EECS 16A

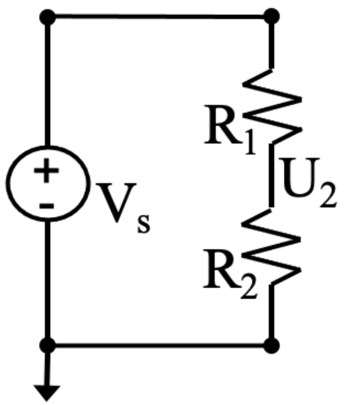
Spring 2023 - Profs. Muller & Waller

Lecture 11A – Op Amp Circuit Analysis

Toolbox

KVL: Voltage drops around a loop sum to 0

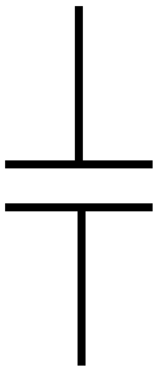
KCL: All currents coming out of a node sum to 0



$$I = \frac{V_S}{R_1 + R_2}$$

$$U_2 = \frac{V_S R_2}{R_1 + R_2}$$

C



$$Q = CV$$

$$I = C\frac{dV}{dt}$$

$$C = \frac{\epsilon A}{d}$$

$$V = IR$$

$$P = IV$$

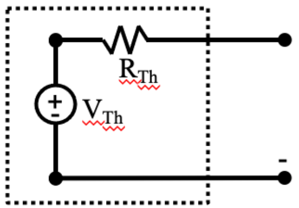
$$R = \frac{\rho L}{A}$$

$$R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

V_{source}(off) → short

I_{source}(off) → open

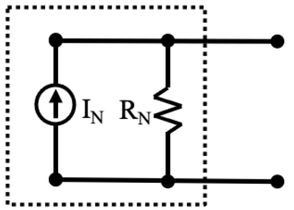
Thevenin Equivalent Circuit



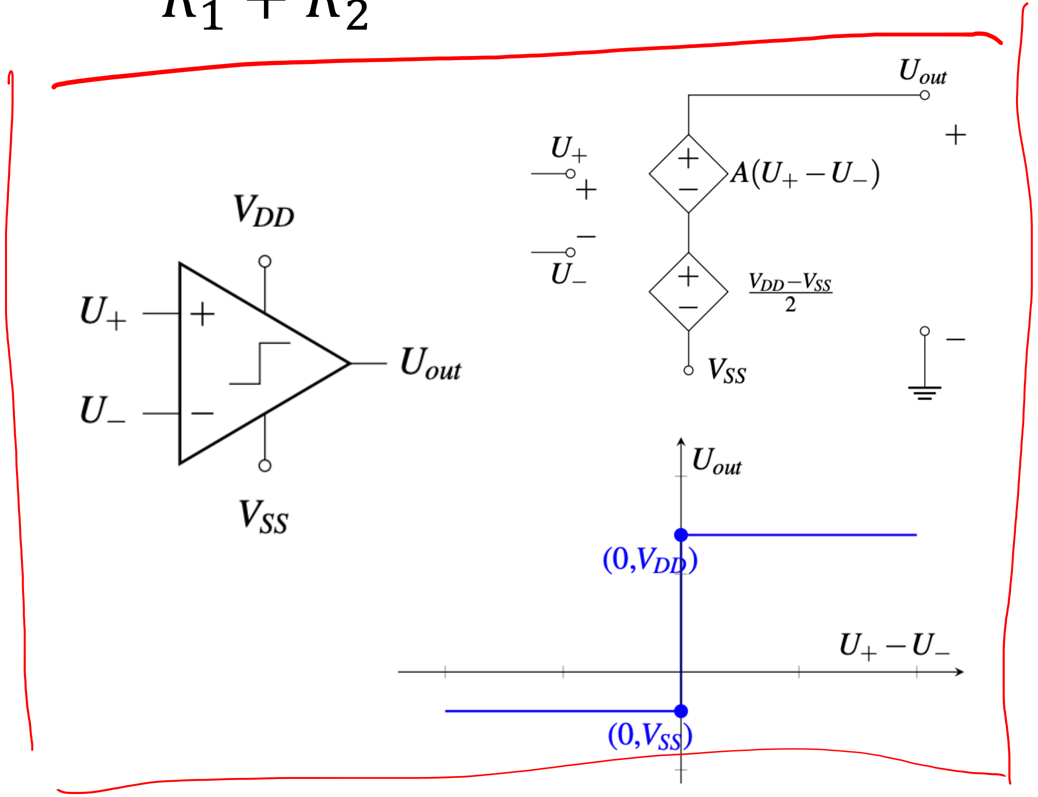
Measure V with open

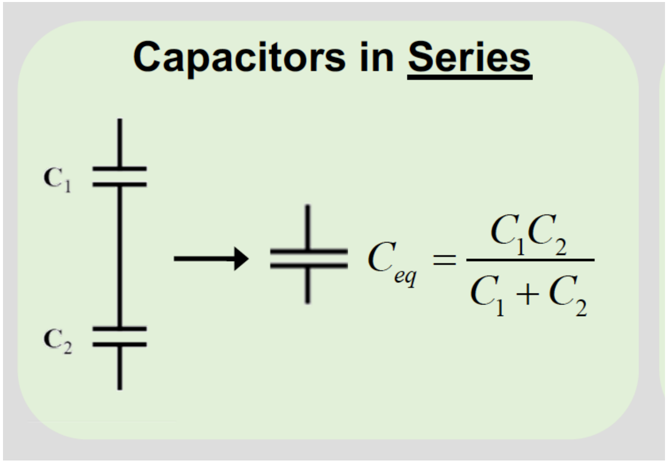
$R_{Th} = V_{Th}/I_N$

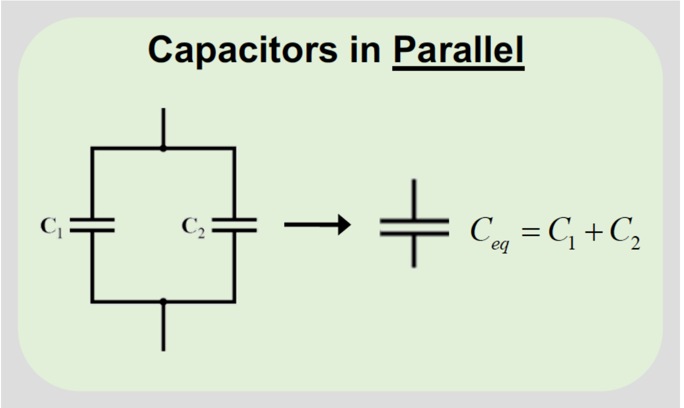
Norton Equivalent Circuit



Measure I with short

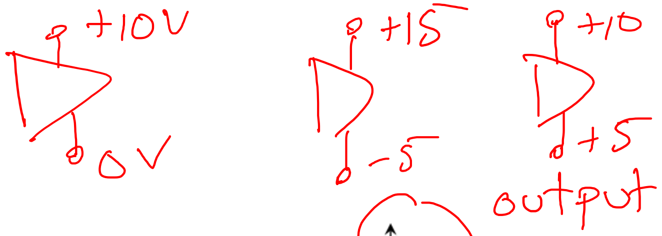


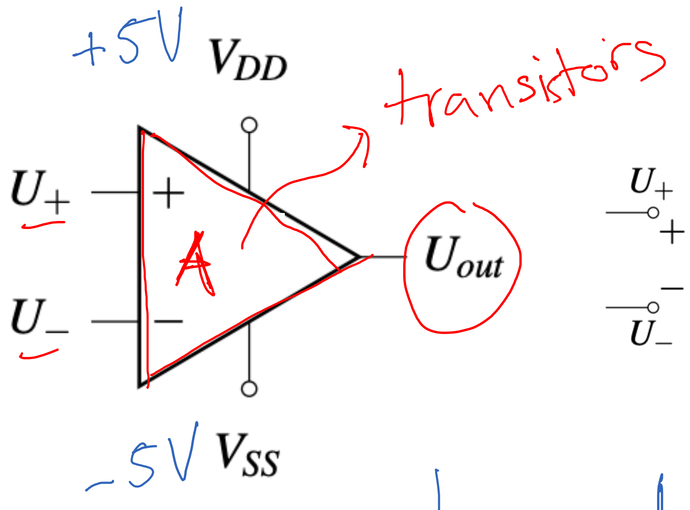


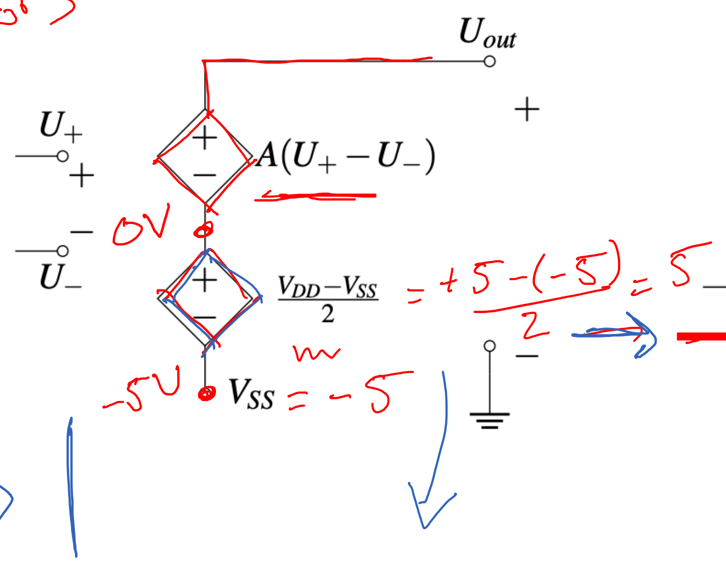


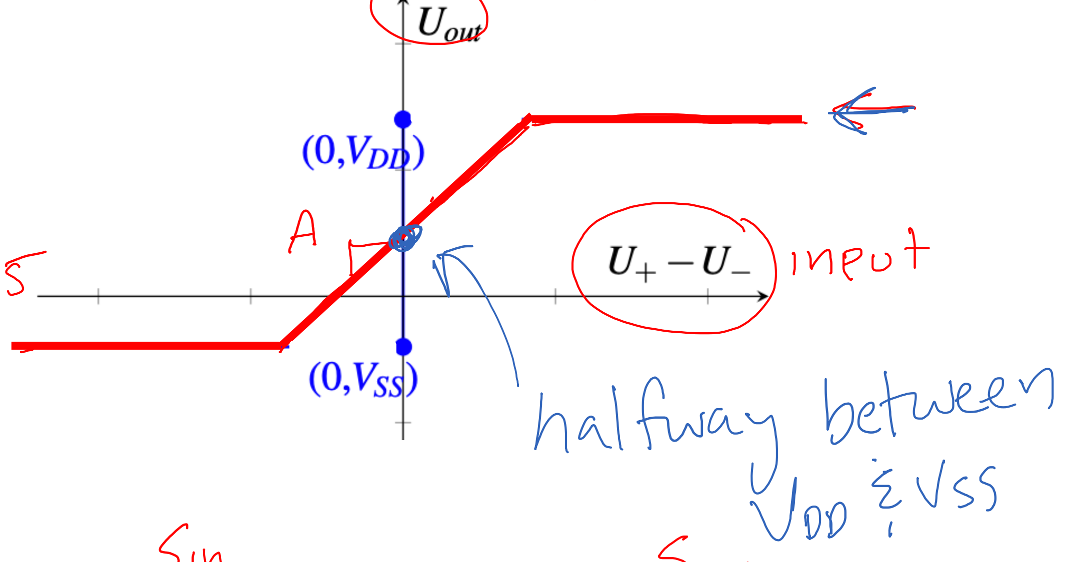
Recap: Op Amps

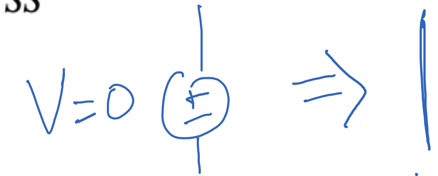
$V_{DD} > V_{SS}$

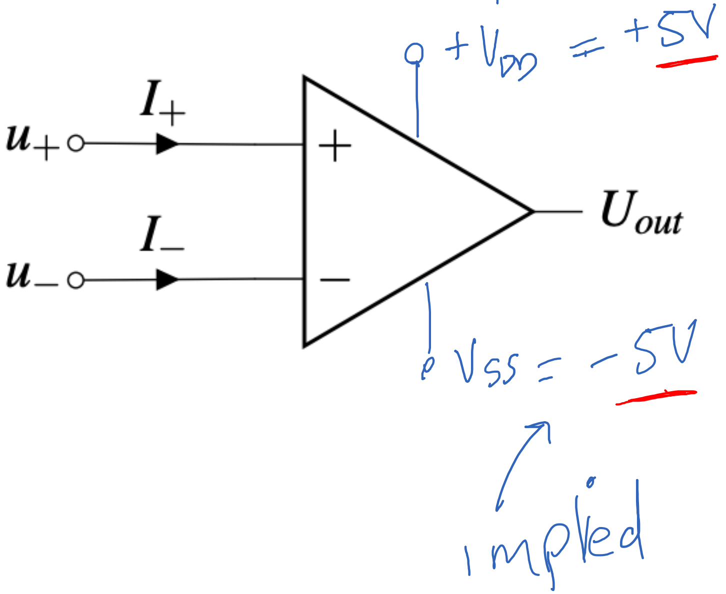




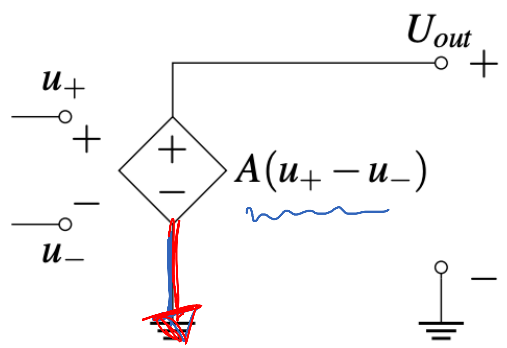


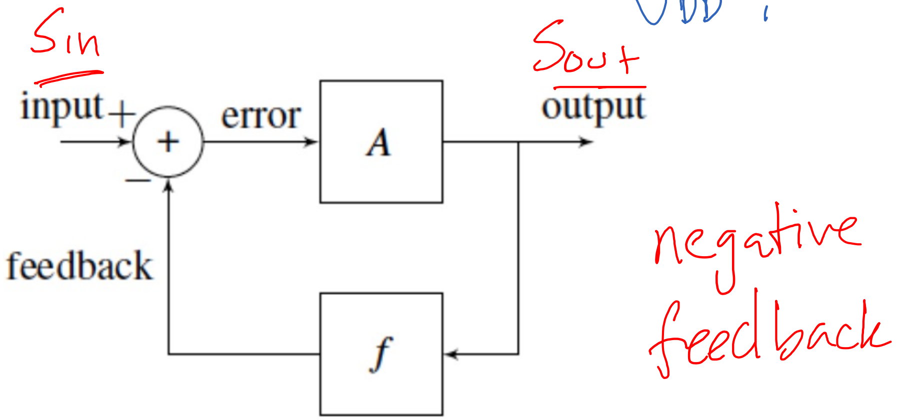






Simplified model:

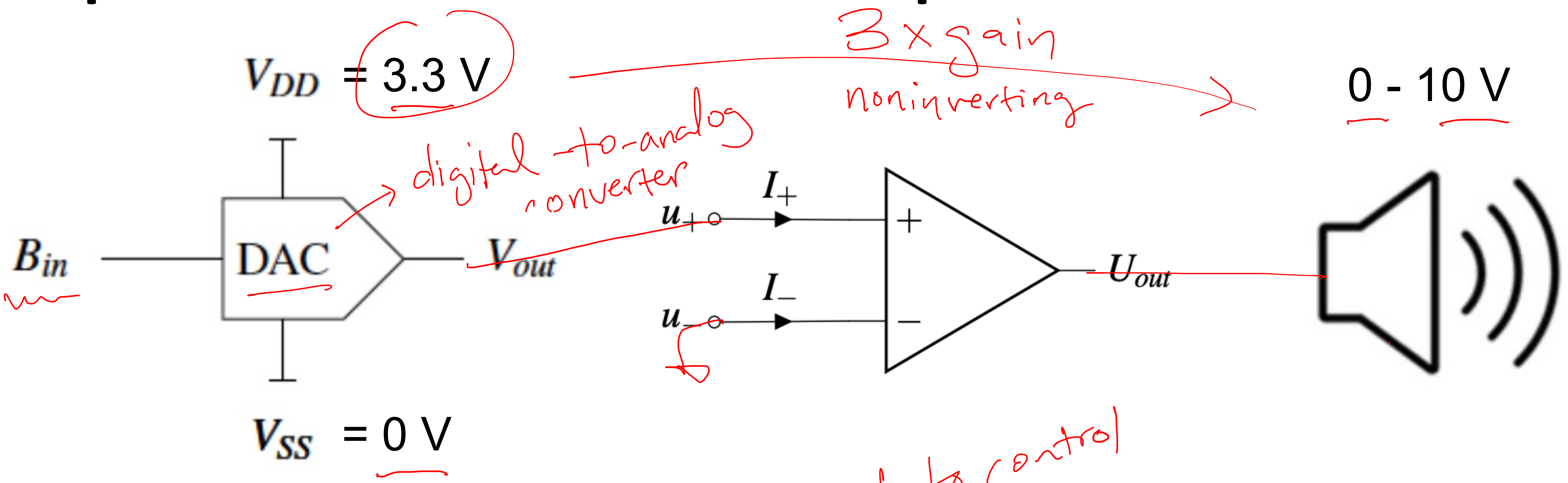


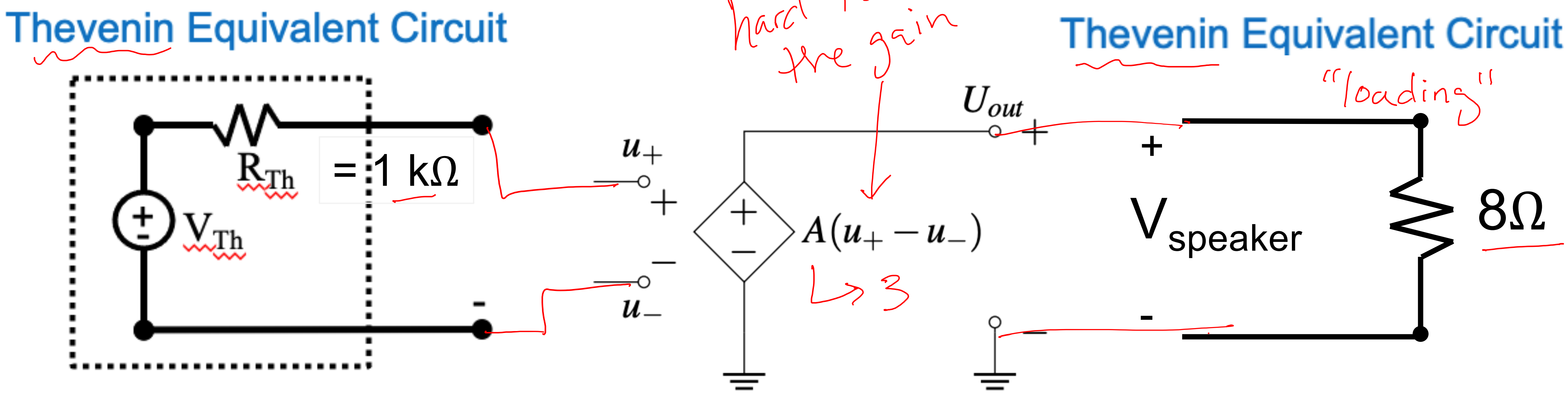


$$\boxed{\frac{S_{out}}{S_{in}} = \frac{A}{1 + Af}}$$

Black's formula

Recap: Need to Isolate DAC and Speaker!



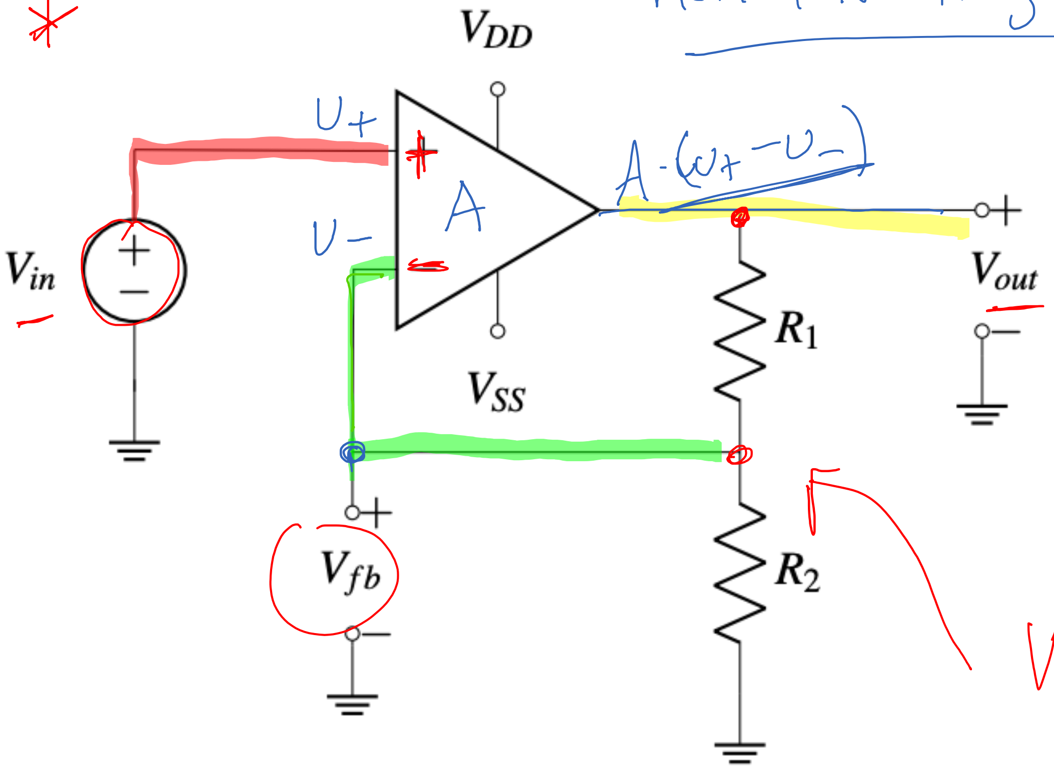


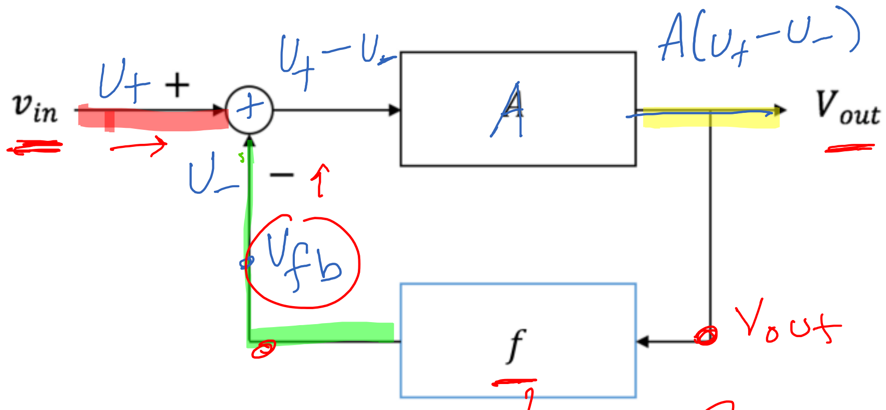
Recap: An Op Amp with Negative Feedback!

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A}{1+Af}}$$

non-inverting amplifier

controls the gain





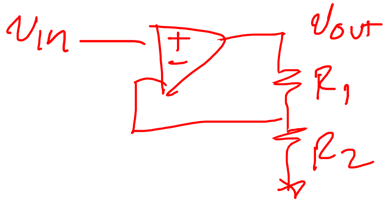
gain

$$V_{fb} = V_{out}\cdot\frac{R_2}{R_1+R_2}$$

$$f \rightarrow \frac{R_2}{R_1+R_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{\cancel{A}^{\infty}}{1+\cancel{A}^{\infty}\left(\frac{R_2}{R_1+R_2}\right)} = \frac{\cancel{A}}{\cancel{A}\left(\frac{R_2}{R_1+R_2}\right)} = \frac{R_1+R_2}{R_2} = \boxed{\frac{R_1}{R_2}+1}$$

Need to Gain and Isolation



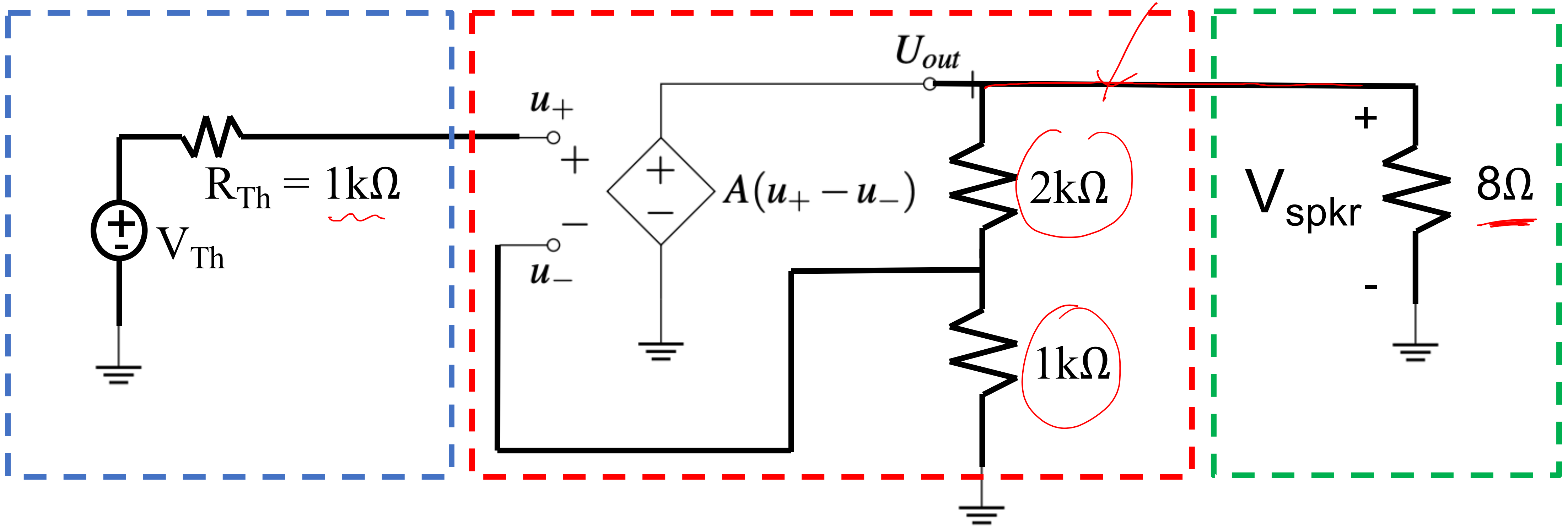
$$v_{out} = \left(\frac{R_1}{R_2} + 1\right) v_{in}$$

$$= \frac{2k}{1k} + 1 = 3!$$

DAC

Amplifier, Non-inverting Gain = 3

Speaker



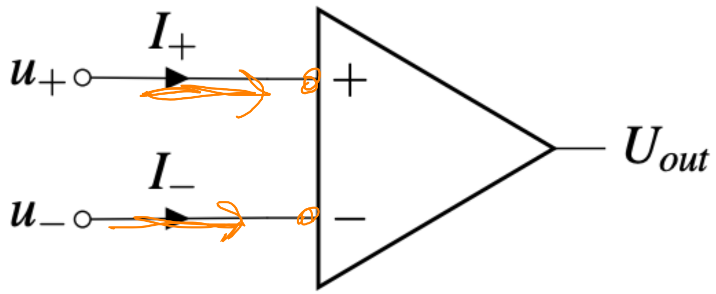
* attenuation

gain < 1

Why does this work? + amplifies

why is 8 Ω not attenuating?

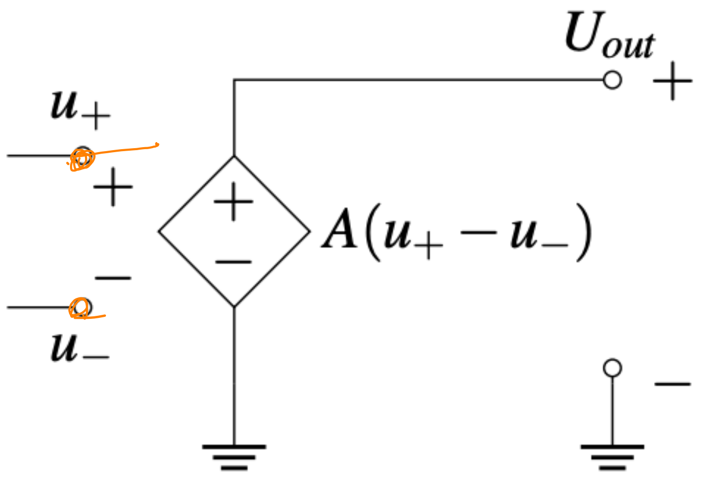
The Golden Rules of (Ideal) Op Amps #1



Inputs are open circuits

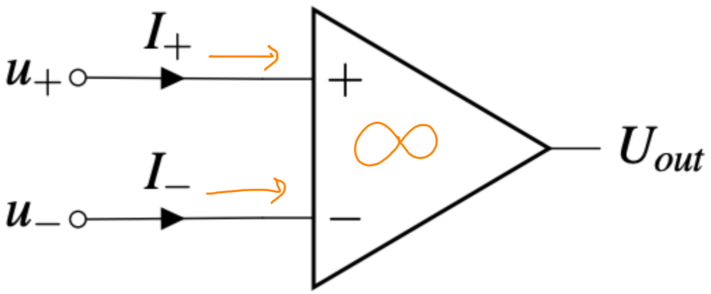
* $R_{in} = \infty$

#1: $I_+ = I_- = 0$

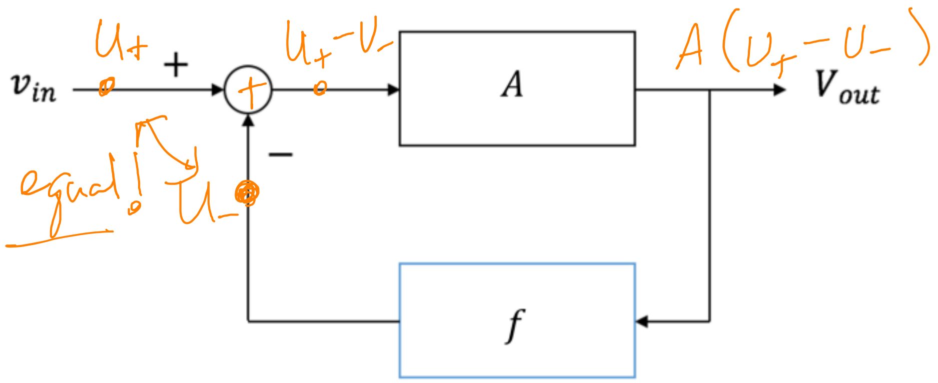


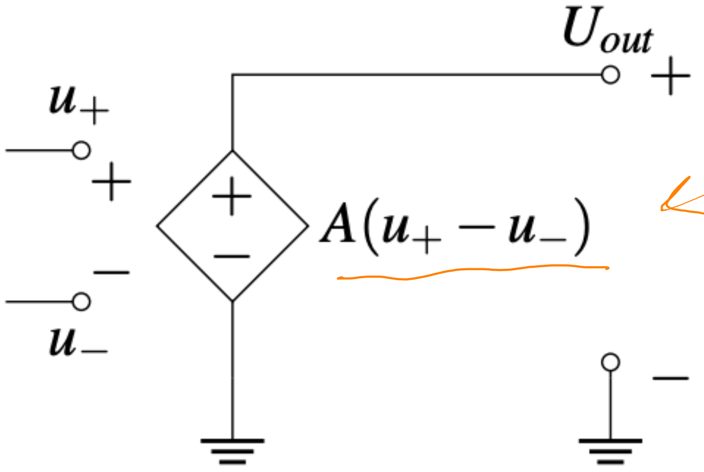
The Golden Rules of (Ideal) Op Amps #2

(applies to negative feedback)



The gain of an ideal op amp is $A = \infty$





$$A(u_+ - u_-) = U_{out}$$

$$(u_+ - u_-) = \frac{U_{out}}{A} \rightarrow \infty \Rightarrow 0$$

$$\Rightarrow (u_+ - u_-) = 0$$

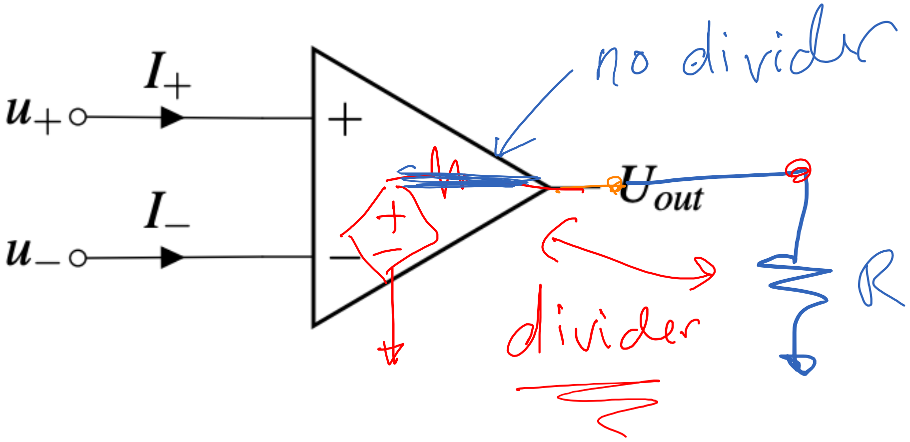
$$\boxed{\#2: \; \boxed{u_+ = u_-}}$$

Only in negative feedback and A= ∞

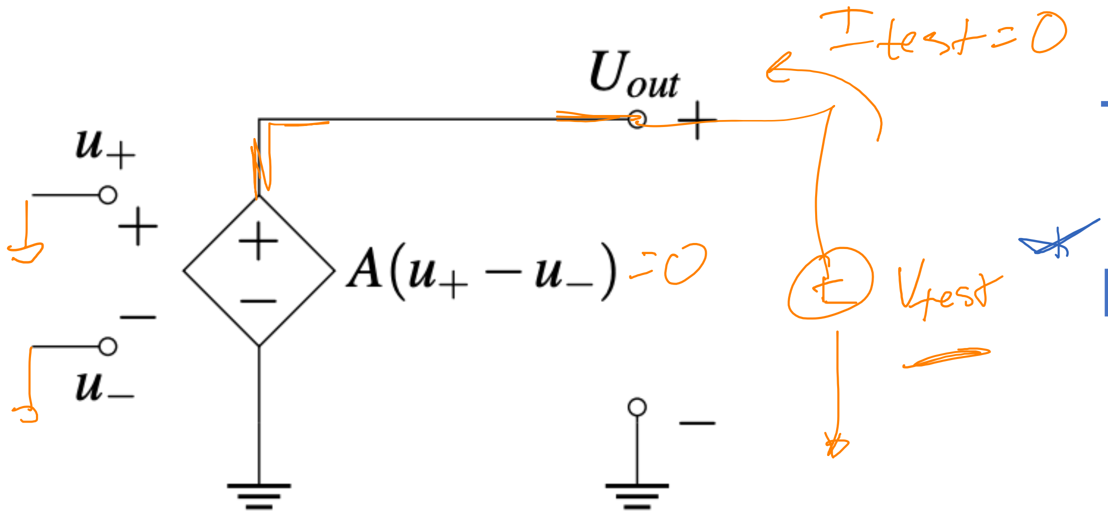
An Extra Attribute of (Ideal) Op Amps

Golden rules

1. $A = \infty$
2. $R_{out} = 0\ \Omega$



any output won't attenuate the signal



The output resistance of an op-amp is

$R_{out} = 0\ \Omega$ (like a short circuit)

$\infty \cdot 0 \rightarrow$ finite

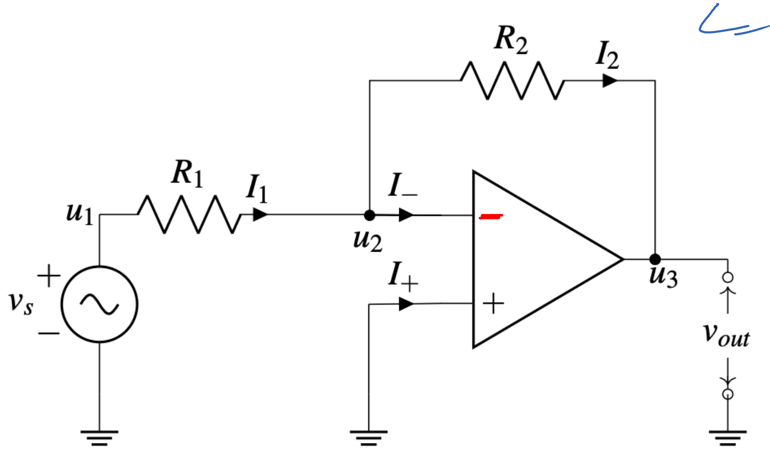
$$V_{out} = \overset{\to\infty}{A}\underbrace{(U_+ - U_-)}_{0} = 0$$

Useful Configurations

good for speaker

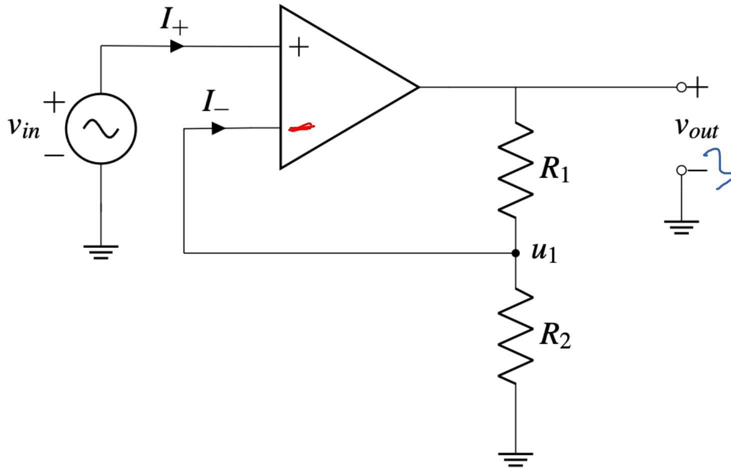
Trans-impedance Amplifier

* Inverting Amplifier



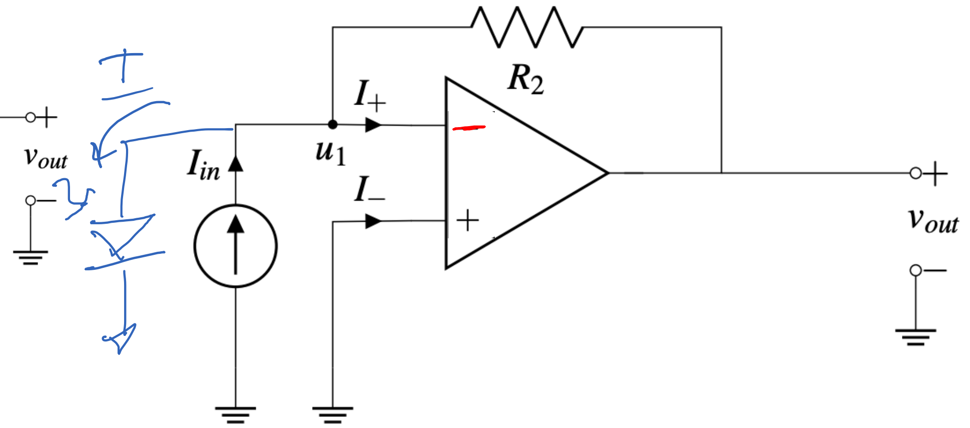
input → V
output → V
gain → −

* Non-inverting Amplifier



Input → V
output → V
gain → +

* Trans-resistance Amplifier

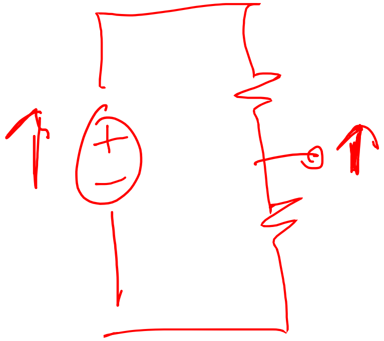


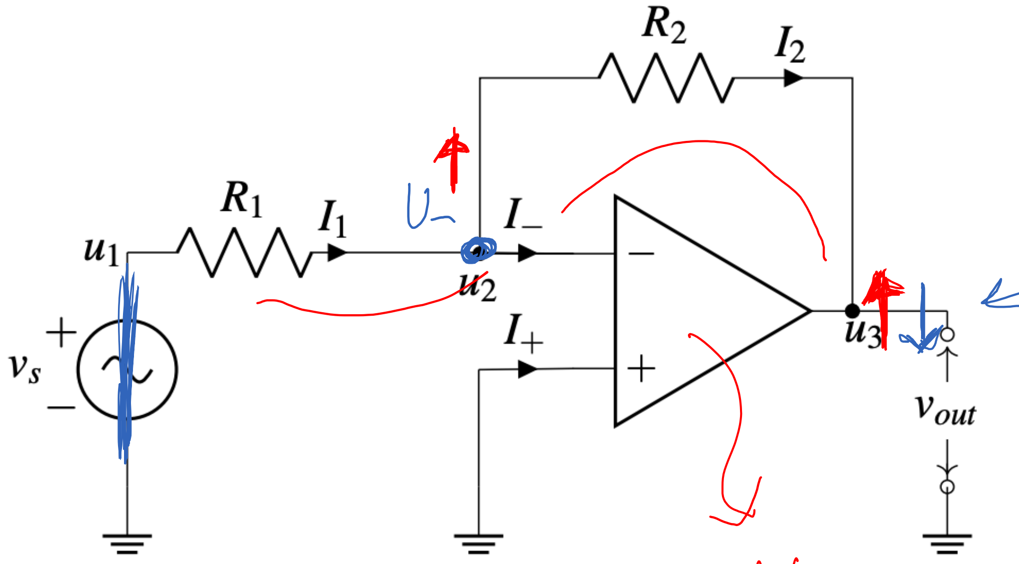
input → I
output → V
gain → −

Check for Negative Feedback

How do we check?

want negative feedback





in negative FB ✓

$V_{out} = A(U_+ - U_-)$

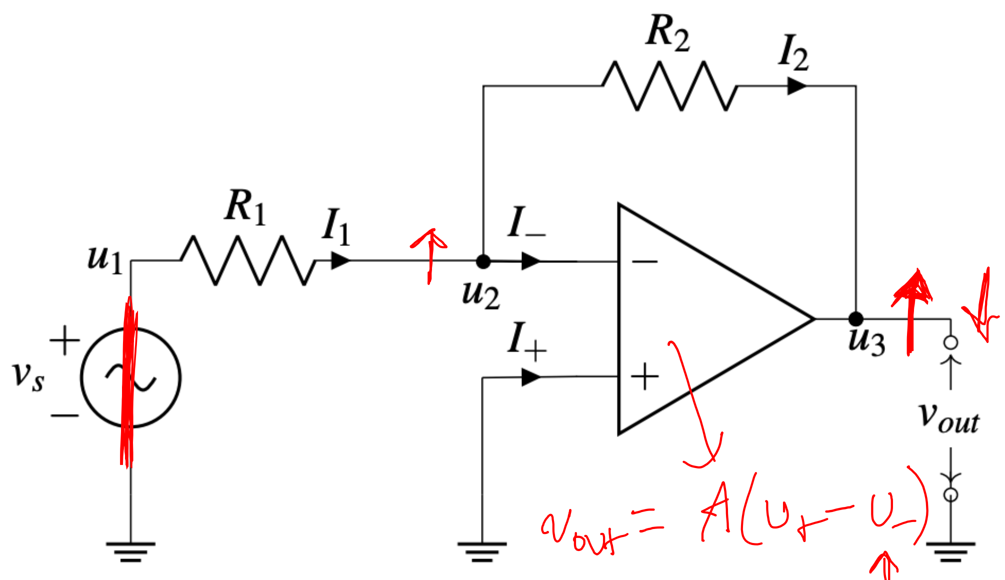
✓ Step 1: Turn off all independent sources

- *Voltage source becomes wire; Current source becomes open*

Step 2: Check how the feedback loop responds to a change in the ouput

- *Negative feedback: Increase output, loop tries to decrease it*
- *Positive feedback: Increase output, loop tries to increase it further*

Example 1: Inverting Amplifier



$v_{out} = A(u_+ - u_-)$

✓ NF

#1: $I_+ = I_- = 0$

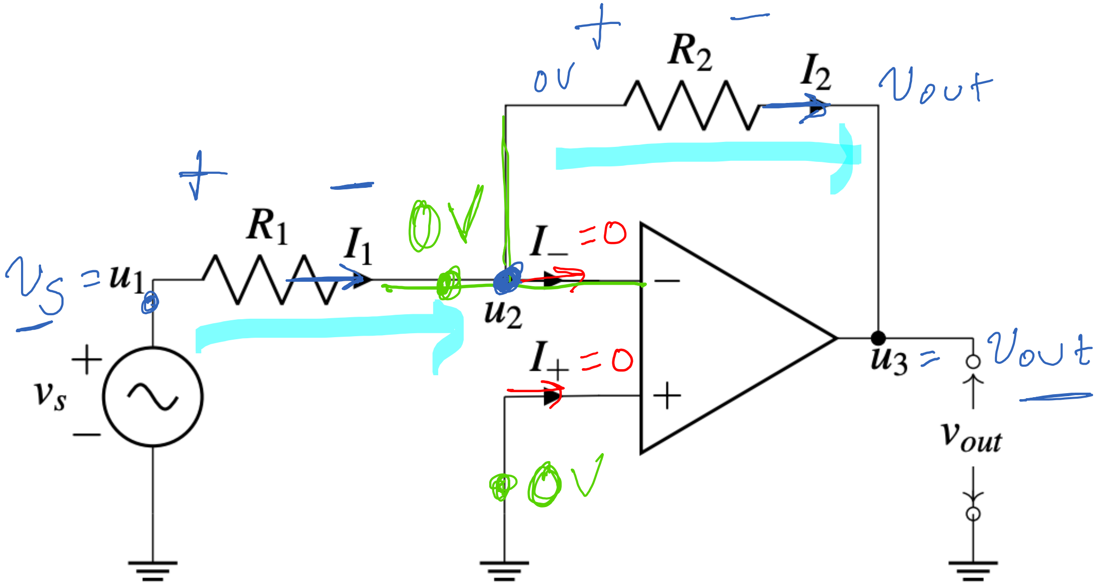
#2: $u_+ = u_-$

Only in negative feedback and A= ∞

✓ $A = \infty$

✓ $R_{out} = 0$

Example 1: Inverting Amplifier



#1: $I_+ = I_- = 0$

#2: $u_+ = u_-$

Only in negative feedback and A= ∞

Looking for:

$V_{out} = \alpha V_S$

$\frac{V_{out}}{V_S} = \alpha \; [V/V]$

$A = \infty$

$R_{out} = 0$

KCL: $I_1 = I_2 + \cancel{I_-}^{\,0}$

① $I_1 = I_2$

Ohm's law: $\frac{V_S - 0V}{R_1} = I_1$ $\quad \frac{0V - V_{out}}{R_2} = I_2$

② $\frac{V_S}{R_1} = I_1$ $\quad -\frac{V_{out}}{R_2} = I_2$

$\frac{V_S}{R_1} = \frac{-V_{out}}{R_2}$

$V_{out} = -\frac{R_2}{R_1} \cdot V_S$

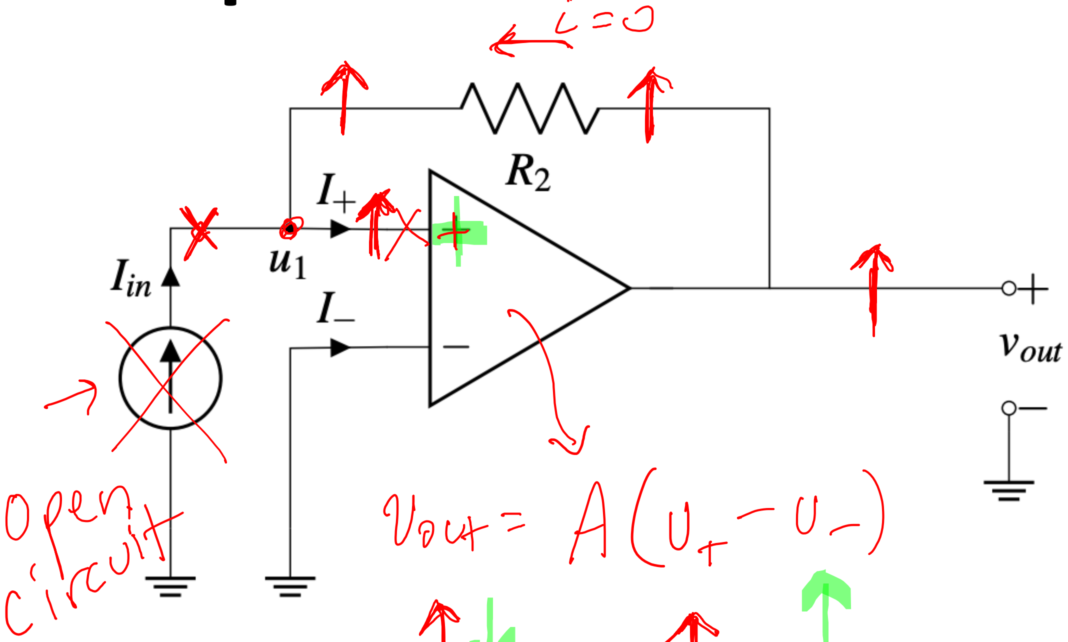
$\boxed{\frac{V_{out}}{V_S} = -\frac{R_2}{R_1}}$ *

Example 2: Trans-resistance Amplifier

#1: $I_+ = I_- = 0$

#2: $u_+ = u_-$

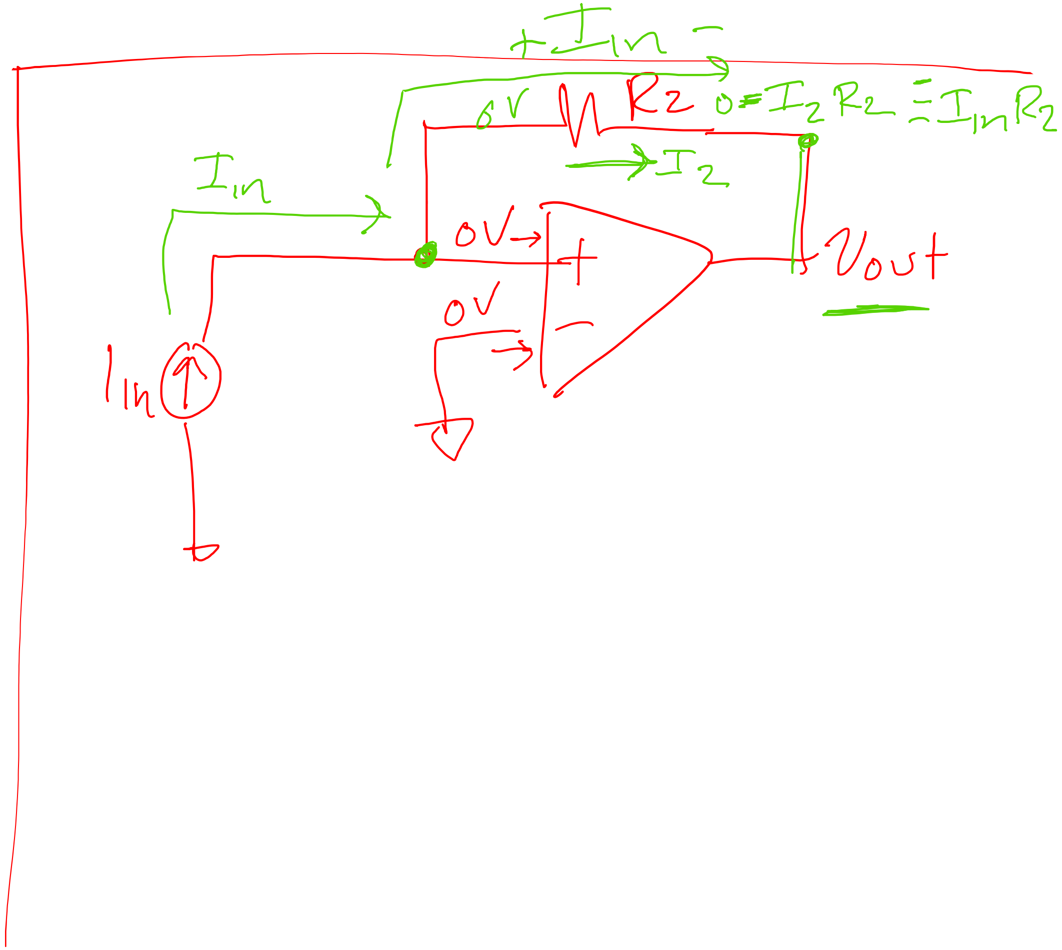
Only in negative feedback and A= ∞



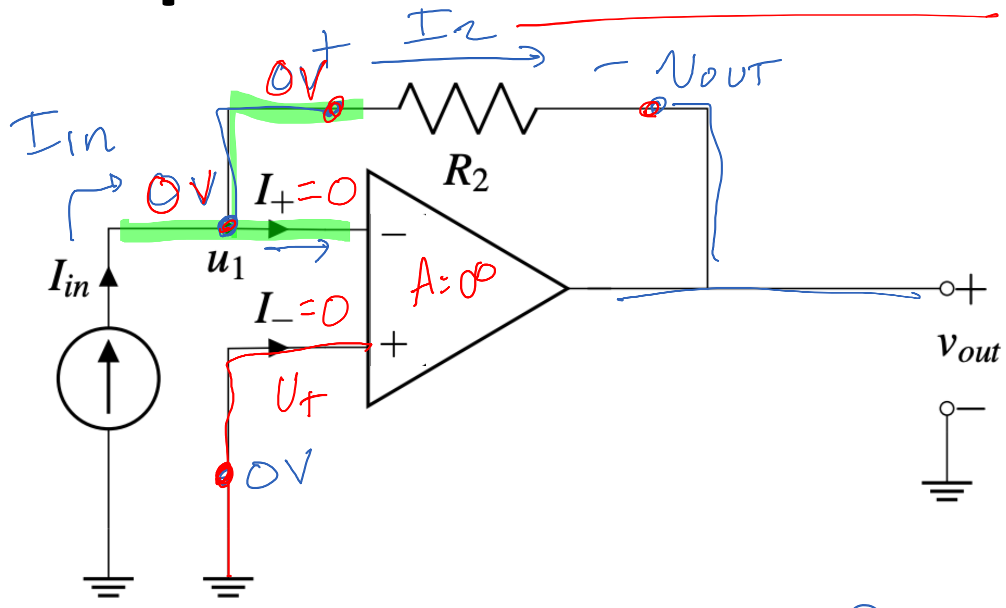
$V_{out} = A(U_+ - U_-)$

positive feedback!

doesn't work!!!



Example 2: Trans-resistance Amplifier



#1: $I_+ = I_- = 0$

#2: $u_+ = u_-$

Only in negative feedback and A= ∞

looking for:

$V_{out} = \alpha I_{in}$

$\frac{V_{out}}{I_{in}} = \alpha \quad \left[\frac{V}{A}\right] = [\Omega] \leftarrow$ resistance!

KCL: $I_{in} = I_2 + \cancel{I_+}^{\,0}$

① $I_{in} = I_2$

Ohm's Law: $\Rightarrow \frac{0V - V_{out}}{R_2} = I_2$

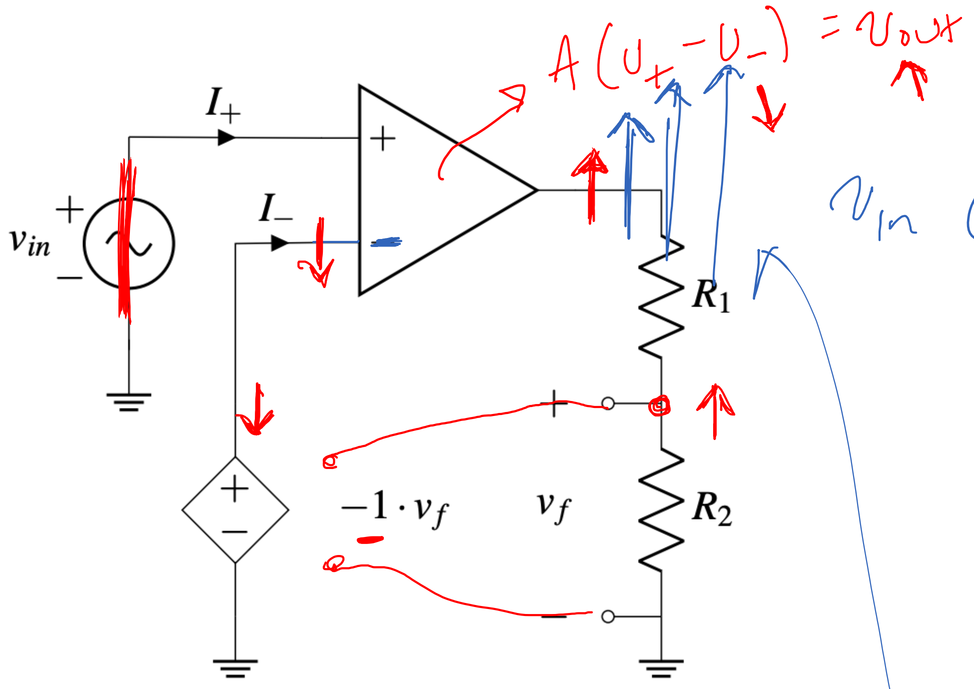
② $\frac{-V_{out}}{R_2} = I_2$

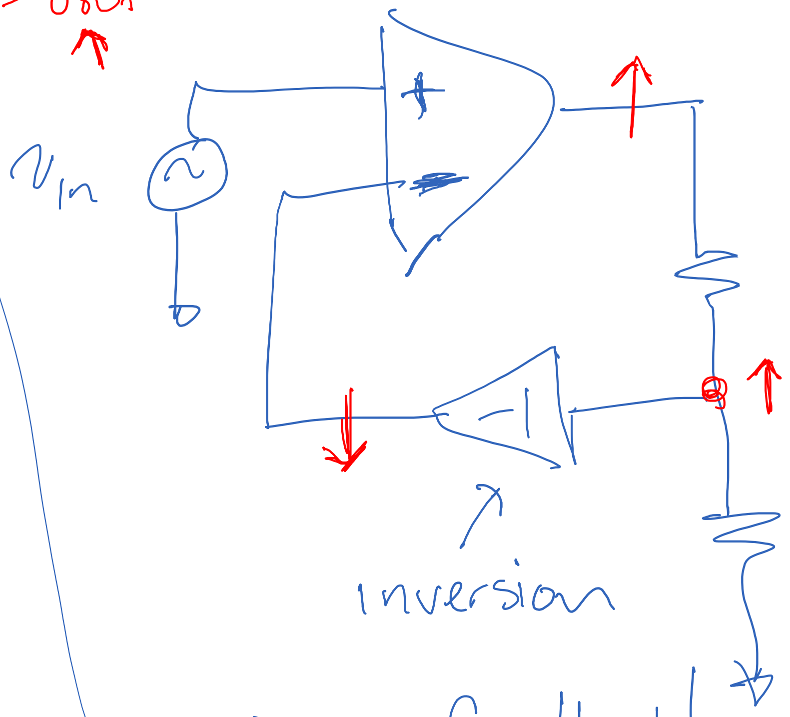
$\frac{-V_{out}}{R_2} = I_2 = I_{in}$

$V_{out} = -R_2 I_{in}$

$$\boxed{\frac{V_{out}}{I_{in}} = -R_2}$$

Example 3: Non-Inverting Amplifier





positive feedback!
doesn't work

#1: $I_+ = I_- = 0$

#2: $u_+ = u_-$

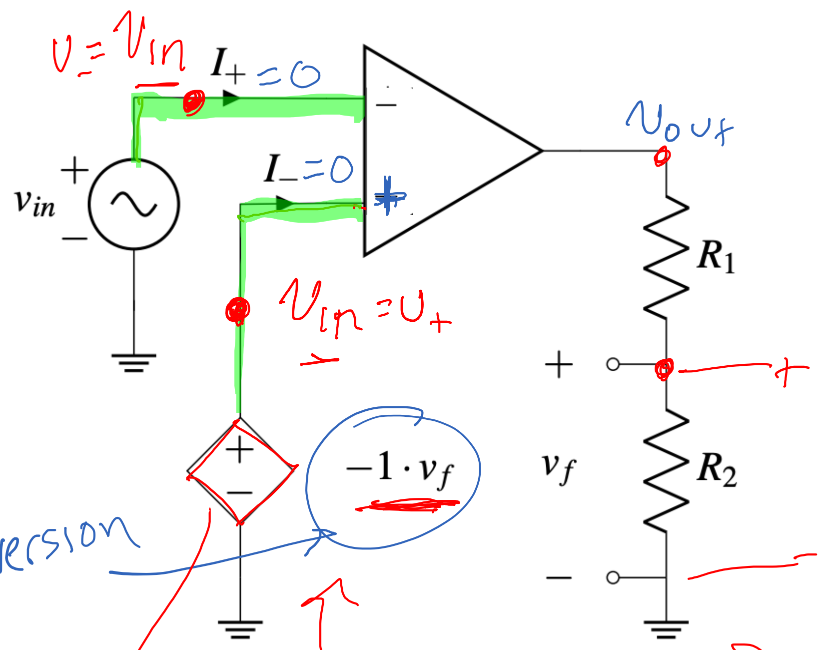
Only in negative feedback and A= ∞

Example 3: Non-Inverting Amplifier ?

#1: $I_+ = I_- = 0$

#2: $u_+ = u_-$

Only in negative feedback and A= ∞



Want $v_{out} = \alpha v_{in}$

or $\dfrac{v_{out}}{v_{in}} = \alpha$

$-1\, v_f = v_{in}$

$v_f = -v_{in}$

$-1 \cdot v_f = v_{in}$

$v_f = -v_{in}$

$v_f = -v_{in}$

Voltage divider

$\dfrac{v_{out} \cdot R_2}{R_1 + R_2} = v_f = -v_{in}$

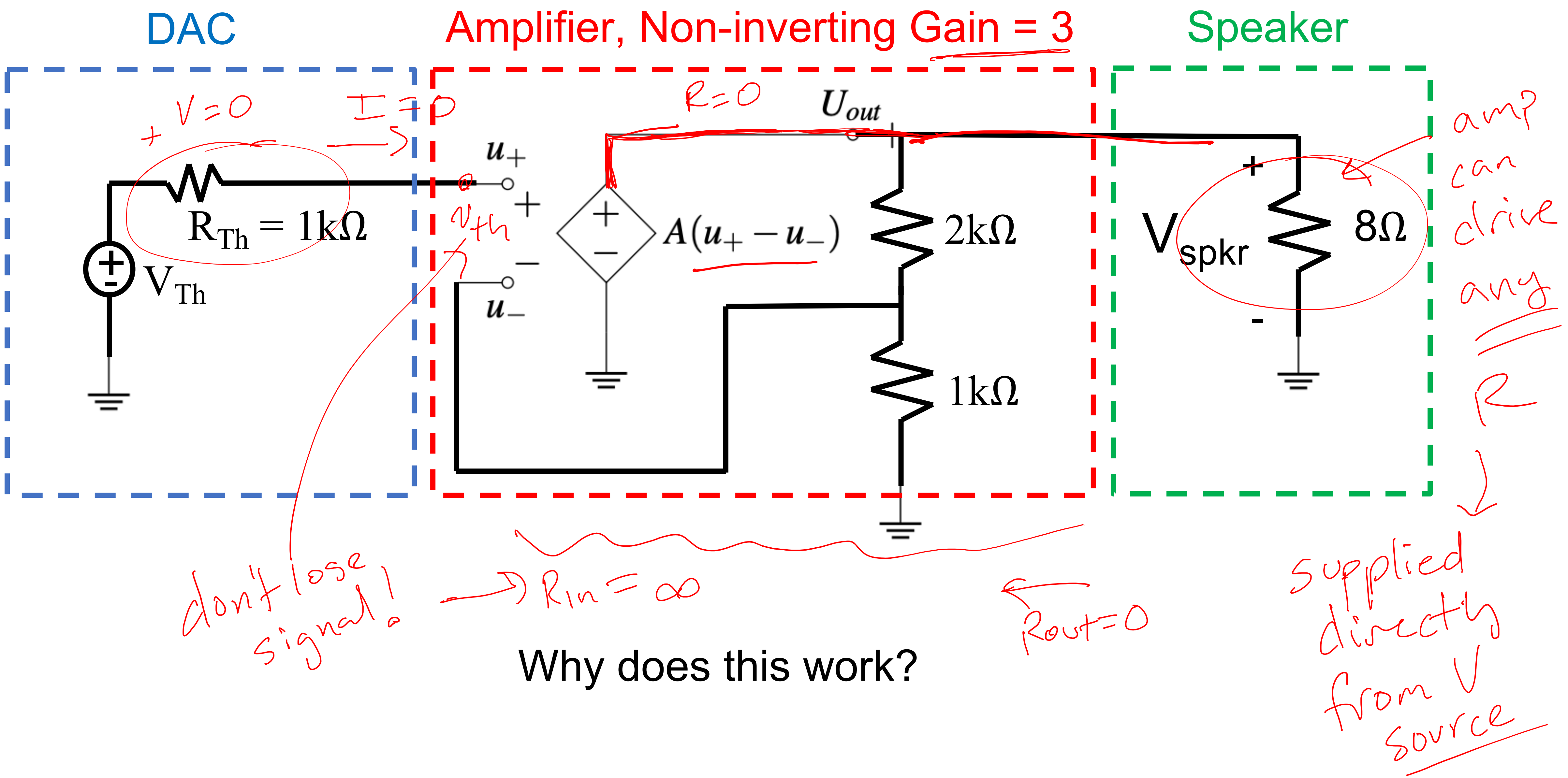
$\dfrac{v_{out} \cdot R_2}{R_1 + R_2} = -v_{in}$

$v_{out} = -\dfrac{R_2 + R_1}{R_2} \cdot v_{in}$

$\dfrac{v_{out}}{v_{in}} = -\left(1 + \dfrac{R_1}{R_2}\right)$

Need to Gain and Isolation

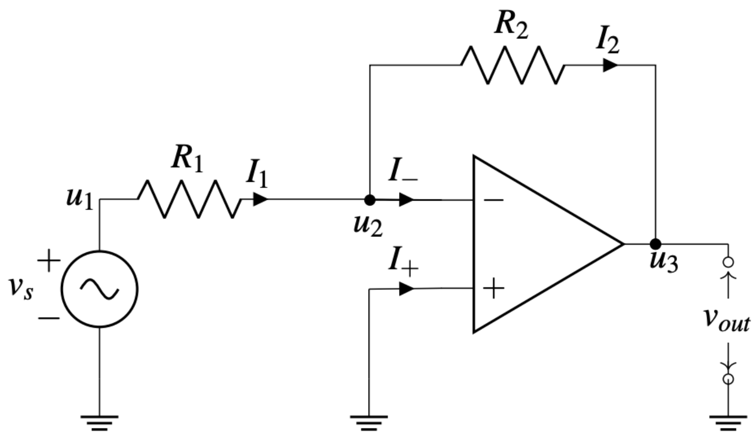
No attenuation



Why does this work?

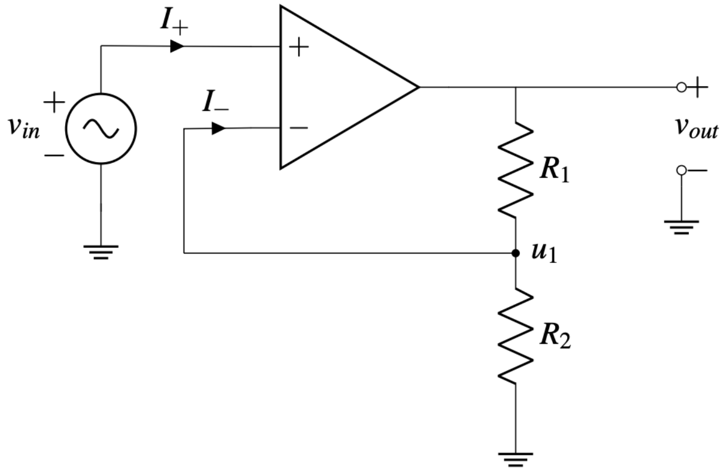
Summary of Useful Configurations

Inverting Amplifier



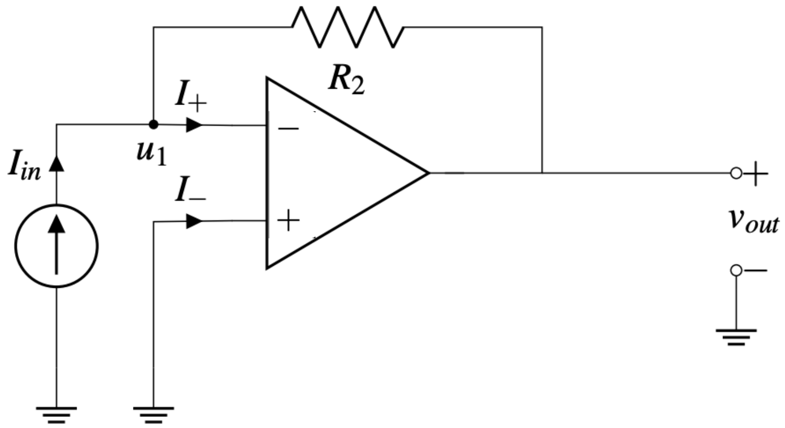
$$v_{out} = -\frac{R_2}{R_1} \cdot v_{in}$$

Non-inverting Amplifier



$$v_{out} = (1 + \frac{R_1}{R_2}) \cdot v_{in}$$

Trans-resistance Amplifier



$$v_{out} = -R_2 \cdot I_{in}$$