_1 and T_2.

(d) We want to build a circuit that creates a voltage waveform with an amplitude that changes based on the height of the train. Your colleague recommends you start with the circuit as shown below, where I_s is a periodic current source, and C_{eq} is the equivalent capacitance between T_1 and T_2. The graph below shows I_s, a square wave with period τ and amplitude I_1, as a function of time.

Find an equation for and draw the voltage $V_{C_{\text{eq}}}(t)$ as a function of time. Assume the capacitor C_{eq} is discharged at time $t = 0$, so $V_{C_{\text{eq}}}(0) = 0\,\text{V}$.

Hint: Your final expression should resemble a periodic function.





(e) Based on your answer to the previous 2 parts, how does the voltage $V_{C_{eq}}(t)$ change with the height of the train h?

(f) What if we change the period of the square wave from τ to $\frac{\tau}{2}$ (and keep its amplitude I_1 the same)? How would $V_{C_{eq}}(t)$ change?

6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

