EECS 16A Designing Information Devices and Systems I
Spring 2023 Homework 11

This homework is due April 7, 2023, at 23:59.
Self-grades are due April 14, 2023, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw11.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit each file to its respective assignment on Gradescope.

1. **Reading Assignment**

For this homework, please read Note 17 and Note 18 to learn about comparators and op-amps. You are always encouraged to read beyond this as well.

(a) What is the purpose of a comparator? How can we use a comparator circuit to detect a touch for a capacitive touchscreen?

(b) If the op-amp supply voltages are $V_{DD} = 5\,\text{V}$ and $V_{SS} = 0\,\text{V}$, then what are the minimum and maximum value of V_{out}?

(c) What does the internal gain of an op-amp, A, mean? What is its value for an ideal op-amp? What about for a non-ideal one?

2. **It's finally raining!**

A lettuce farmer in the Salinas Valley has grown tired of imprecise online rainfall forecasts. They decide to take matters into their own hands by building a rain sensor. They place a square tank outside and attach two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.

Note: In practice, water is conductive. However for this problem, assume the metal plates are properly insulated so that no current flows through the water and we can treat it like a dielectric material. In other words, the electric circuit is better modeled as a capacitance and not a resistance.

The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) What is the capacitance between terminals a and b when the tank is full? What about when it is empty? The permittivity of air is $\varepsilon_{\text{air}} = \varepsilon_0$, and the permittivity of rainwater is $\varepsilon_{\text{H}_2\text{O}} = 75\varepsilon_0$.

(b) Suppose the height of the water in the tank is $h_{\text{H}_2\text{O}}$. Model the tank as a pair of capacitors in parallel, where one capacitor has a dielectric of air, and one capacitor has a dielectric of water. Find the total capacitance C_{tank} between the two metal walls/plates using circuit equivalence.

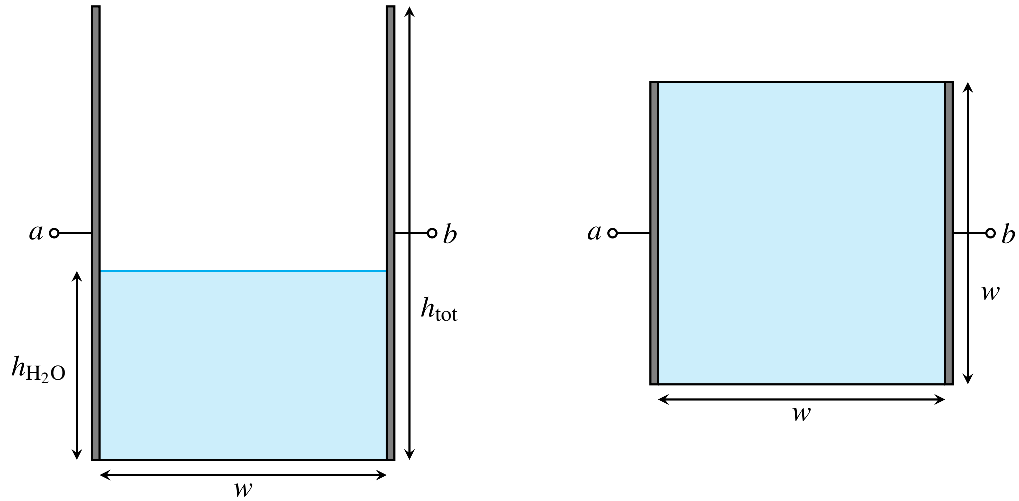
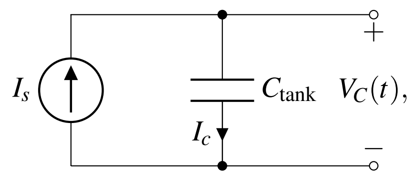


Figure 1: Tank side view (left) and top view (right).

(c) After building this tank, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends building the following circuit:



where C_{tank} is the total tank capacitance between terminals a and b calculated in part (b), and I_s is a known current supplied by a current source.

The user suggests measuring $V_C(t)$ for a brief interval of time, compute the rate of change of V_C, and determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank} is initialized to $0\,V$, i.e. $V_C(0) = 0$.

(d) Using the equation you derived for $V_C(t)$, describe how you can use this circuit to determine C_{tank} and h_{H_2O}.

3. Op-Amp in Negative Feedback

In this question, we analyze op-amp circuits that have finite op-amp gain A. We replace the op-amp with an equivalent circuit model with parameterized gain, A, and observe the gain's effect on the terminal and output voltages as the gain approaches infinity. **Note here that $V_{SS} = -V_{DD}$.**

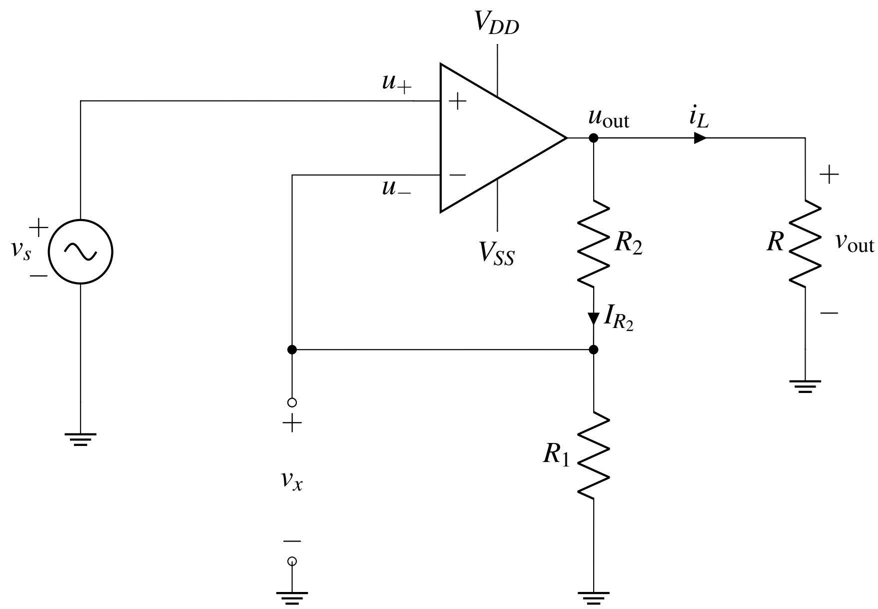


Figure 2: Non-inverting amplifier circuit using an op-amp for feedback

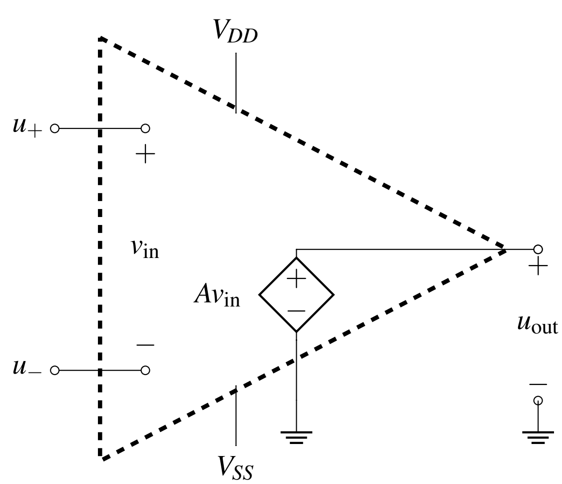


Figure 3: Op-amp model with finite gain, A

(a) Let us first examine when gain A is finite (i.e. not ∞). To understand what happens in this case, first draw an equivalent circuit for the first op amp circuit in Fig. 2, **by replacing the ideal op-amp in the non-inverting amplifier with the op-amp model in Fig. 3.**

(b) Now, using this setup, calculate v_{out} and v_x in terms of A, v_s, R_1, R_2, and R. Is the magnitude of v_x larger or smaller than the magnitude of v_s? Do these values depend on R? *Hint: Note that the currents into the input terminals of the op-amp are zero.*

(c) Using your solution to the previous part, calculate the limits of v_{out} and v_x as $A \to \infty$.

(d) Are there maximum and minimum values v_{out} can be for this op-amp circuit? If so, determine these values.

Hint: Consider the voltage supply rails.

4. Transistor Equivalent

Consider the amplifier circuit in Fig. 4 which amplifies input V_{in} to output V_{out}. The circuit accomplishes this by using a bipolar junction transistor or BJT. The BJT is a three-terminal circuit element with nodes B, C, and E.

In some situations, the BJT can be modeled with an equivalent linear circuit containing a voltage-dependent current source as shown in Fig. 5.

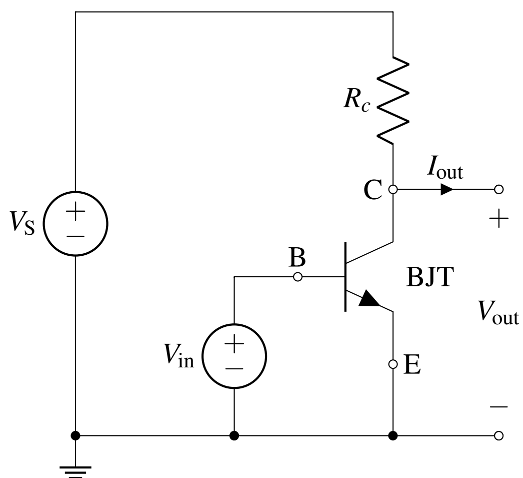


Figure 4: Amplifier circuit with BJT

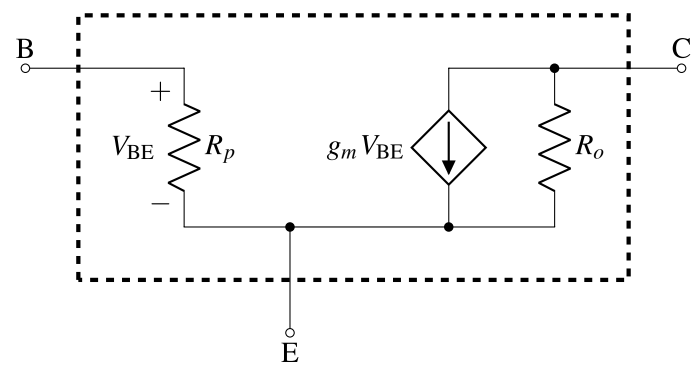


Figure 5: Equivalent circuit model for BJT

We want to find the Thevenin and Norton equivalent of the amplifier circuit across the terminals V_{out} (i.e., between nodes C and E).

Note: You can use the parallel operator (||) in your final answers.

(a) Redraw the original amplifier circuit in Fig. 4 but with the BJT equivalent circuit model in Fig. 5 substituted.

(b) Use the open circuit test to find the Thevenin voltage, V_{th}, between nodes C and E.

Recall the open circuit test finds $V_{out} = V_{oc}$ when an open circuit is connected across the terminals, then $V_{th} = V_{oc}$.

(c) Use the short circuit test to find the Norton current, I_{no}, between nodes C and E.

Recall the short circuit test finds $I_{out} = I_{sc}$ when a short circuit is connected between the terminals, then $I_{no} = I_{sc}$.

(d) Find the Thevenin/Norton resistance $R_{th} = R_{no}$ using $R_{th} = \frac{V_{th}}{I_{no}}$.

(e) We can also find R_{th} by turning off all of the independent sources (but *not* the dependent sources) and deriving the equivalent resistance seen from the terminals. Derive R_{th} with this method. Does it match your answer from part (c)?

Hint: To simplify the dependent source, focus on first finding V_{BE}.

5. Digital to Analog Converter (DAC)

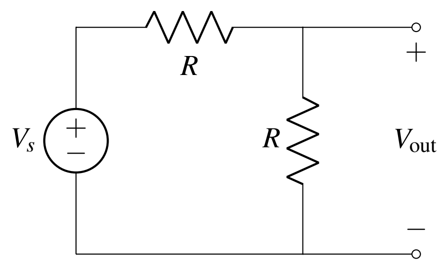
In many electronics applications, such as audio speakers, we need to produce an analog output, or any voltage between 0 to V_s. These analog voltages must be produced from digital voltages that can only be values of V_s or 0. A circuit that does this is known as a Digital to Analog Converter (DAC). It takes a binary representation of a number and turns it into an analog voltage.

The output of a DAC can be represented with the equation shown below:

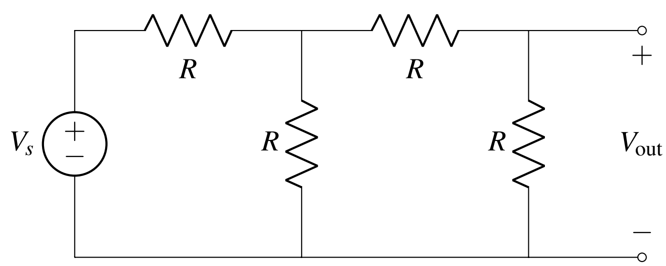
$$V_{\text{out}} = V_s \sum_{n=0}^{N} \frac{1}{2^n} b_n$$

where each binary digit b_n is multiplied by $\frac{1}{2^n}$.

(a) We know how to take an input voltage and divide it by 2:



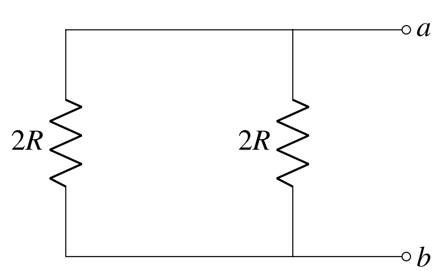
To divide by larger powers of two, we might hope to just "cascade" the above voltage divider. For example, consider:



Calculate V_{out} in the above circut. Is $V_{\text{out}} = \frac{1}{4}V_s$?

(b) The R-$2R$ ladder, shown below, has a very nice property. For each of the circuits shown below, find the equivalent resistance looking in from points a and b. Do you see a pattern?

i.



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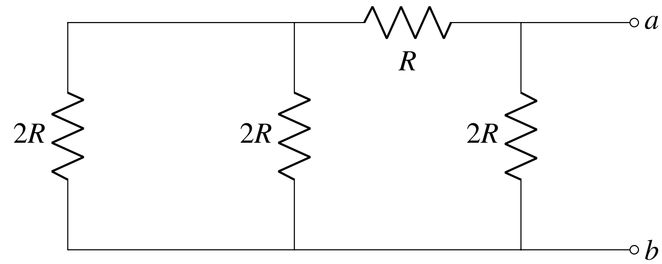
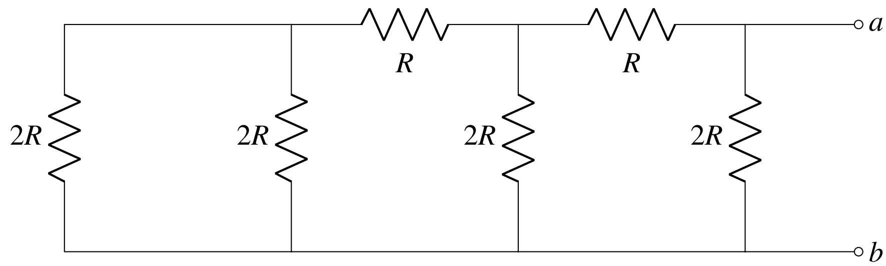
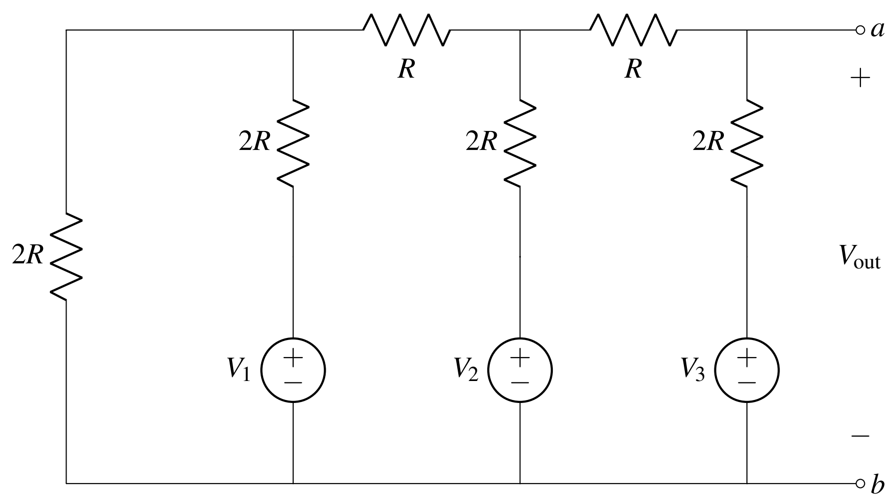


iii.



(c) The following circuit is an R-$2R$ DAC. To understand its functionality, use superposition to find V_{out} in terms of each V_1, V_2, and V_3.



(d) We've now designed a 3-bit R-$2R$ DAC. What is the output voltage V_{out} if $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$?

6. Pre-Lab Questions

These questions pertain to the Pre-Lab reading for the Touch 3B lab. You can find the reading under the Touch 3B Lab section on the 'Schedule' page of the website.

(a) Why can't we have an ideal current source?

(b) For the integrator circuit, if $V_{in}(t)$ is a triangle wave, what kind of wave will $I_{in}(t)$ be?

(c) Does V_{out} increase or decrease when you touch the touchscreen?

7. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.