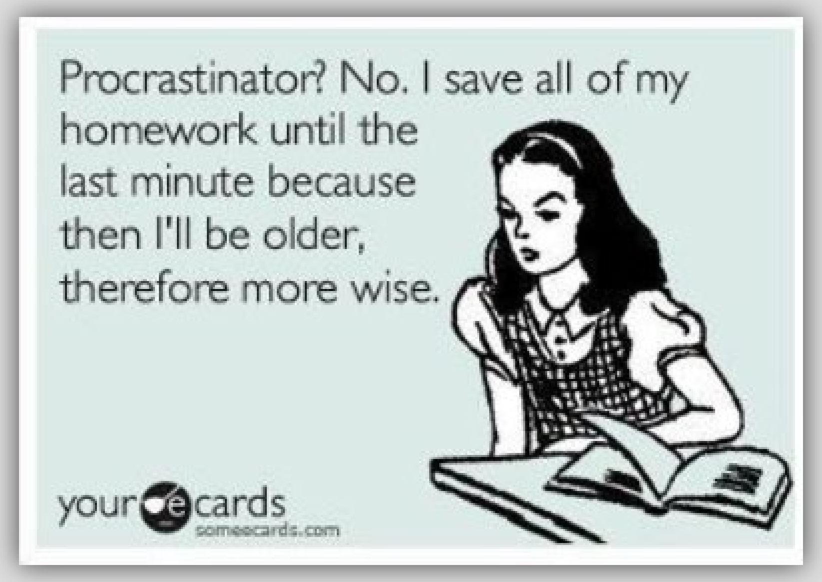

# EECS 16A

Correlations

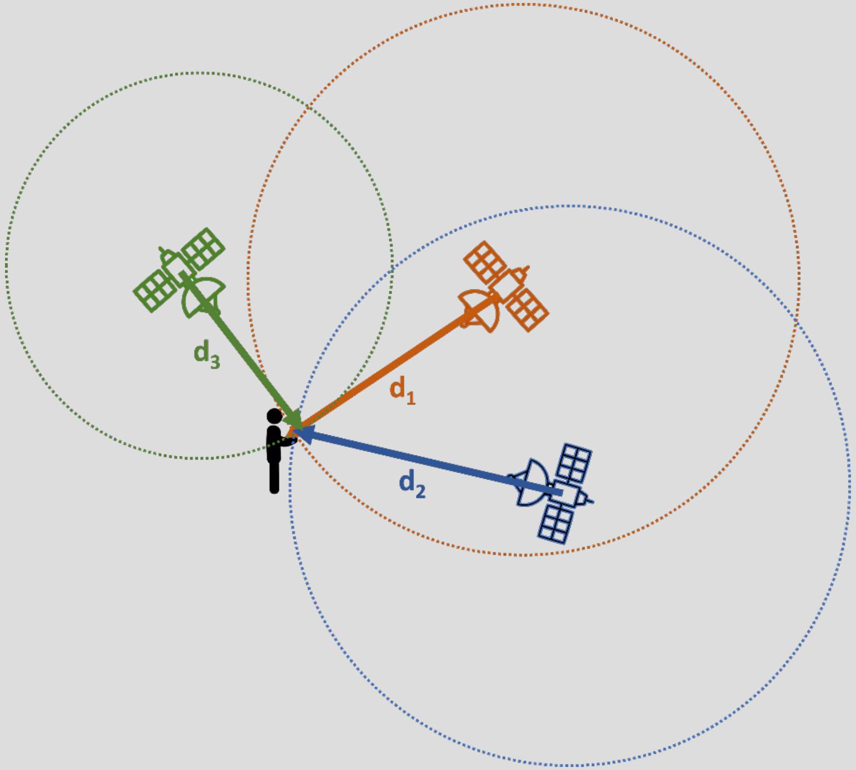
# Admin

- MT2 is on MONDAY!

ARE YOU READYYYYY????

# GPS positioning uses distances from satellites

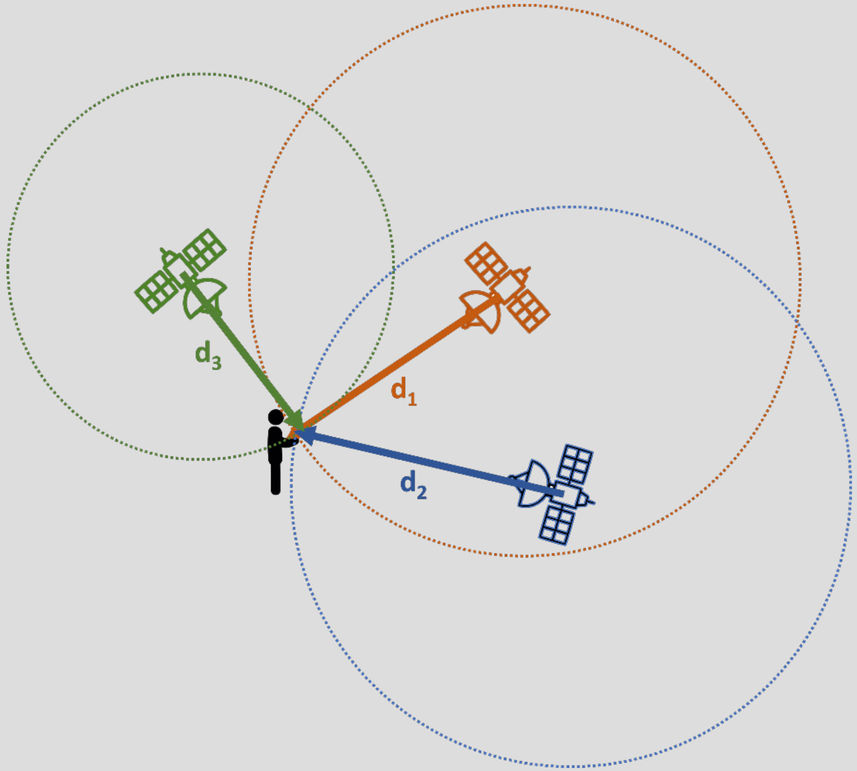



How does knowing distances to satellites tell me my position?

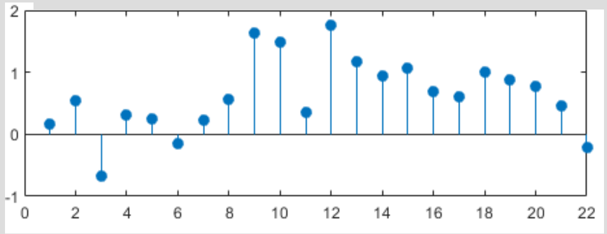
How can I measure those distances?

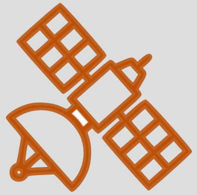
How many satellites to I need?

# Last time: Classification – which satellite is 'on'?

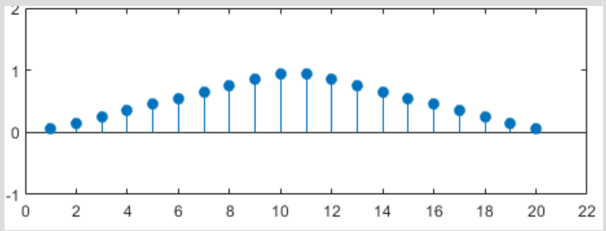


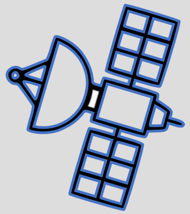

Transmitting signal $\vec{s}_A$



Transmitting signal $\vec{s}$:



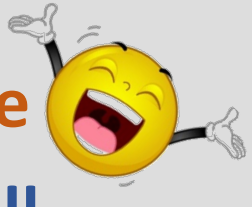
Received signal $\vec{r}$

$\vec{r} =$

Take inner product of the received signal and each transmitted signal:

$\langle \vec{r}, \vec{s}_A \rangle =$ large

$\langle \vec{r}, \vec{s}_B \rangle =$ small



# Inner Product

- Provide a measure of "similarity" between vectors
- (Euclidian) inner product is also called 'dot product'

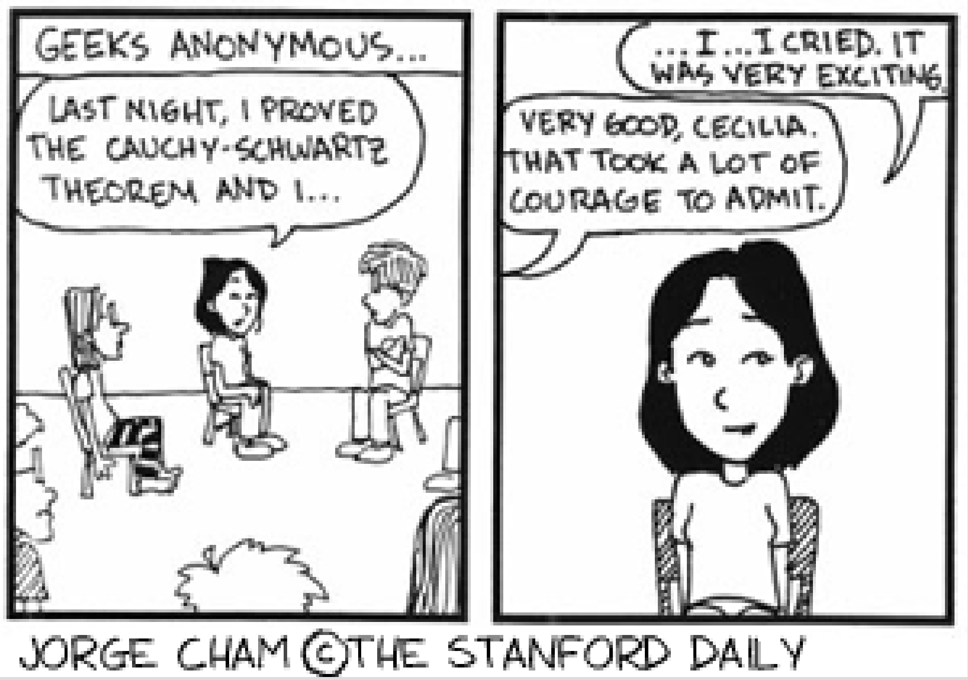
For two vectors $\vec{x}, \vec{y} \in \mathbb{R}^n$, the inner product is:

$$< \vec{\boldsymbol{x}}, \vec{\boldsymbol{y}} >= \vec{\boldsymbol{x}}^{\boldsymbol{T}} \vec{\boldsymbol{y}}$$

$$= x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots x_n y_n$$

$$= \sum_{i=1}^{n} x_i y_i$$

$$= \|\vec{x}\| \|\vec{y}\| \cos\theta$$

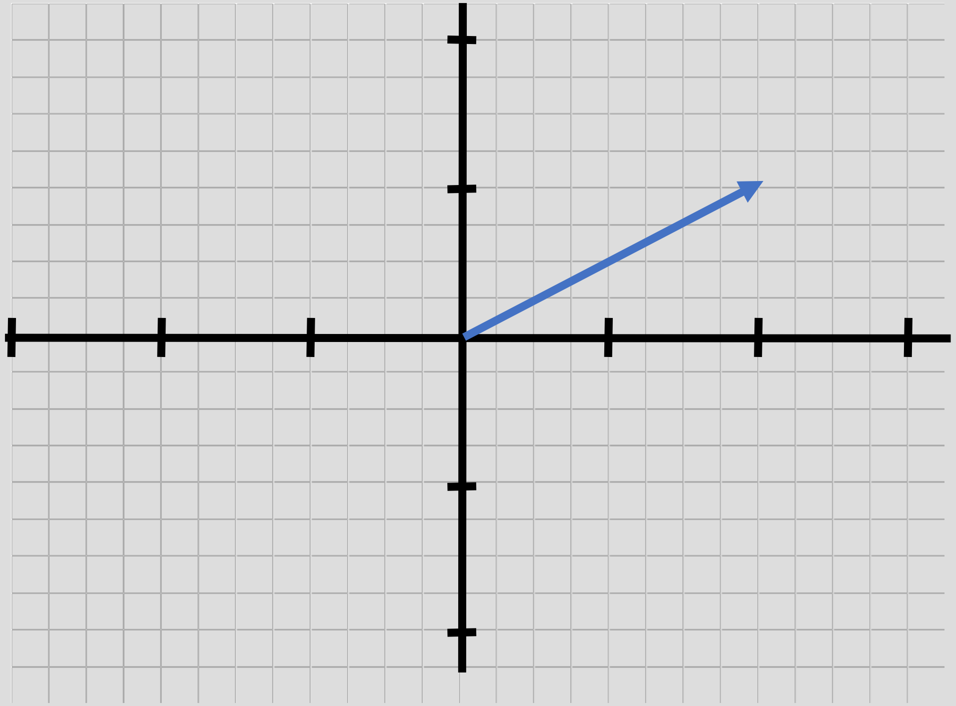


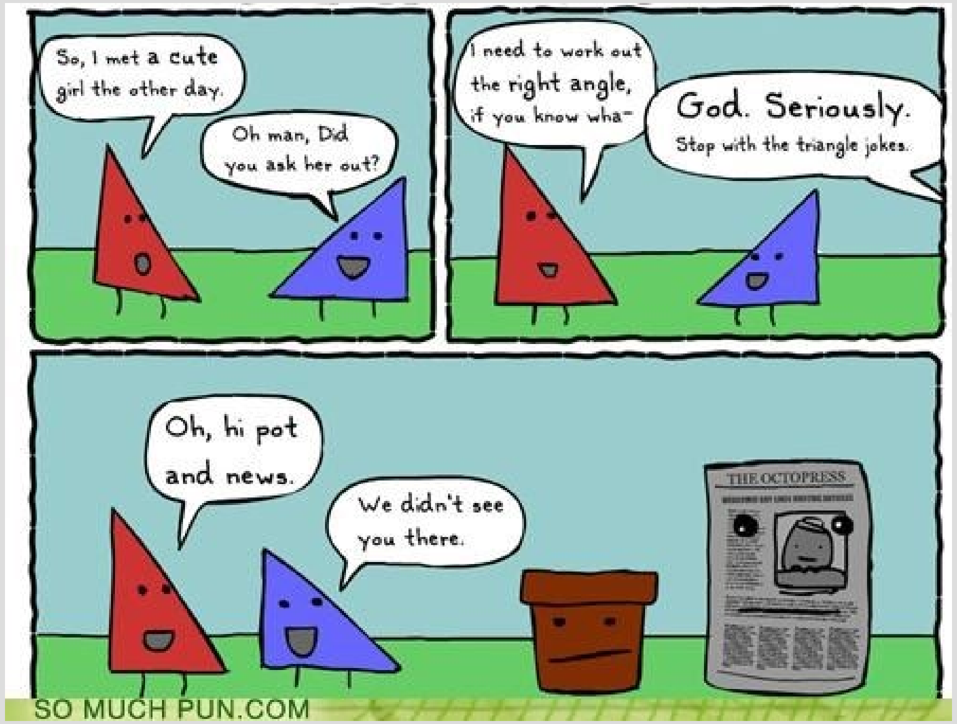

**Cauchy-Schwartz Inequality:** $\langle \vec{x}, \vec{y} \rangle \leq \|\vec{x}\| \|\vec{y}\|$

# Norm

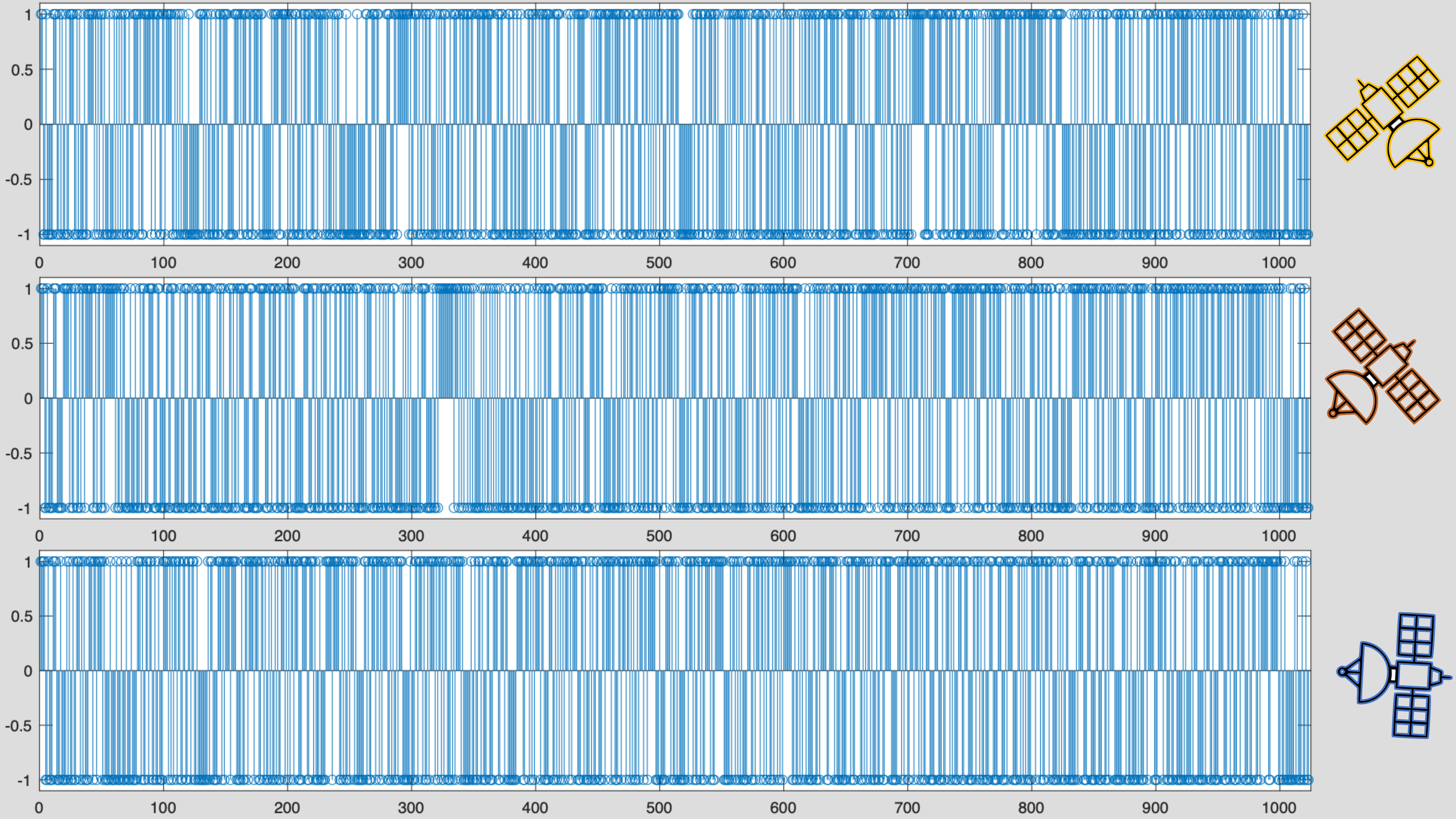
- Provides a measure of “length” of elements in the vector space

$$||\vec{v}|| = \sqrt{< \vec{v}, \vec{v} >}$$



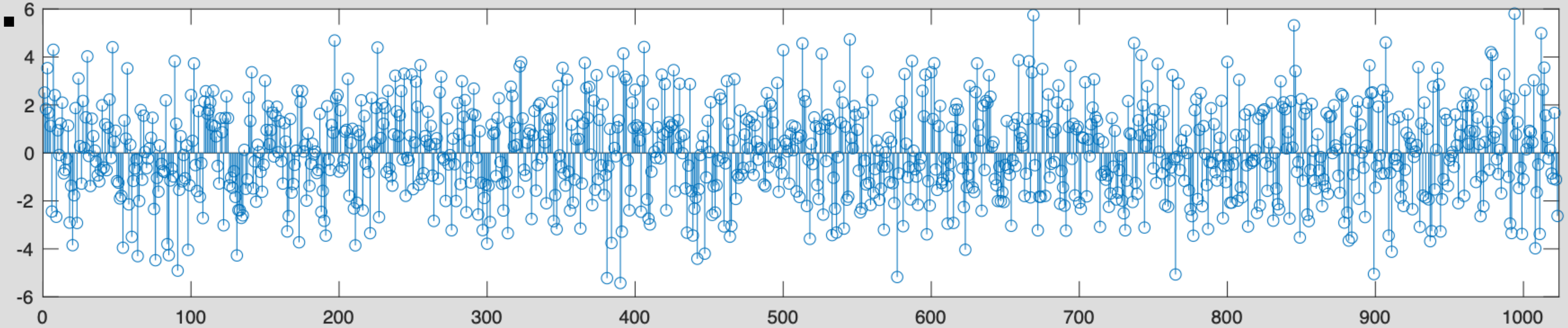


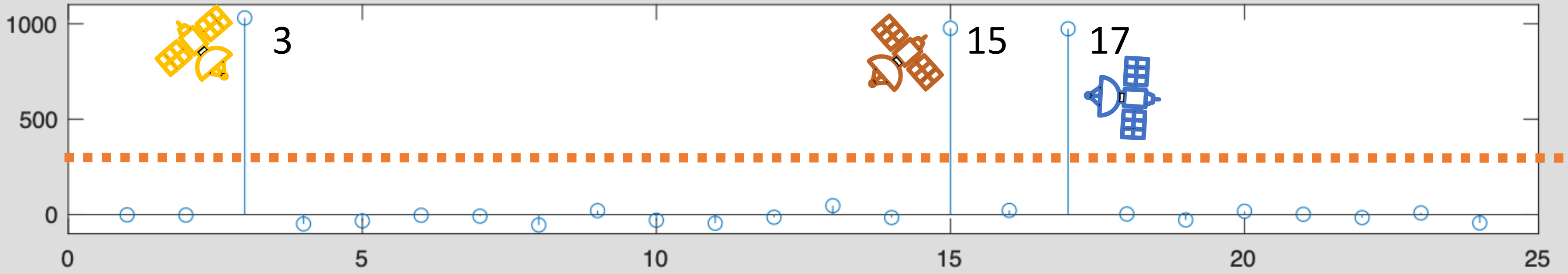
# GPS Gold Codes



# Example:

$\vec{r} =$



# What’s next?

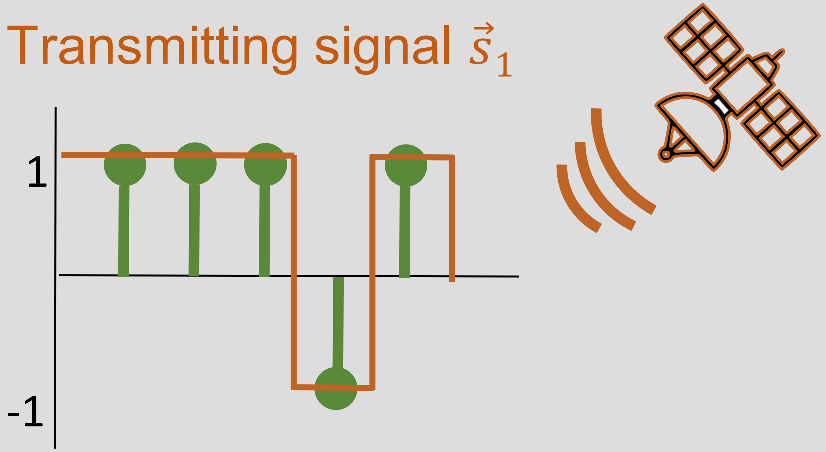
Now I know which satellites are ‘on’.
Next I need to figure out my distance from each.

How?

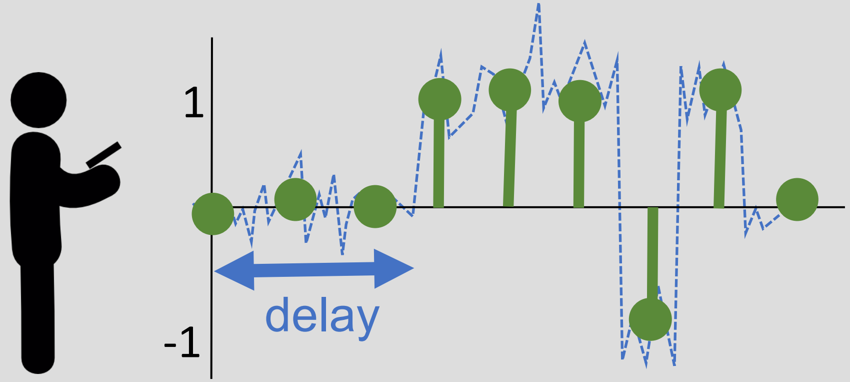
# Timing: how far away is the satellite?

- Satellites transmit a unique code (radio signal )
- Signal is received and digitized by a receiver

Transmitting signal $\vec{s}_1$

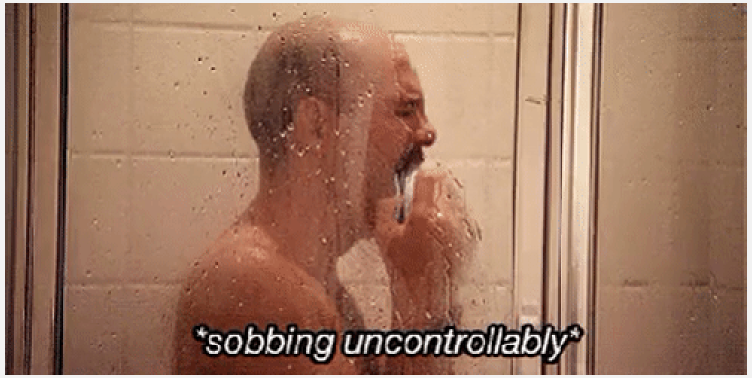



Received signal $\vec{r}$



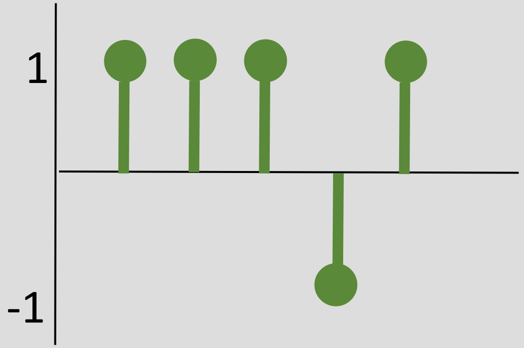

distance = $3[\text{ms}] \times C \approx 900[\text{km}]$

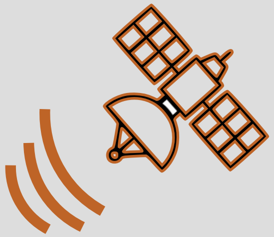
**Calculate the inner product of $\vec{r}$ and $\vec{s}$?**



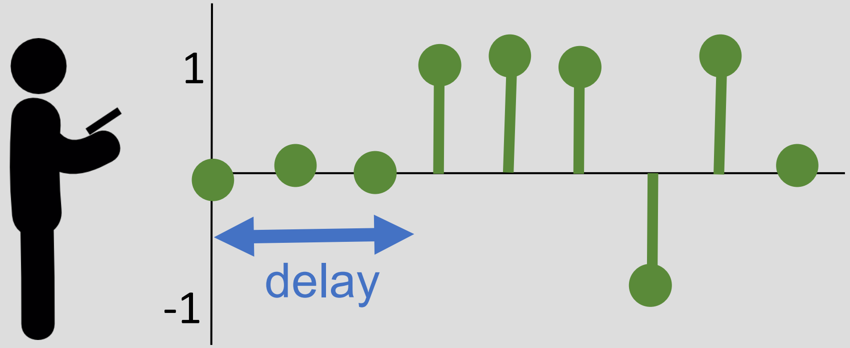
# Timing: how far away is the satellite?

Transmitting signal $\vec{s}_1$



Received signal $\vec{r}$

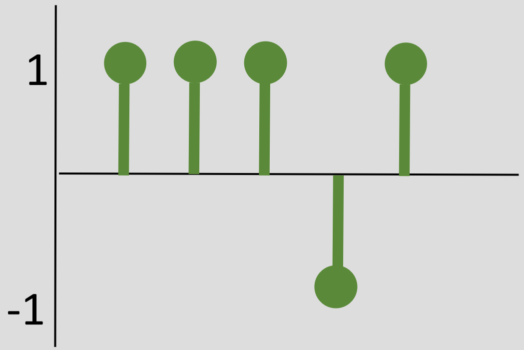


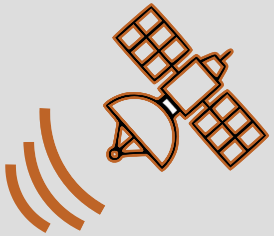

distance = $3[\mathrm{ms}] \times C \approx 900[\mathrm{km}]$

**How can I calculate what the delay/shift is?**

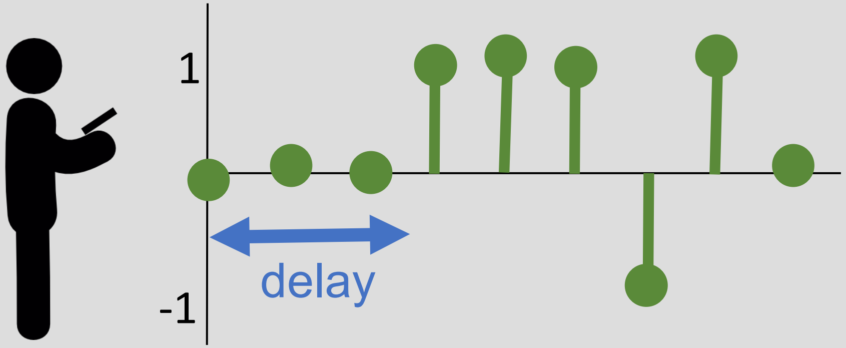
# How can I figure out what the shift is?

Transmitting signal $\vec{s}_1$



Received signal $\vec{r}$




distance = $3[\text{ms}] \times C \approx 900[\text{km}]$

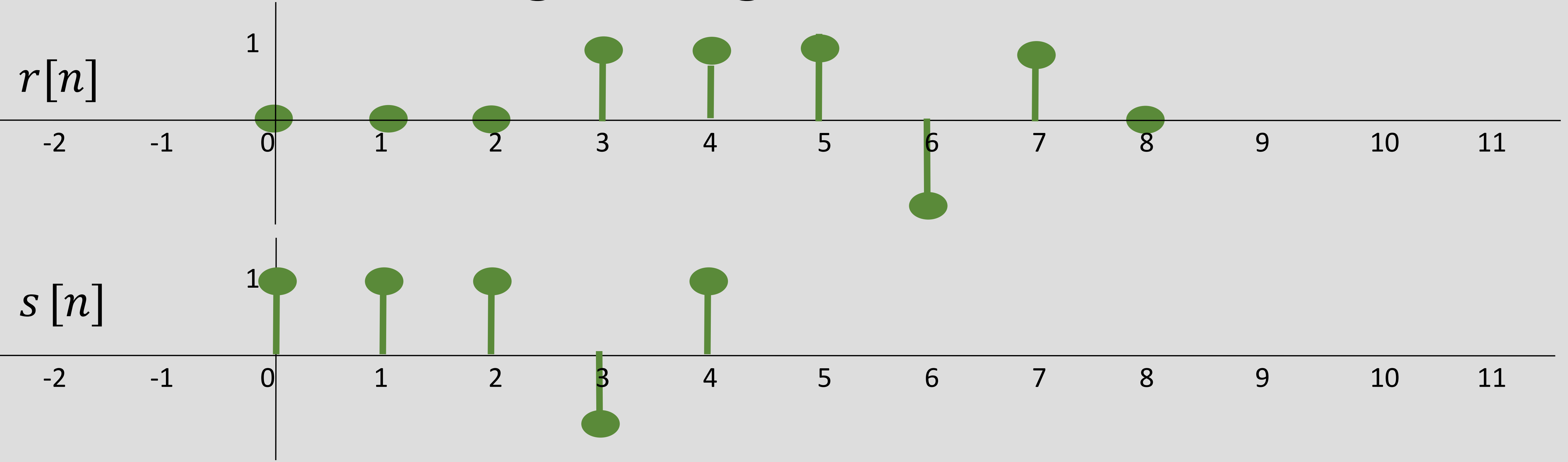
**Problem: $\vec{r}$ and $\vec{s}_1$ are not the same length!**

**Solution: we can 'zero pad' them**

$$\vec{r} = [r_0 \quad r_1 \quad r_2 \quad \cdots \quad r_8]^T \quad \Rightarrow r[n] = \begin{cases} r_n & 0 \le n \le 8 \\ 0 & \text{elsewhere} \end{cases}$$

$$\vec{s} = [s_0 \quad s_1 \quad s_2 \quad \cdots \quad s_4]^T \quad \Rightarrow s[n] = \begin{cases} s_n & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

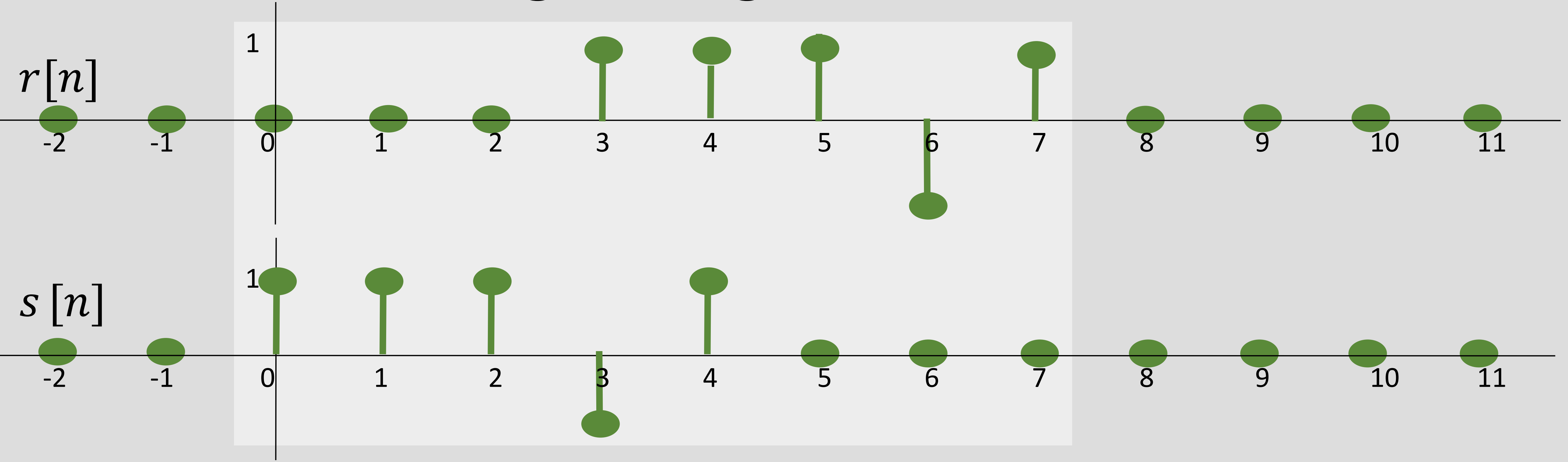
# "Pattern Matching" of Signals




$$r[n] = \begin{cases} r_n & 0 \leq n \leq 8 \\ 0 & \text{elsewhere} \end{cases}$$

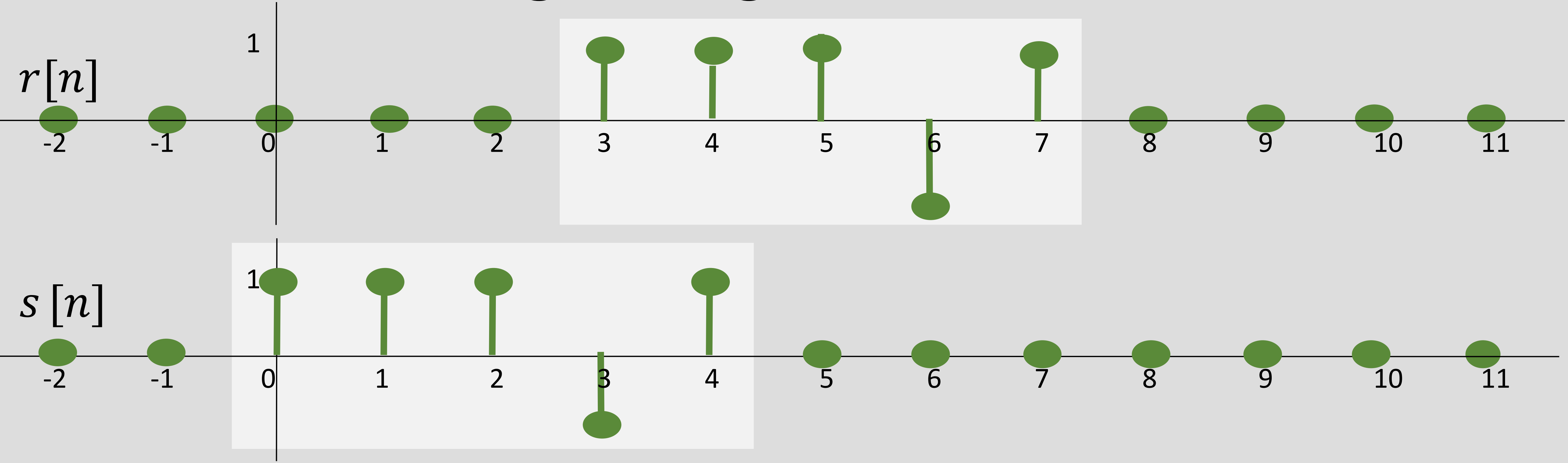
$$s[n] = \begin{cases} s_n & 0 \leq n \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

# "Pattern Matching" of Signals




$$\langle r[n], s[n] \rangle = \sum_{n=-\infty}^{\infty} r[n]s[n]$$

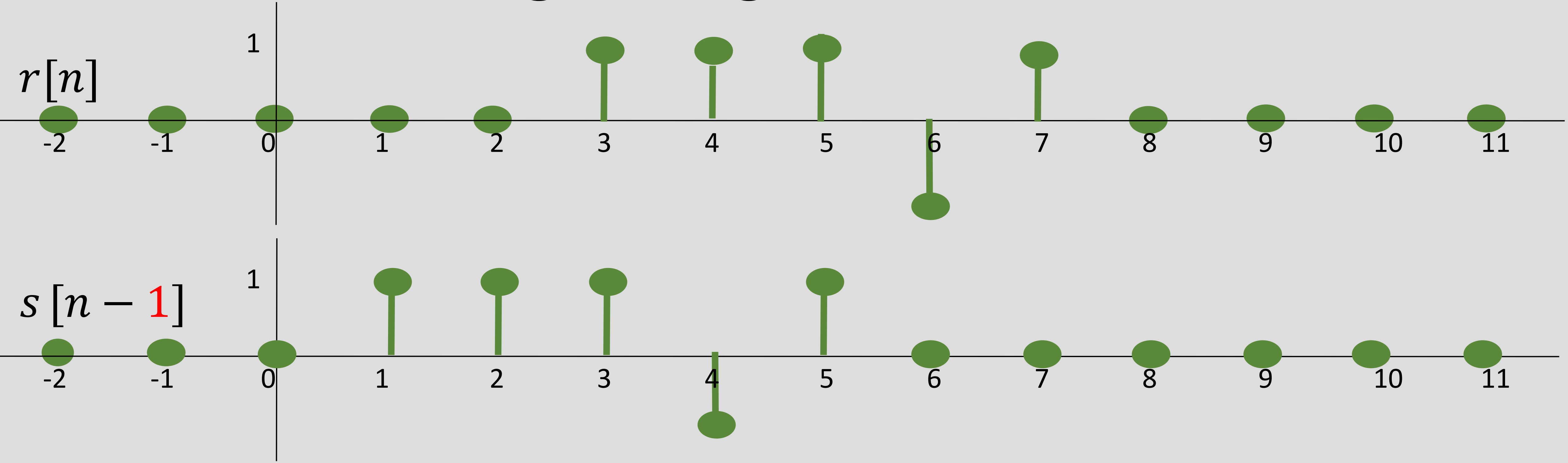
# “Pattern Matching” of Signals




Q: When will I get a large inner product?

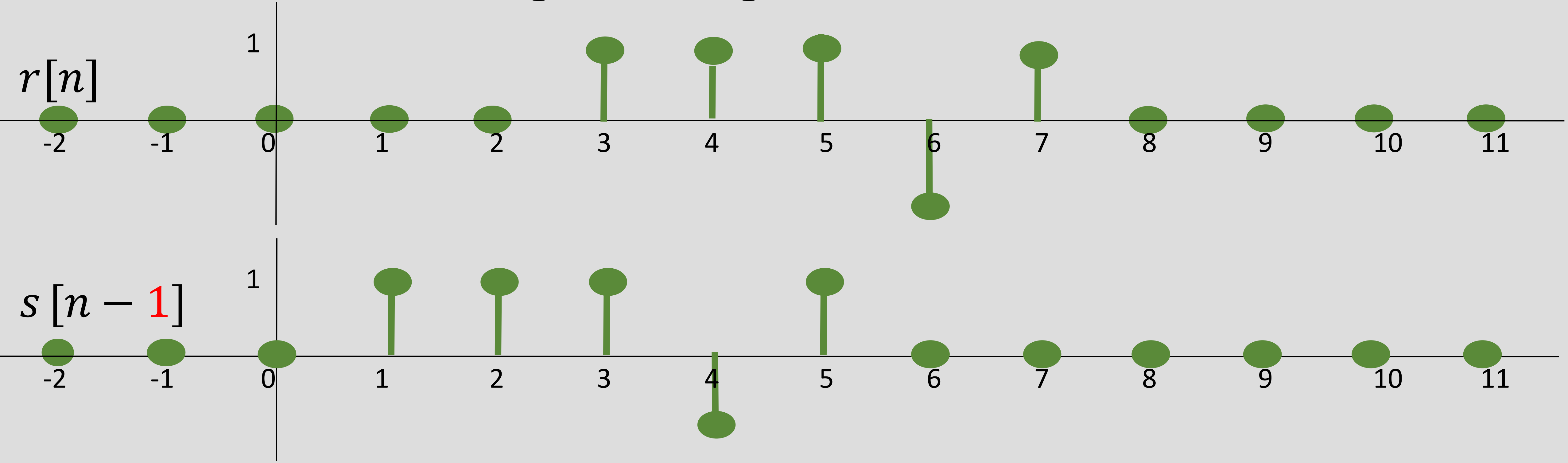
A: when I do an inner product of r[n] with a shifted version of s[n]

# "Pattern Matching" of Signals




$$\langle r[n], s[n-1] \rangle$$

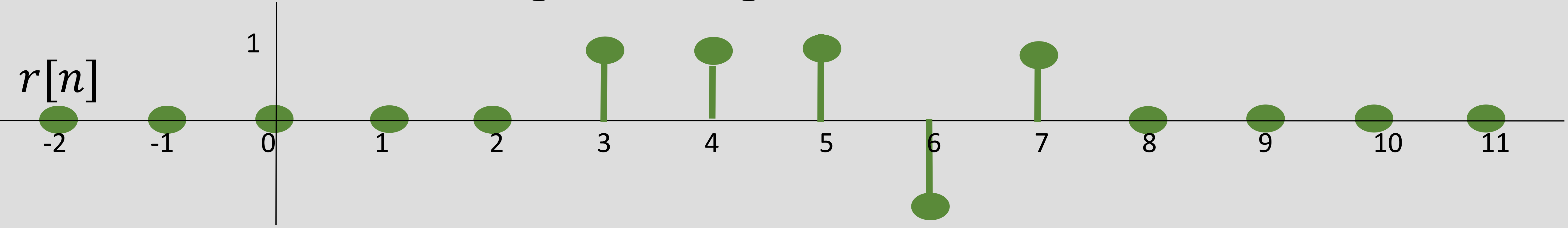
# "Pattern Matching" of Signals

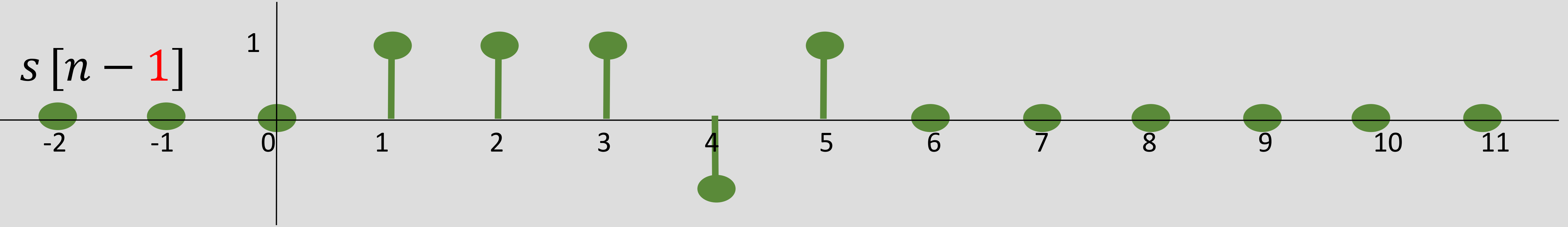



$$\mathrm{corr}_r(\vec{s})[1] = \langle r[n], s[n-1] \rangle = \sum_{n=-\infty}^{\infty} r[n]s[n-1] \ = 1$$

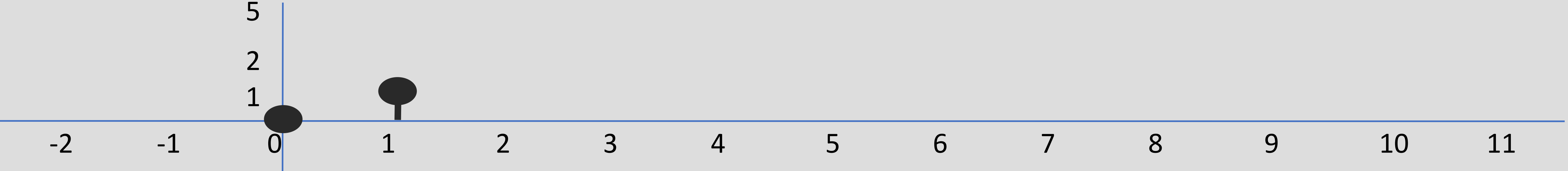
**This is one element of the "correlation" of r[n] and s[n]!**

# "Pattern Matching" of Signals

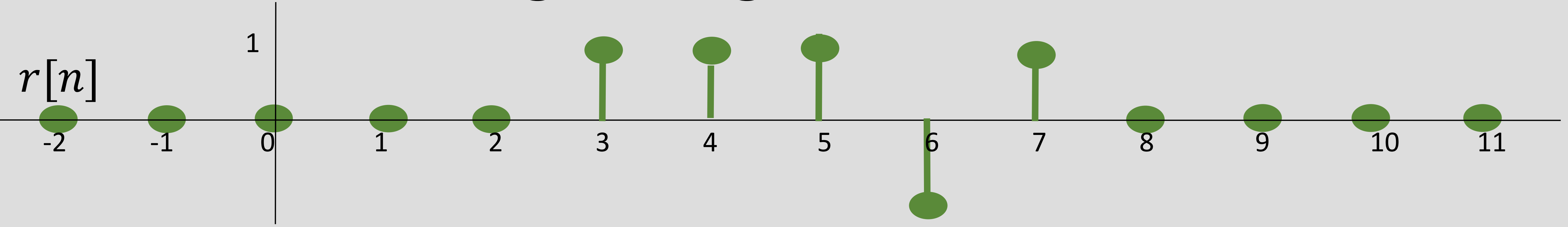


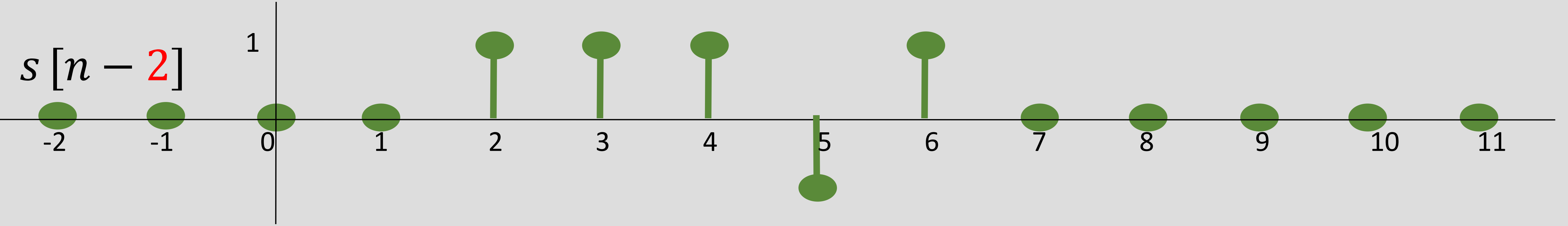




$$\mathrm{corr}_r(\vec{s})[1] = \langle r[n], s[n-1] \rangle = \sum_{n=-\infty}^{\infty} r[n]s[n-1] \quad = 1$$

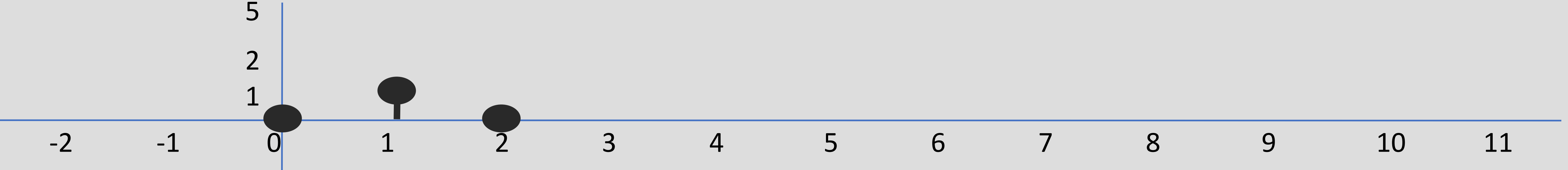


# “Pattern Matching” of Signals

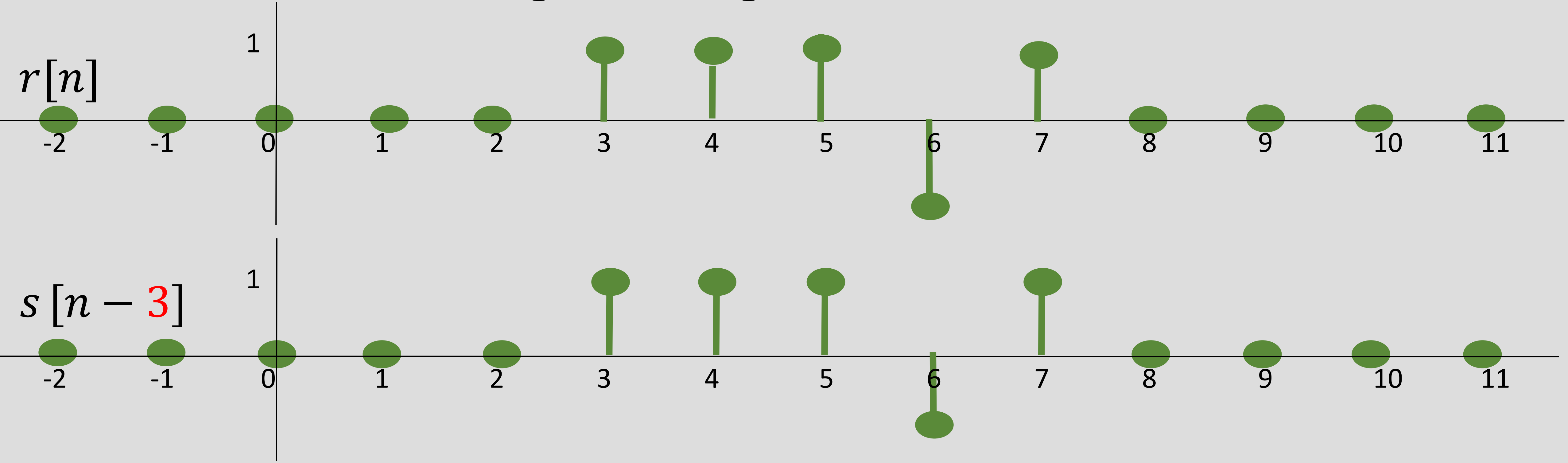


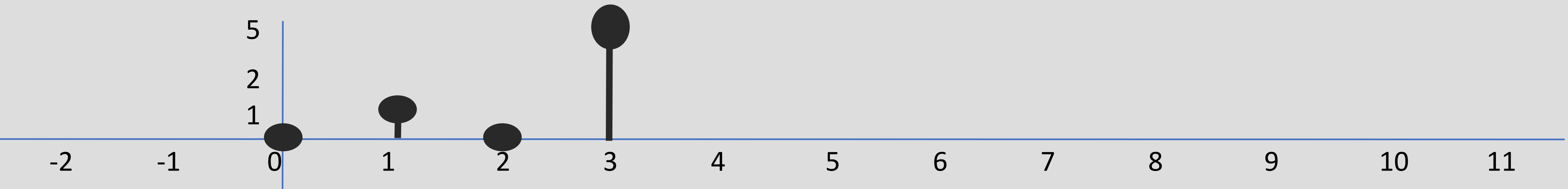

$$\text{corr}_r(\vec{s})[2] = \langle r[n], s[n-2] \rangle = 0$$



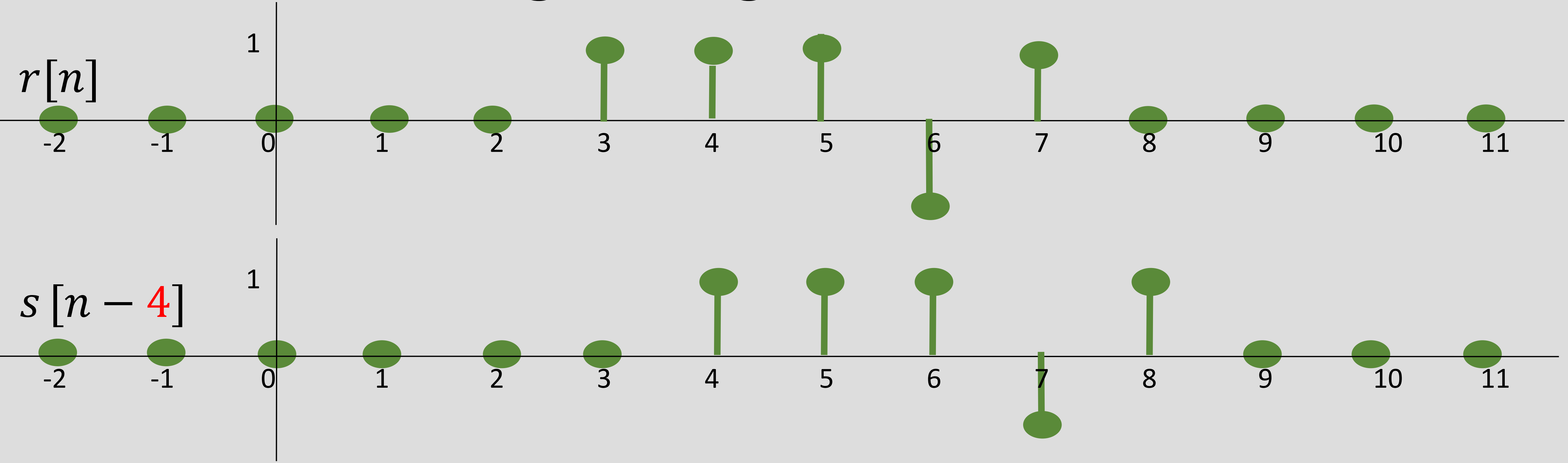
# "Pattern Matching" of Signals



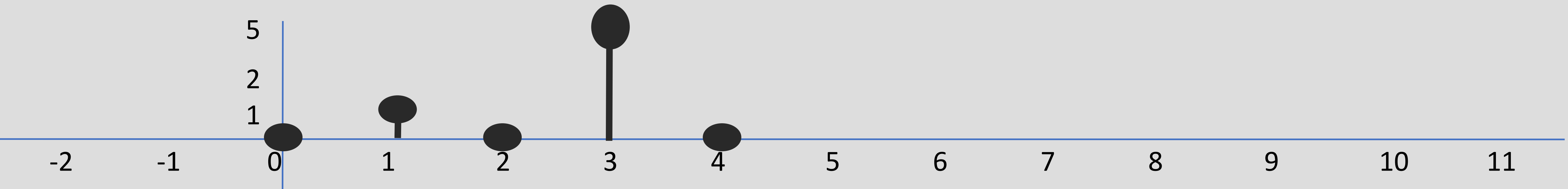

$$\text{corr}_r(\vec{s})[3] = \langle r[n], s[n-3] \rangle = 5$$



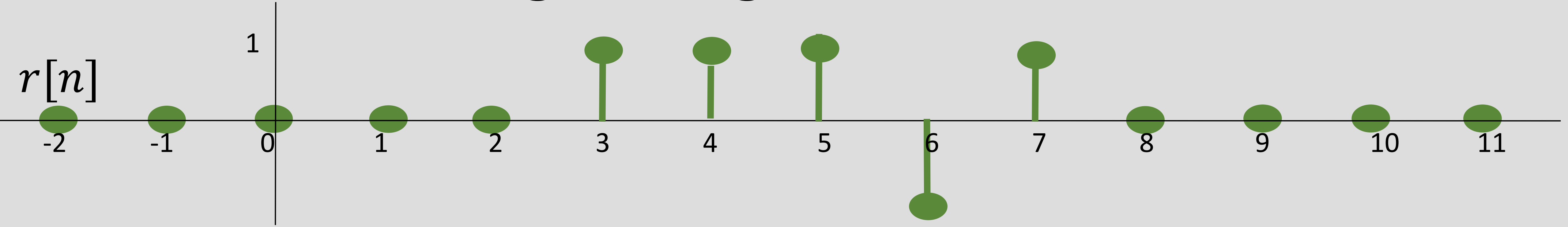
# "Pattern Matching" of Signals

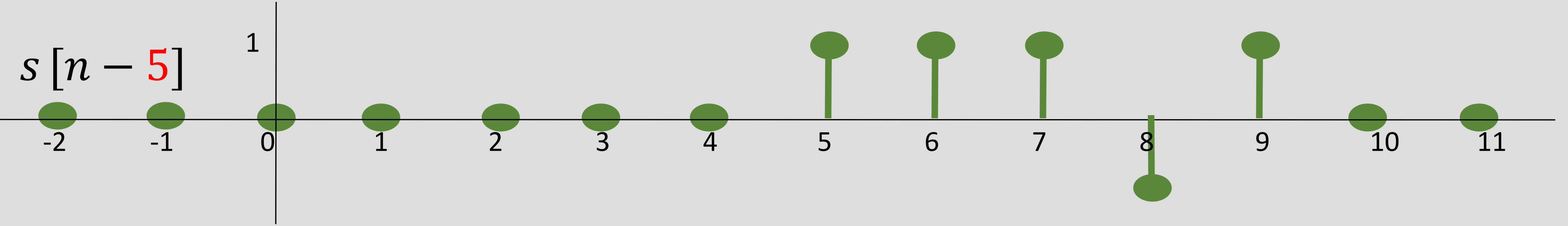



$$\mathrm{corr}_r(\vec{s})[4] = \langle r[n], s[n-4] \rangle = 0$$

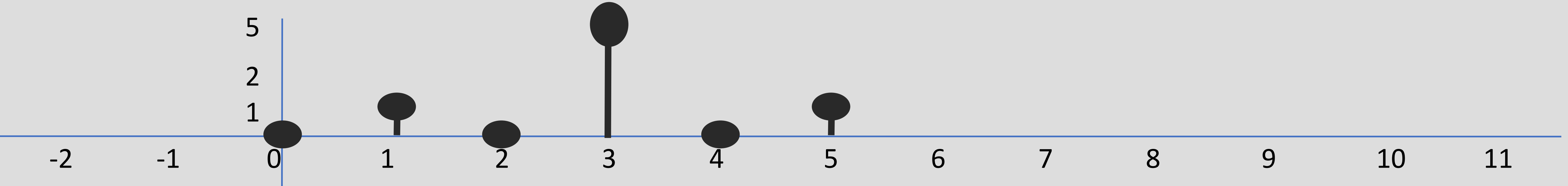


# "Pattern Matching" of Signals

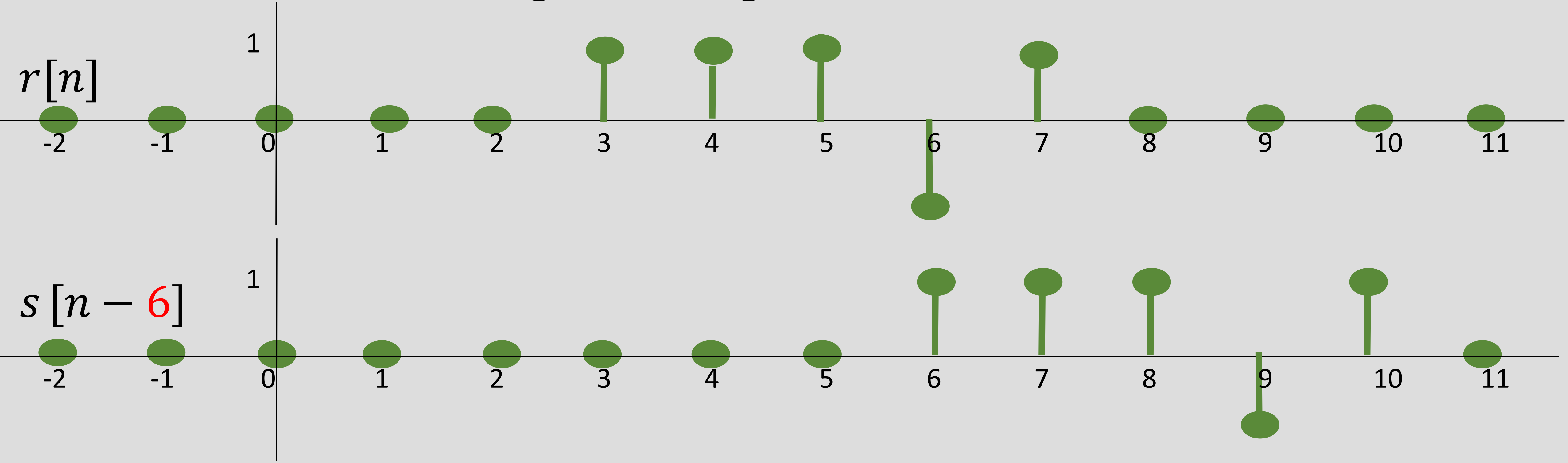


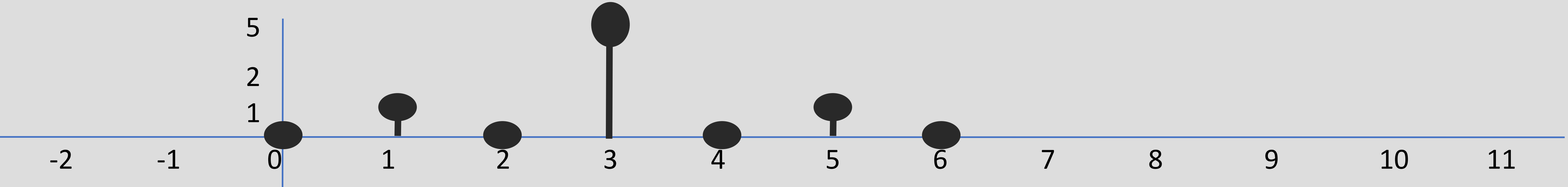

$$\mathrm{corr}_r(\vec{s})[5] = \langle r[n], s[n-5] \rangle = 1$$



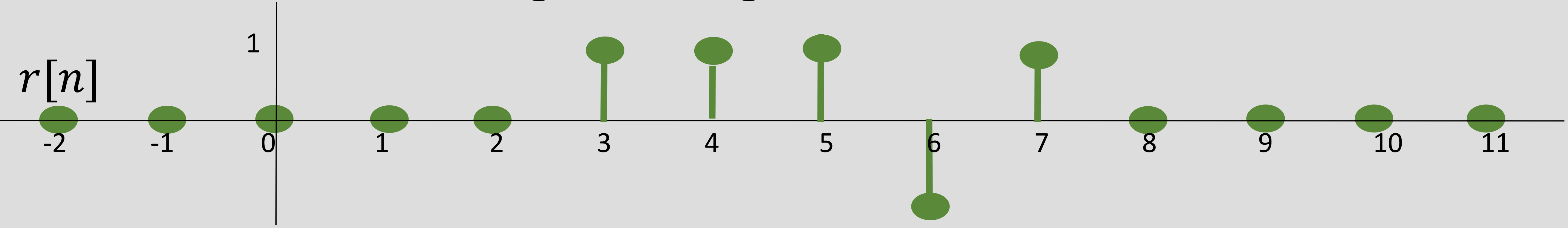
# "Pattern Matching" of Signals

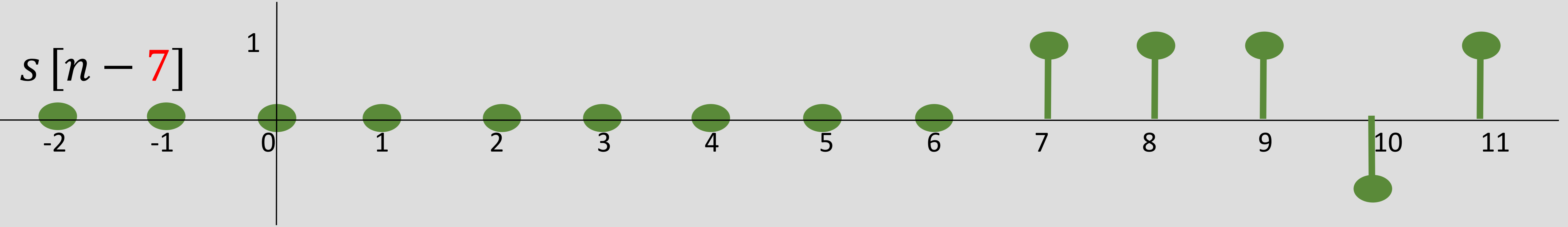



$$\mathrm{corr}_r(\vec{s})[6] = \langle r[n], s[n-6]\rangle = 0$$

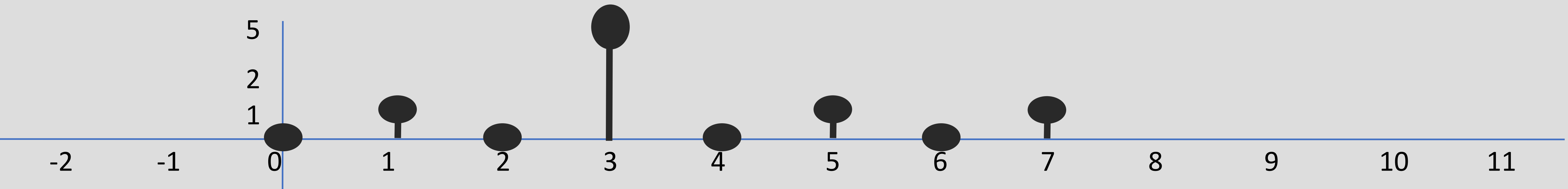


# “Pattern Matching” of Signals

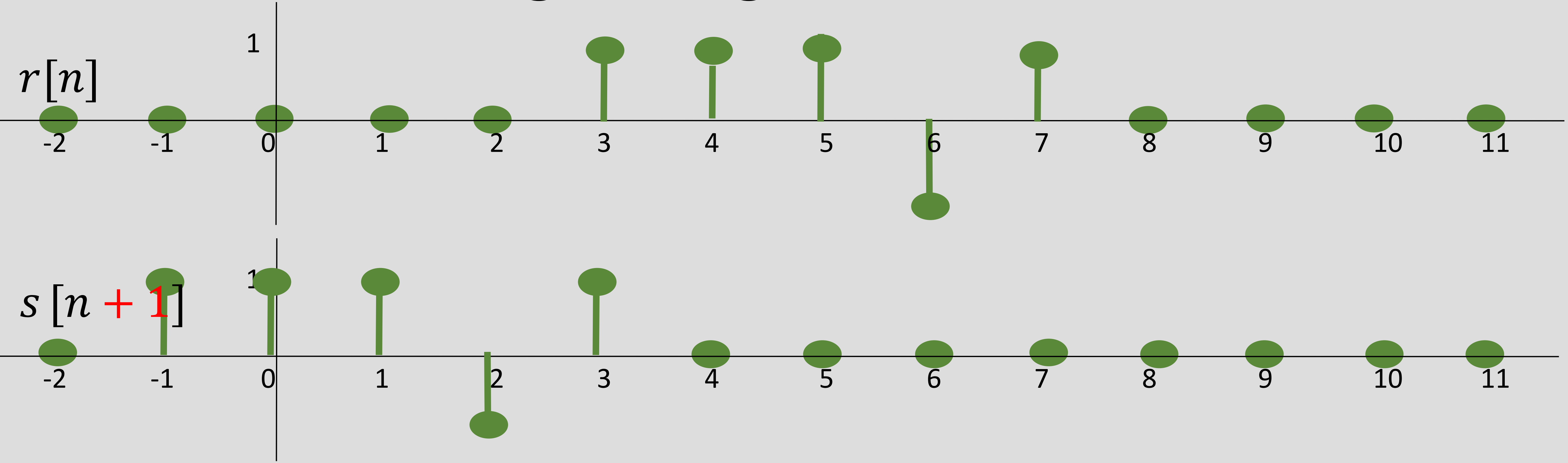


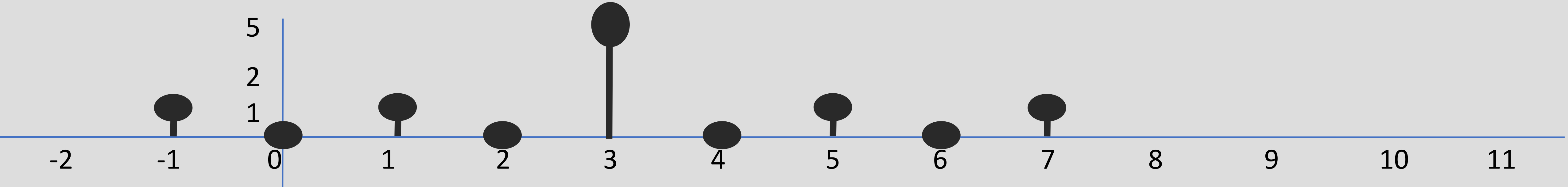

$$\text{corr}_r(\vec{s})[7] = \langle r[n], s[n-7] \rangle = 1$$



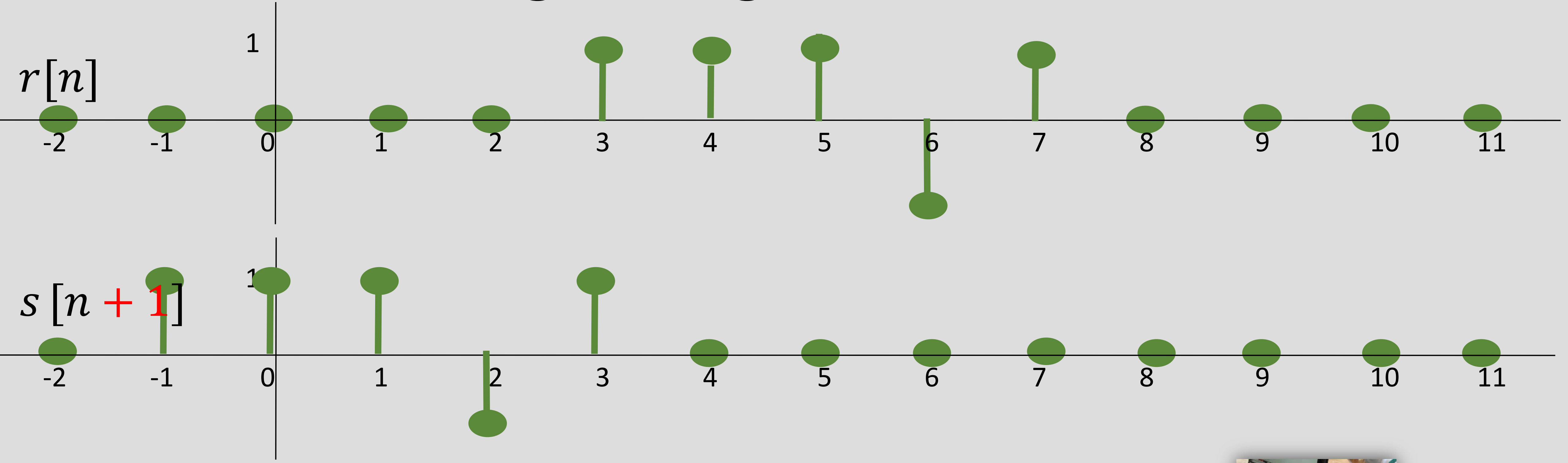
# "Pattern Matching" of Signals



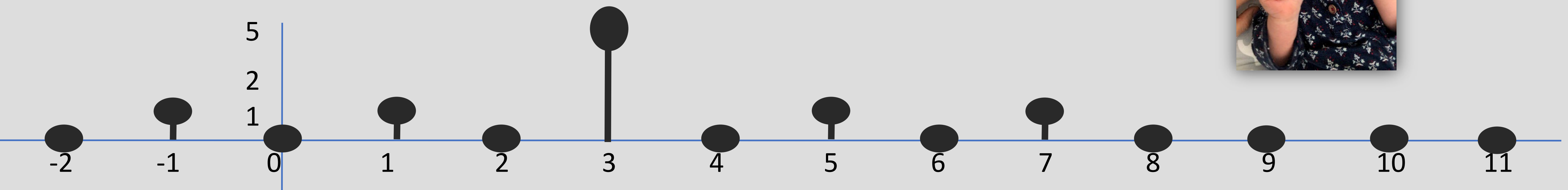

$$\text{corr}_r(\vec{s})[-1] = \langle r[n], s[n+1] \rangle = 1$$



# “Pattern Matching” of Signals



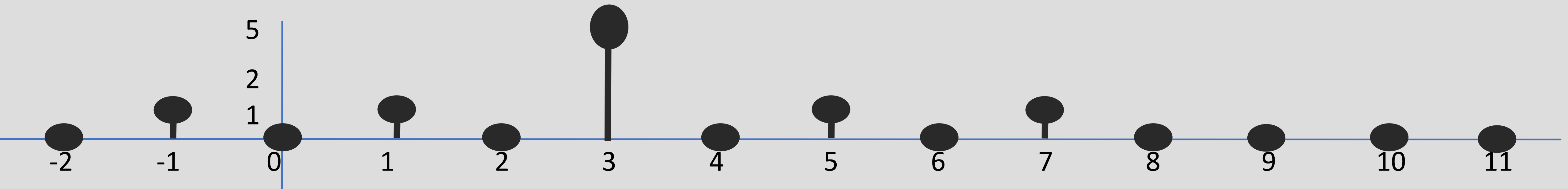

I’m done! The rest are zeros!



# Definition of correlation

**The cross-correlation between vectors $\vec{r}$ and $\vec{s}$ is:**

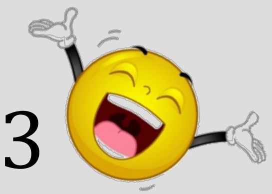
$$\mathrm{corr}_{\vec{r}}(\vec{s})[k] = \langle r[n], s[n-k] \rangle = \sum_{n=-\infty}^{\infty} r[n]s[n-k]$$

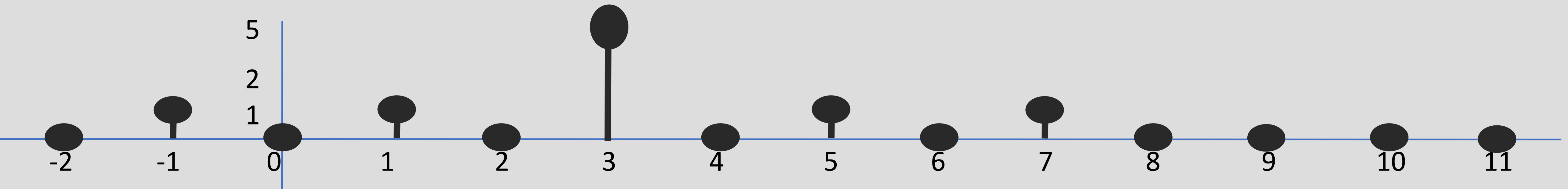


# So what is the delay?

$$k^* = \underset{k}{\operatorname{argmax}} \; \operatorname{corr}_{\vec{r}}(\vec{s})[k]$$

$$k^* = 3$$





# Cross-correlation Properties

## What is the length of the cross-correlation?

- If $\vec{x} \in \mathbb{R}^N$ ,and $\vec{y} \in \mathbb{R}^M$ ,then the length of $\mathrm{corr}_{\vec{x}}(\vec{y})$ is $N + M - 1$

## Can I swap the order of the things?

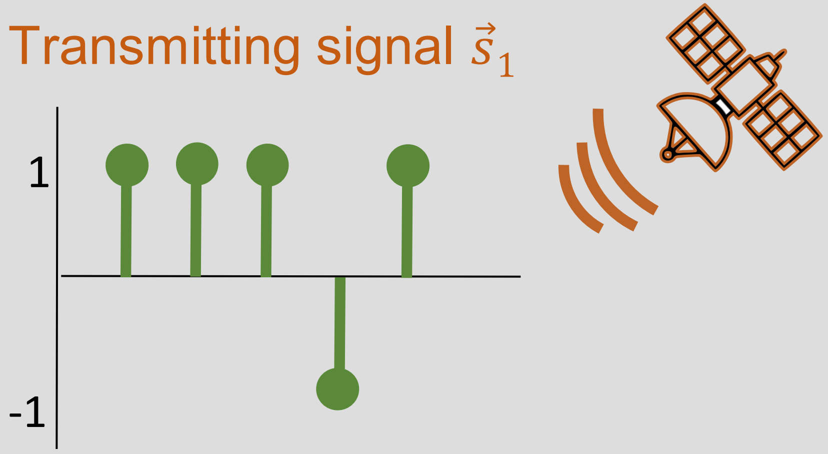
- $\mathrm{corr}_{\vec{x}}(\vec{y}) \neq \mathrm{corr}_{\vec{y}}(\vec{x})$

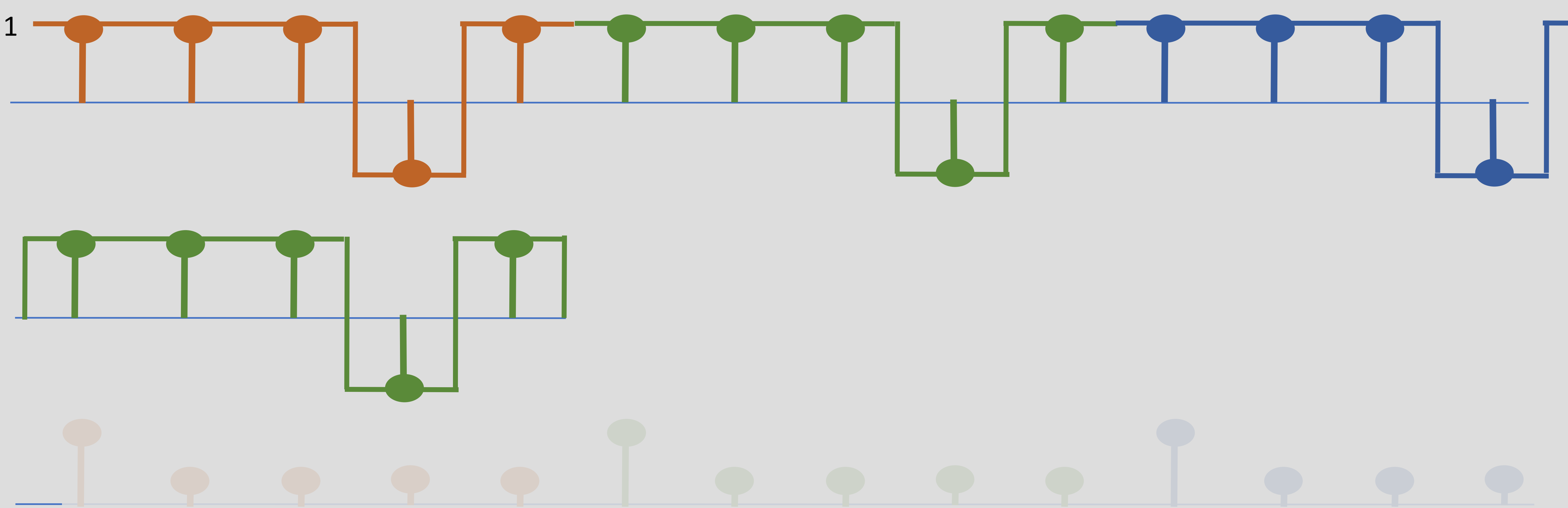
## What's the correlation of something with itself?

- $\mathrm{corr}_{\vec{x}}(\vec{x})$ is called auto-correlation

# Periodic Signals

- Satellites repeat the codes over and over
  - cross-correlation is "periodically expanded" instead of zero-padded
  - result is periodic



# What are good properties for the codes?

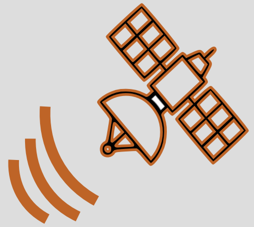

Two problems:

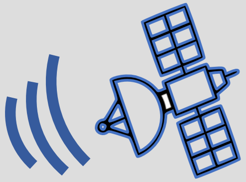
1. Interference
2. Timing

- Shifted versions of self are not very correlated
- Songs for each satellite/beacon are not very correlated

**Can I achieve that with just 1's and -1's?**

# What kind of correlations do we want?



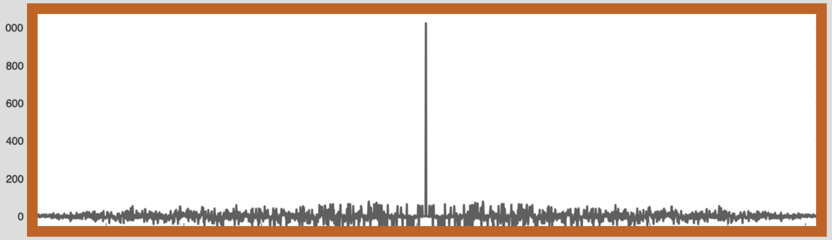


$$r[n] = s_1[n-\tau_1] + s_2[n-\tau_2] + noise[n]$$

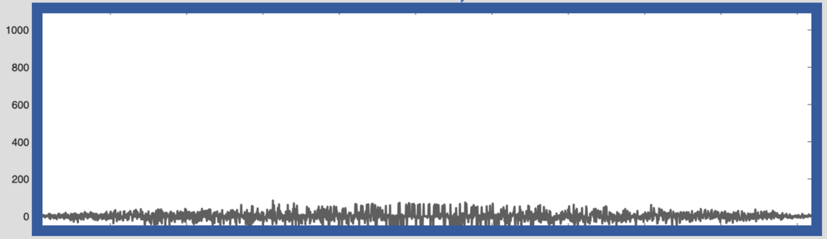
Correlate with $s_1[n]$:

$$\text{corr}_{\vec{r}}(\vec{s}_1)[k] = \langle r[n], s_1[n-k] \rangle$$

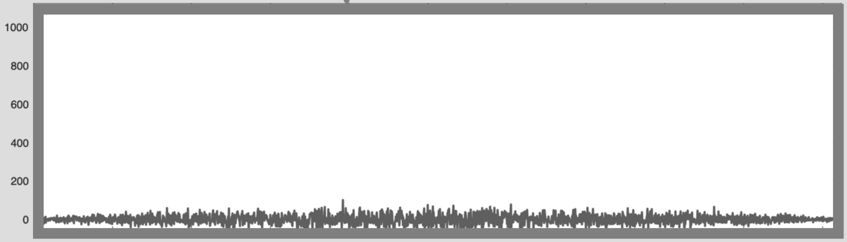
$$= \langle s_1[n-\tau_1], s_1[n-k] \rangle + \langle s_2[n-\tau_2], s_1[n-k] \rangle + \langle noise[n], s_1[n-k] \rangle$$




Auto-correlation should be like an impulse



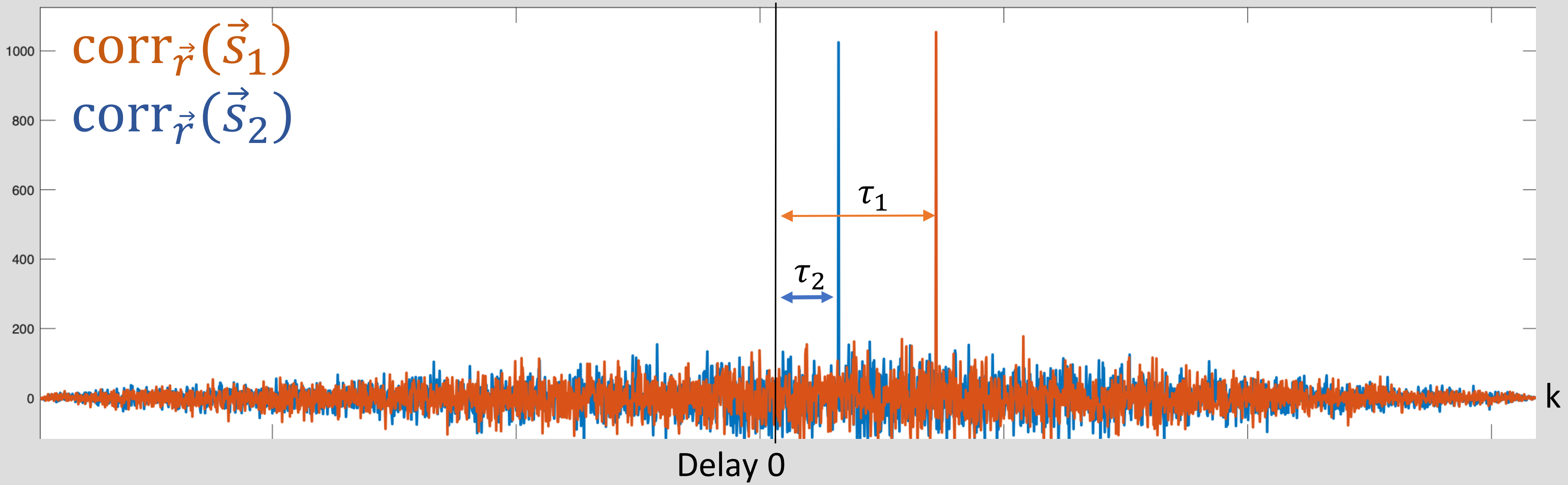

cross-correlation with other satellites should be small




cross-correlation with noise should be small (always true?)

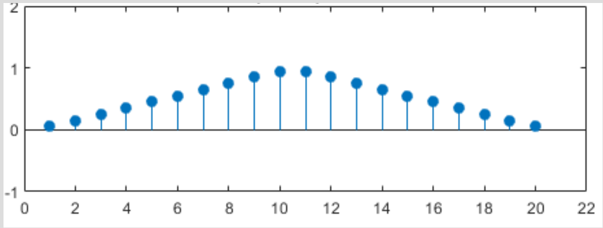
# Received Signal

$$r[n] = s_1[n - \tau_1] + s_2[n - \tau_2] + noise[n]$$

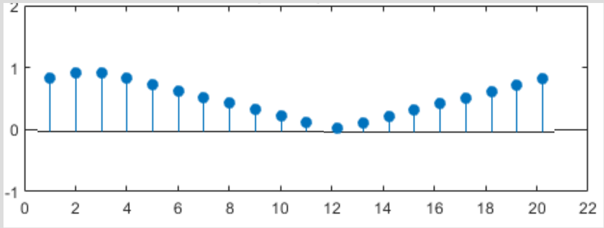


# Is this a good code?

$\vec{\mathbf{s}} =$




$\vec{\mathbf{r}} =$

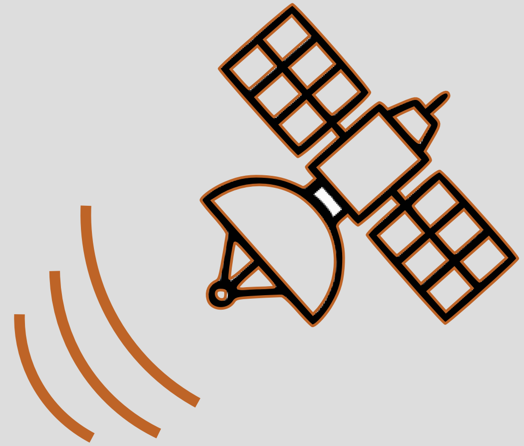


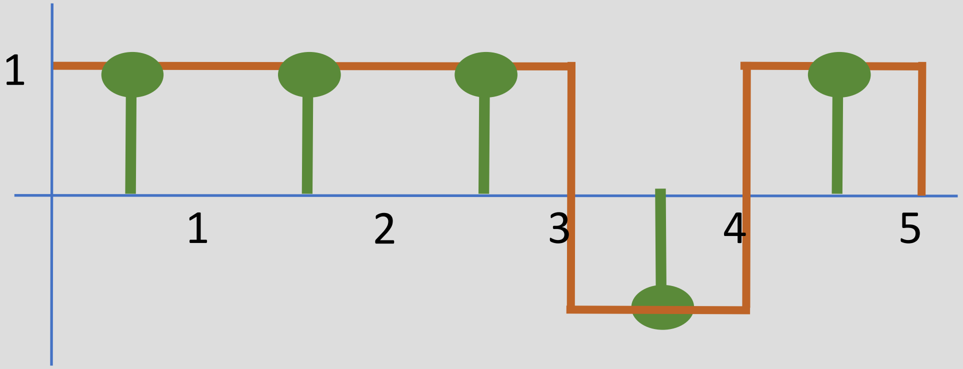

**What will the correlation look like?**

# Timing….

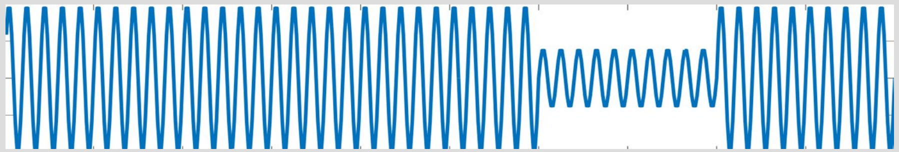
- Satellites transmit a (modulated) unique code
  - Radio signal
- Signal is received (demodulated) and digitized by a receiver

TX: $\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

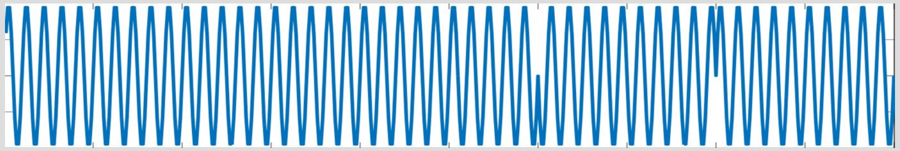





Amplitude Modulation (AM)



Phase Modulation (PM)



# How to solve for GPS coordinates:

1. ***Identify*** which satellites are 'on'
2. Find the ***delay/shift*** for each satellite
3. Use shifts to find ***distances*** to each satellite
4. ***Trilateration*** to find my coordinates