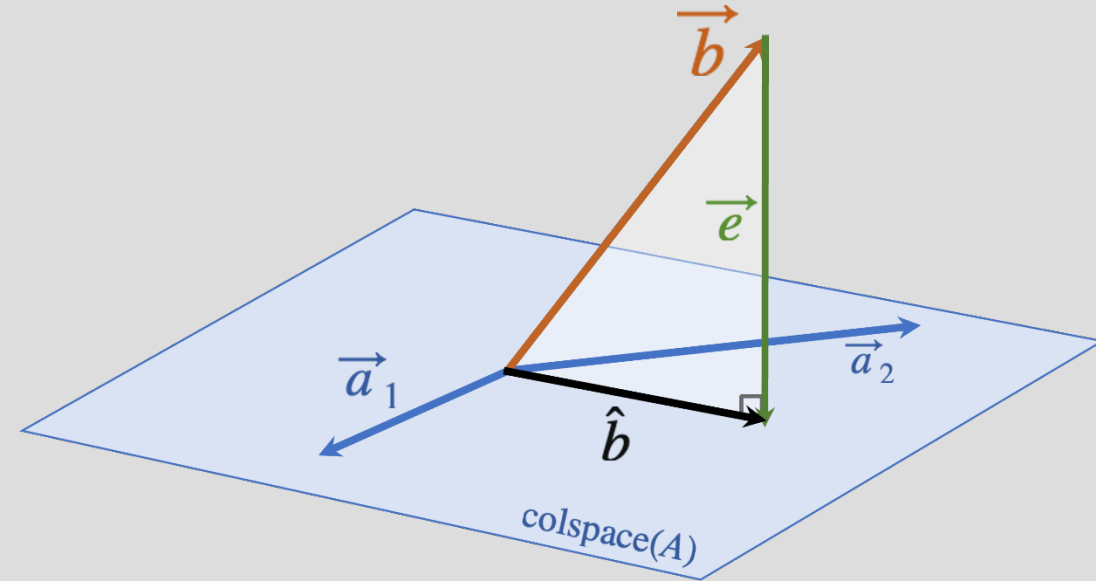


EECS 16A
fun stuff!

Overdetermined system: use least squares

$$A x = b$$



- the least-squares solution “minimally perturbs” b

$$\hat{x} = (A^T A)^{-1} A^T \vec{b}$$

Underdetermined system: ????

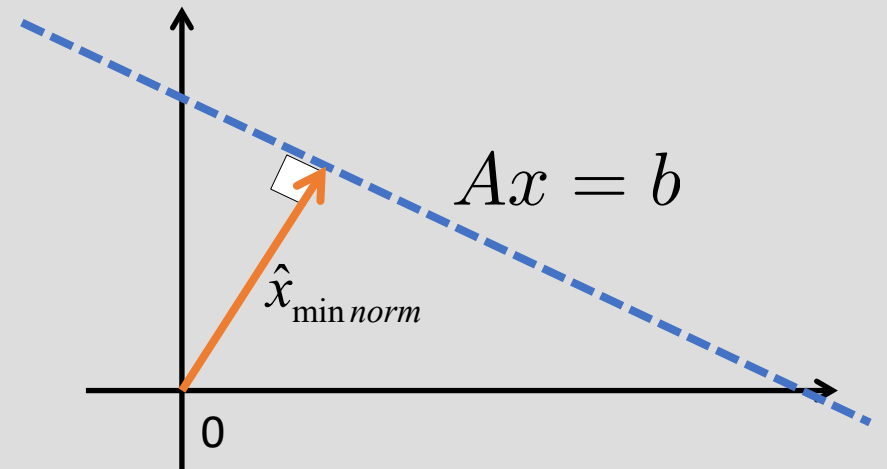
$$A x = b$$

IF TV SCIENCE WAS MORE LIKE REAL SCIENCE



- Can be infinite valid solutions!
- Ideas: pick the 'smallest' one? The 'sparsest'?
- e.g. min norm:

$$\hat{x}_{\min norm} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \vec{b}$$

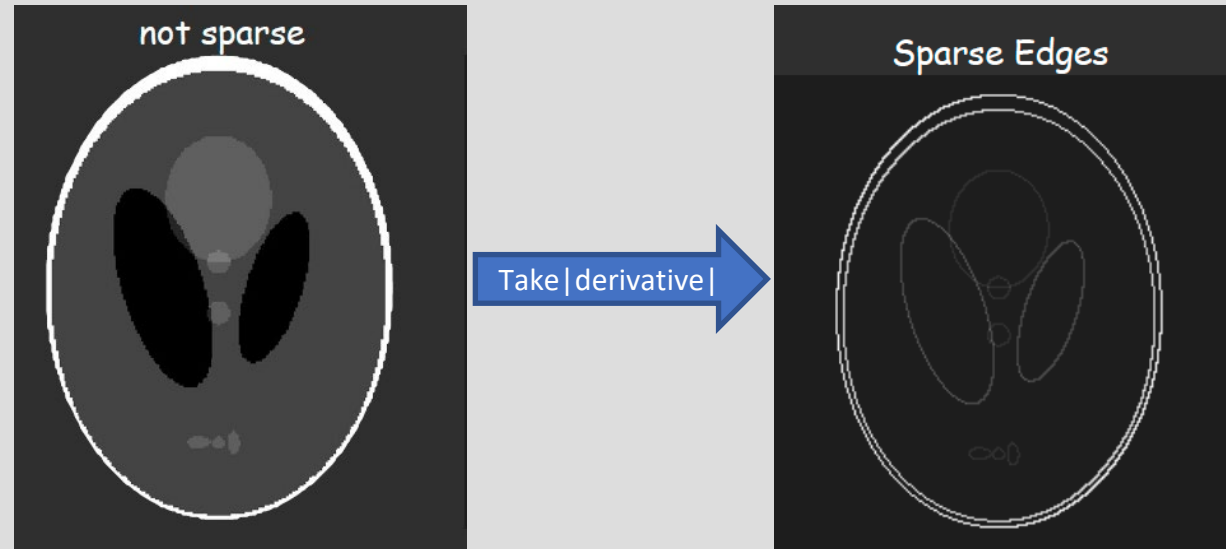


'Sparsity' tells us how 'dense' the solution is

Dense Matrix									
1	2	31	2	9	7	34	22	11	5
11	92	4	3	2	2	3	3	2	1
3	9	13	8	21	17	4	2	1	4
8	32	1	2	34	18	7	78	10	7
9	22	3	9	8	71	12	22	17	3
13	21	21	9	2	47	1	81	21	9
21	12	53	12	91	24	81	8	91	2
61	8	33	82	19	87	16	3	1	55
54	4	78	24	18	11	4	2	99	5
13	22	32	42	9	15	9	22	1	21

Sparse Matrix									
1	.	3	.	9	.	3	.	.	.
11	.	4	2	1
.	.	1	.	.	.	4	.	1	.
8	.	.	.	3	1
.	.	.	9	.	.	1	.	17	.
13	21	.	9	2	47	1	81	21	9
.
.	.	.	.	19	8	16	.	.	55
54	4	.	.	.	11
.	.	2	22	.	21

The fraction of non-zero elements in a matrix is called the *sparsity*



Sometimes things are sparse in a different way

Example: image compression

Reduce memory by smartly choosing which information to throw away



No compression



23:1 compression



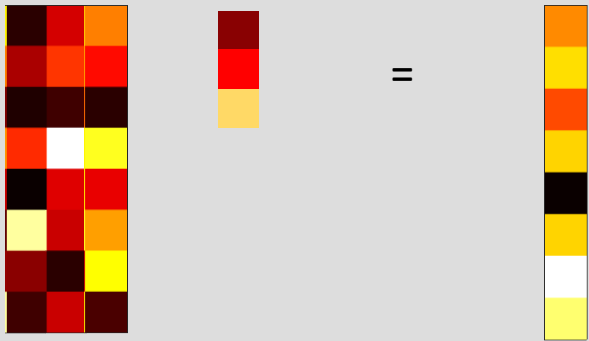
144:1 compression



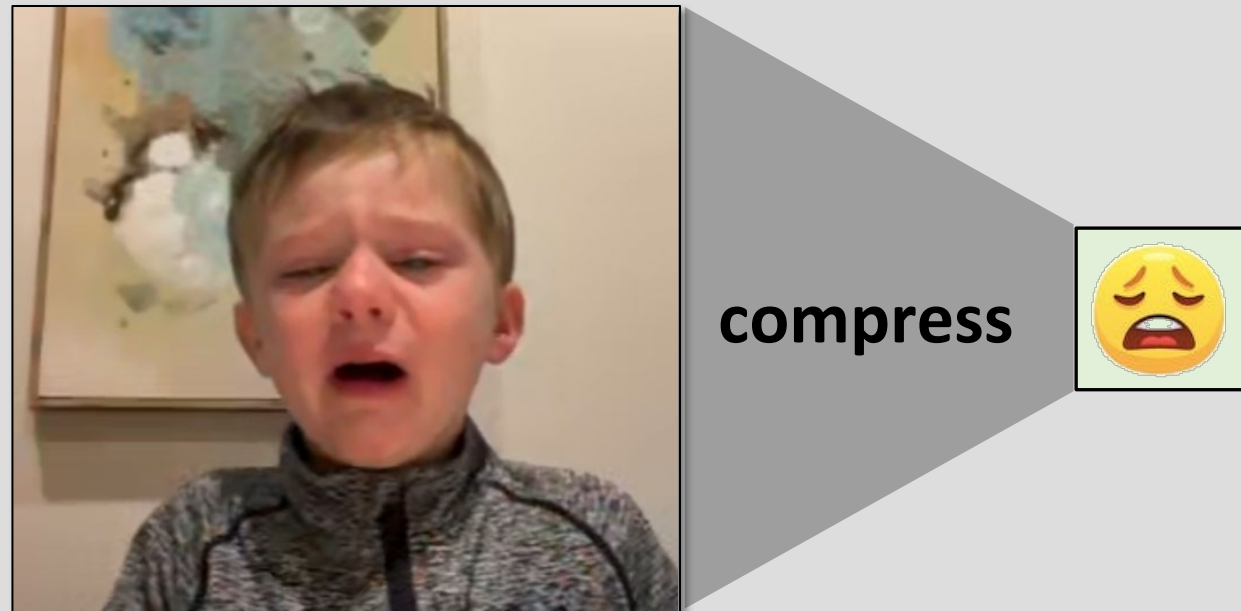
Sparse x means only a few columns of A 'matter'



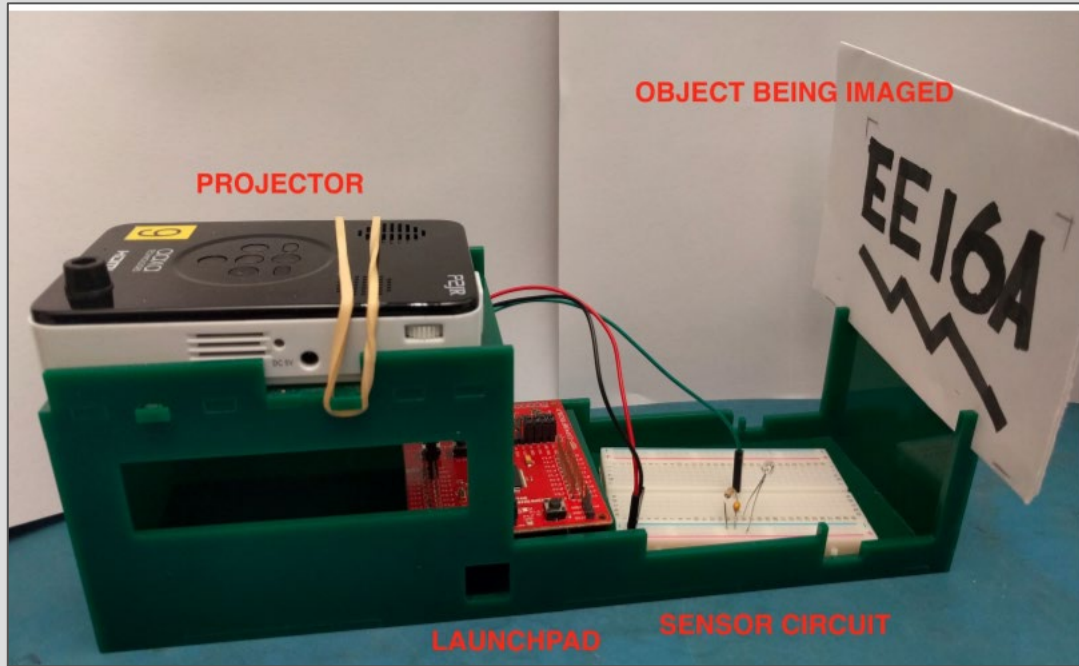
If we knew which elements were non-zero, we could solve a small least squares problem:



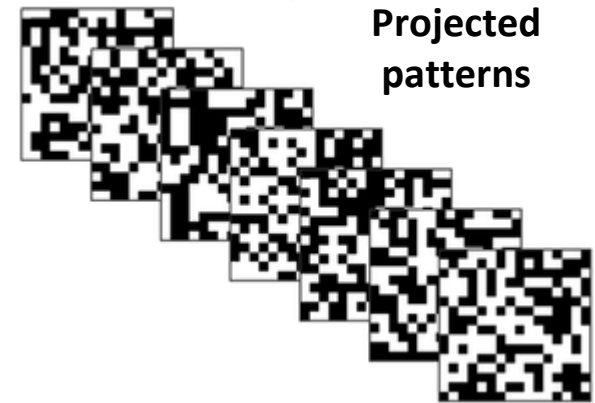
Can we compress data at the capture stage?



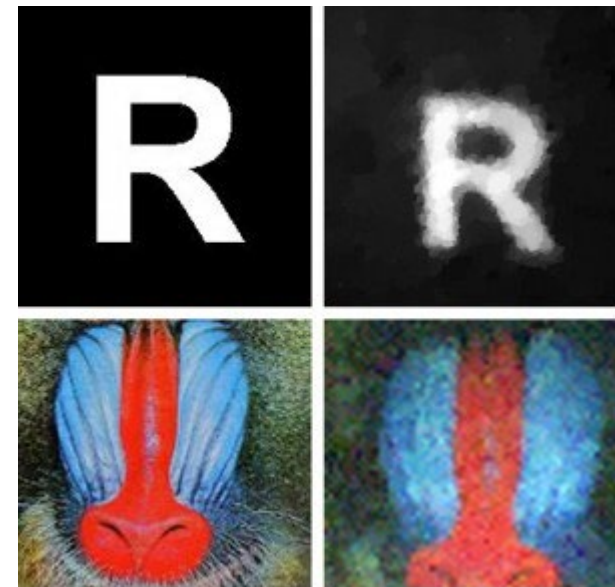
Yes! With compressed sensing! Example: single-pixel camera



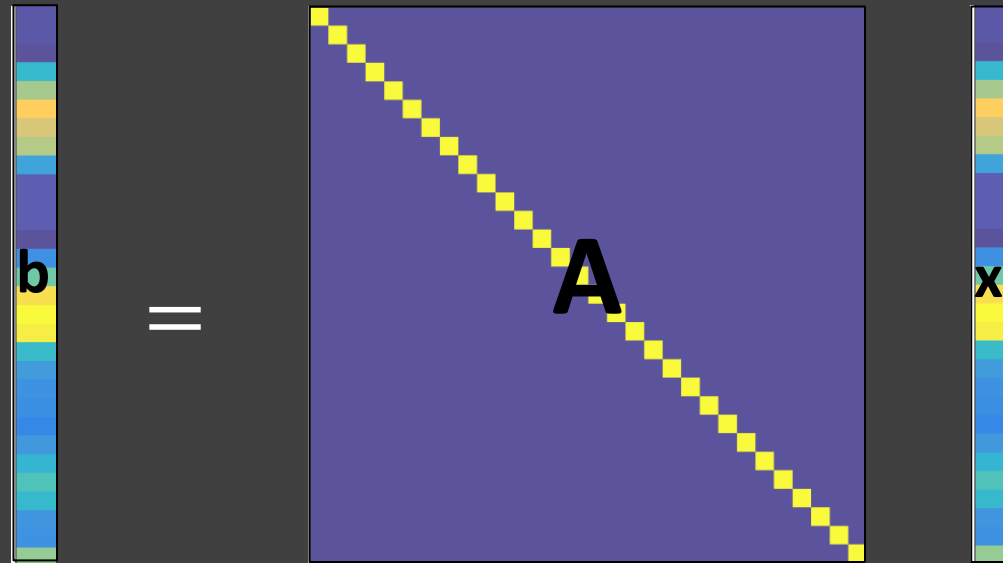
If you design the patterns on your imaging lab well, and images are compressible, you could solve with very little data!



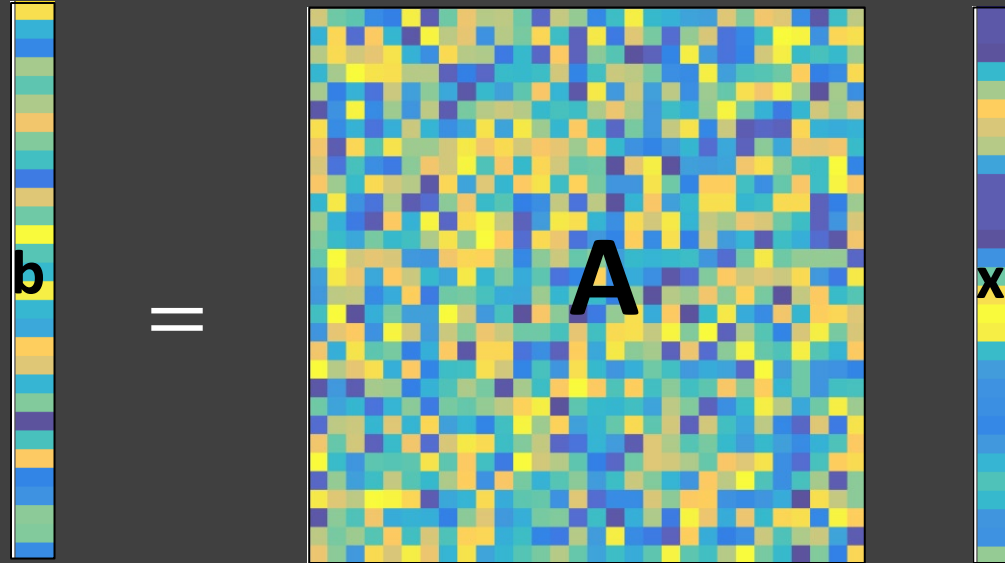
Original image Reconstructed
(2% of data)



We usually take direct measurements



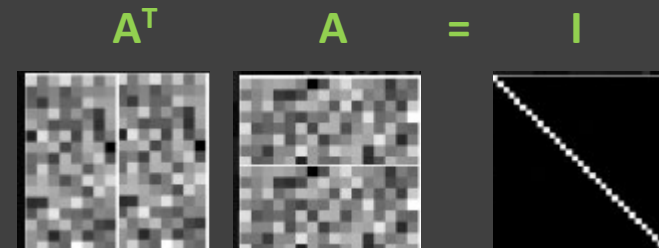
Multiplexed measurements



A diagram illustrating the equation $b = Ax$. On the left is a vertical column vector labeled b , composed of 16 colored segments. In the center is a square matrix labeled A , also composed of 16x16 colored segments. On the right is another vertical column vector labeled x , composed of 16 colored segments. An equals sign is placed between the vector b and the matrix A .

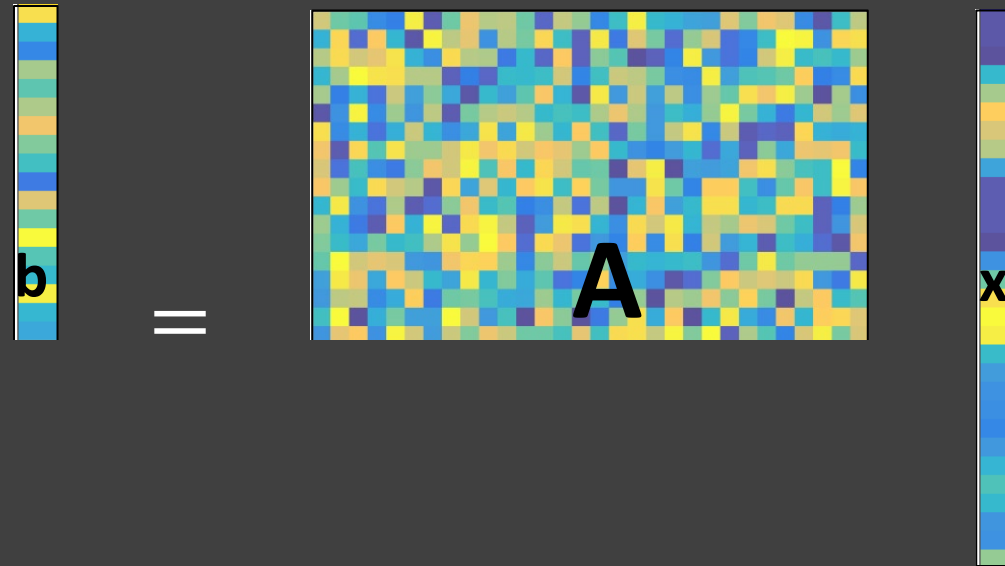
What makes a good A matrix?

A is “orthogonal”



A diagram illustrating the equation $A^T A = I$. On the left is a square matrix labeled A^T , which is a grayscale noise pattern. In the center is a square matrix labeled A , which is also a grayscale noise pattern. On the right is a square matrix labeled I , which is a black square with a white diagonal line. An equals sign is placed between the matrices A^T and A .

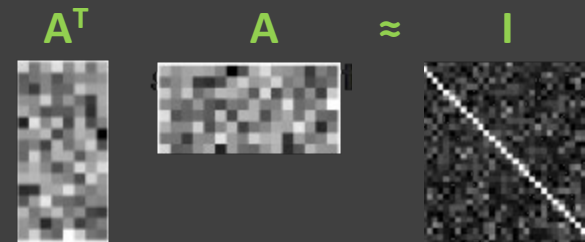
Compressed sensing solves underdetermined problems



A diagram illustrating the equation $b = Ax$. On the left is a vertical column vector labeled b . In the center is a square matrix labeled A . On the right is another vertical column vector labeled x . An equals sign is placed between b and A , and the vector x is positioned to the right of A .

What makes a good A matrix?

A is (almost) orthogonal



A diagram showing three square matrices. The first is labeled A^T , the second is labeled A , and the third is labeled I . An approximation symbol \approx is placed between A and I . The matrices A^T and A are grayscale images of random noise. The matrix I is a grayscale image of an identity matrix, showing a bright diagonal line against a dark background.

Computational Imaging

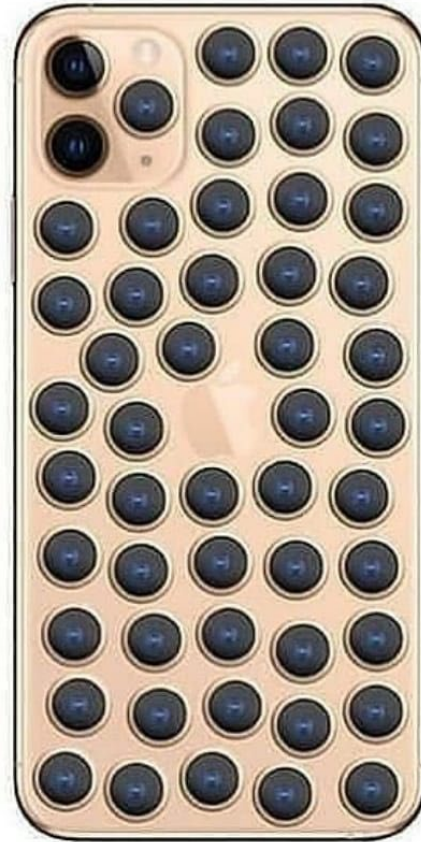
2019

iPhone 11 Pro



2029

iPhone 21 Pro

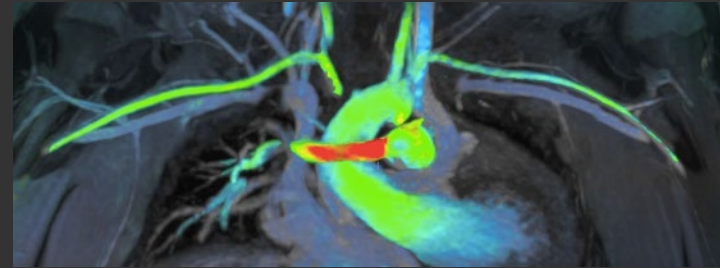


Computational Imaging @ Berkeley

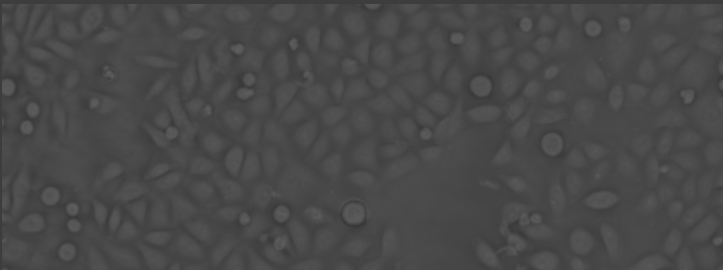
Light Field Cameras
Ren Ng



Compressed Sensing MRI
Michael Lustig



Comp. Illumination Microscopy
Laura Waller



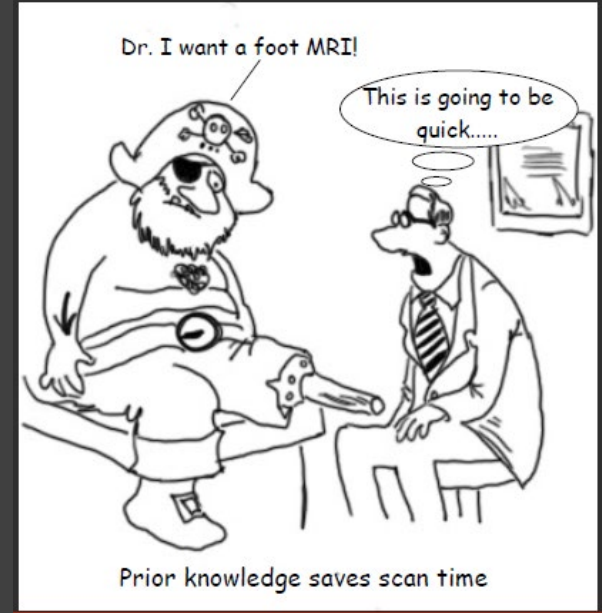
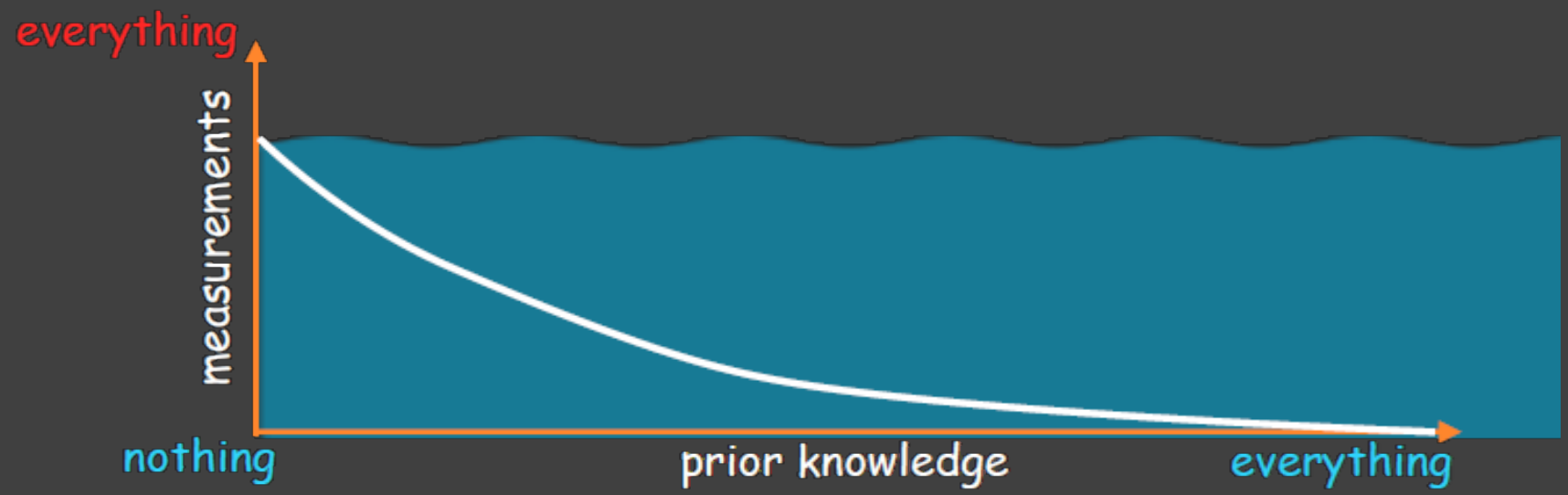
Full-stack Optimization
Ben Recht



$$x(s) = \text{PSF} \circledast \left\{ \sum_{j=1}^k c_j \delta(s - s_j) \right\}$$

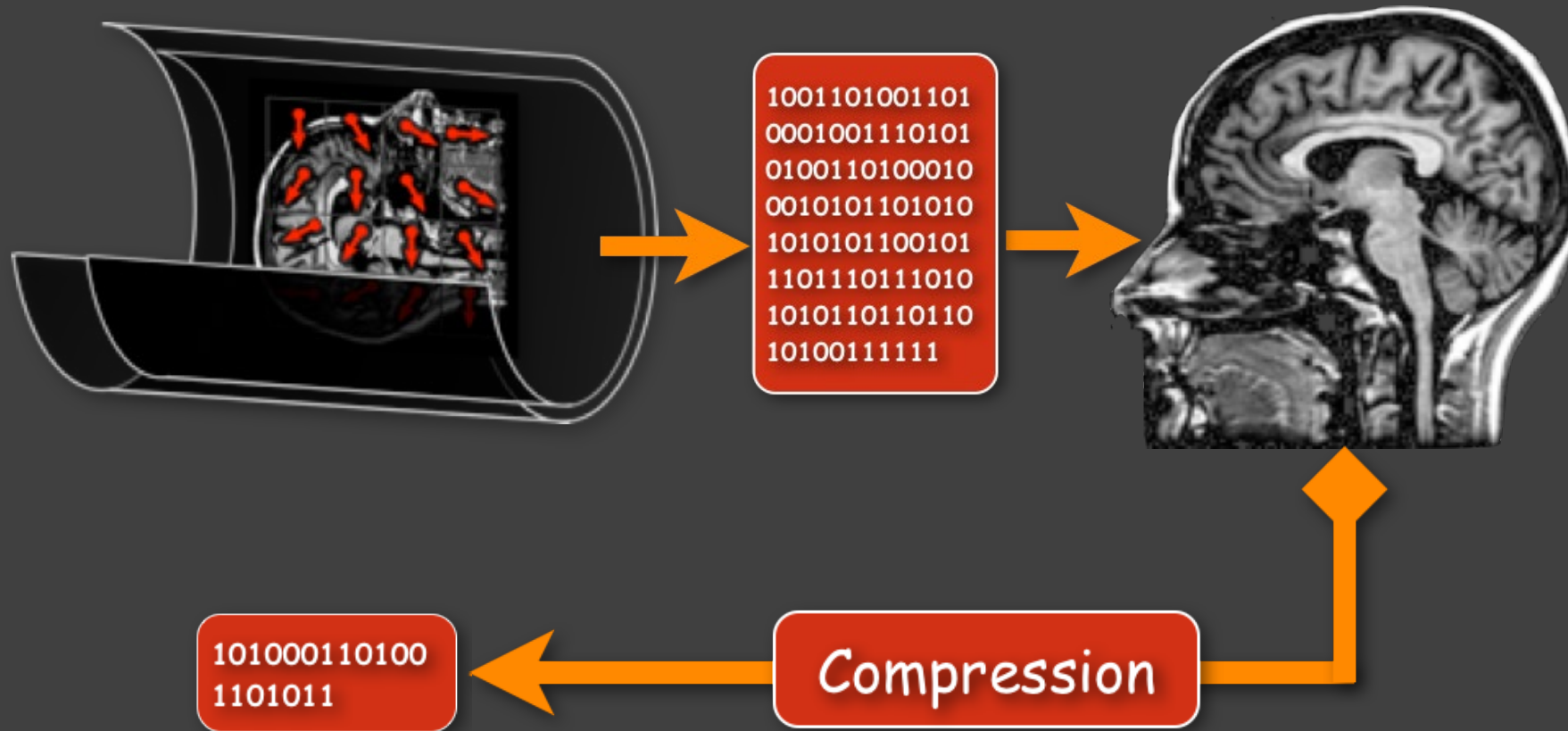
Compressed sensing is all about using prior knowledge

- Redundancy reduces sampling requirements
(The more you know, the less you need)



Compressed Sensing MRI

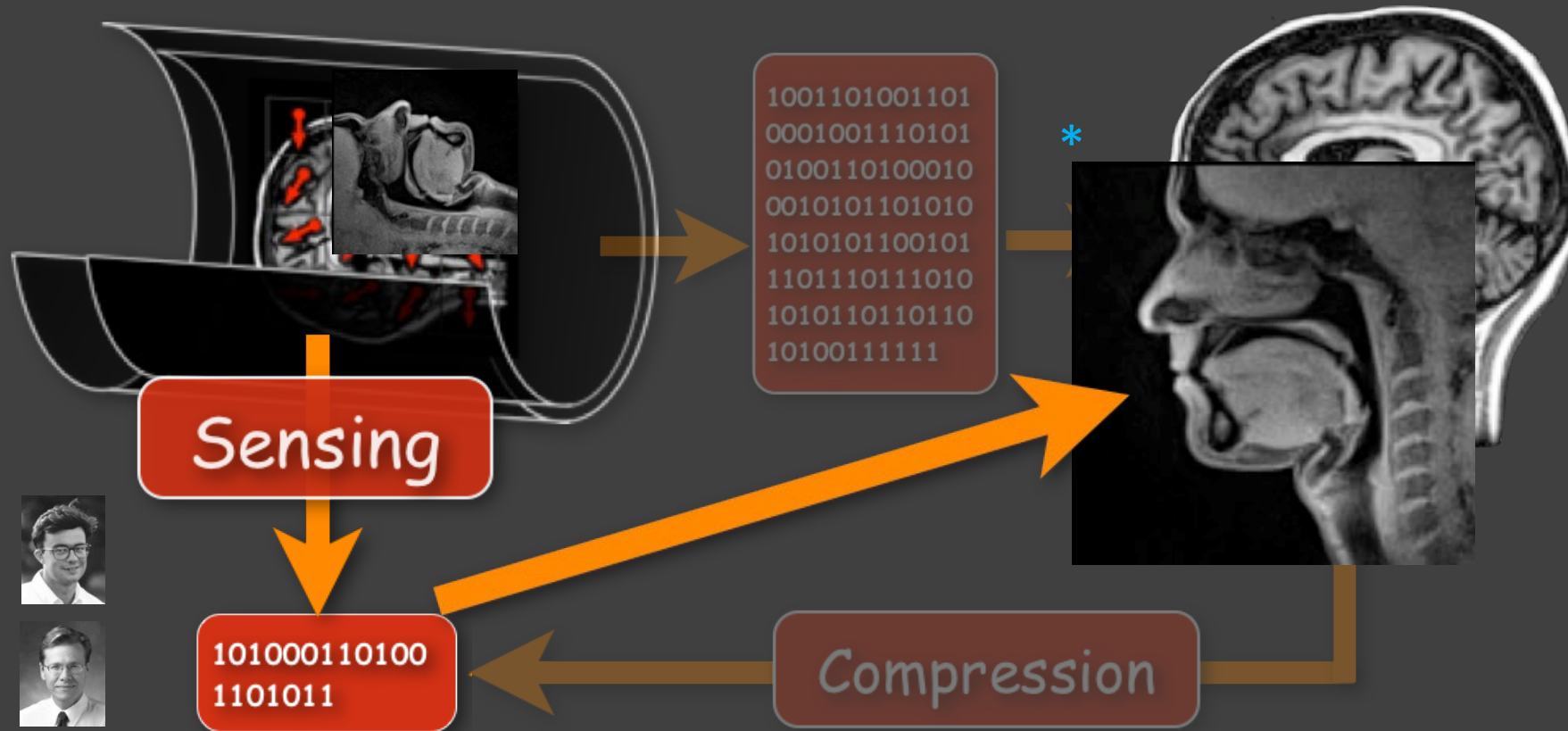
Medical images are compressible
Standard approach: First collect, then compress



Compressed Sensing MRI

Medical images are compressible

New approach: Acquire “compressed” data directly!

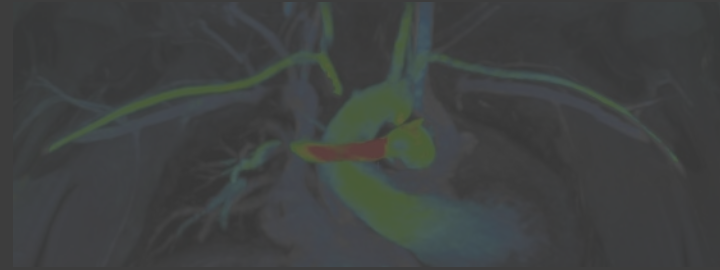


Computational Imaging @ Berkeley

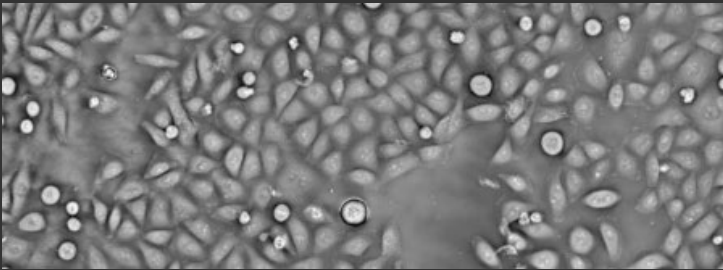
Light Field Cameras
Ren Ng



Compressed Sensing MRI
Michael Lustig



Comp. Illumination Microscopy
Laura Waller

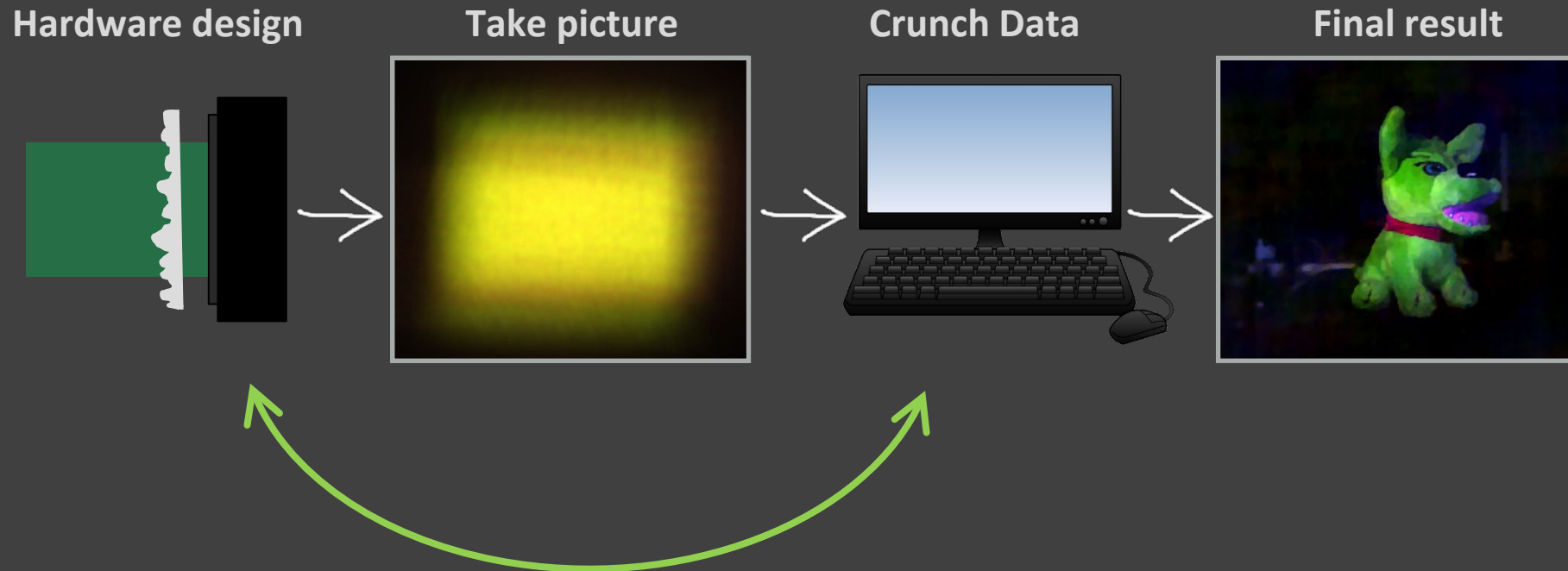


Full-stack Optimization
Ben Recht

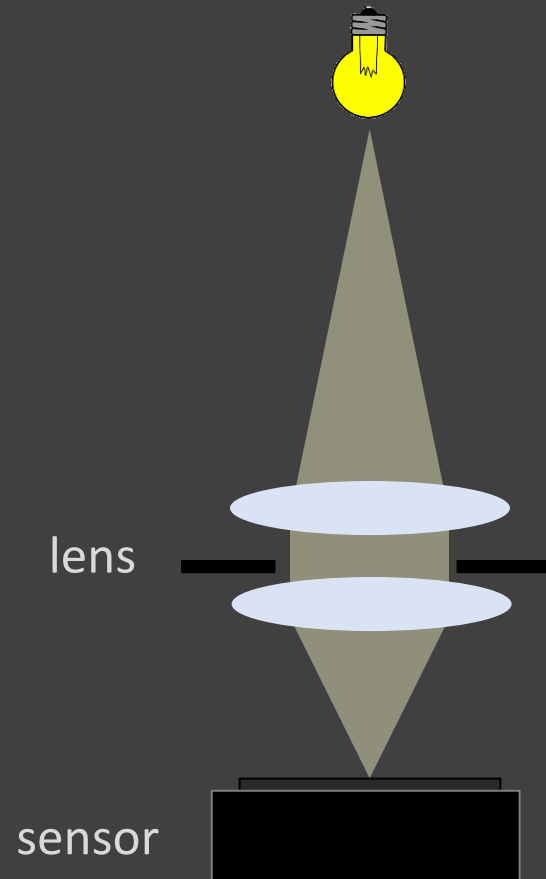


$$x(s) = \text{PSF} \otimes \left\{ \sum_{j=1}^k c_j \delta(s - s_j) \right\}$$

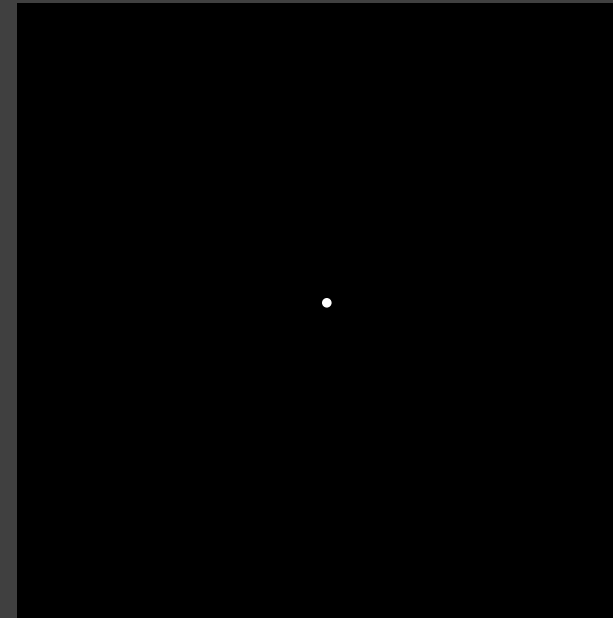
Computational imaging pipeline



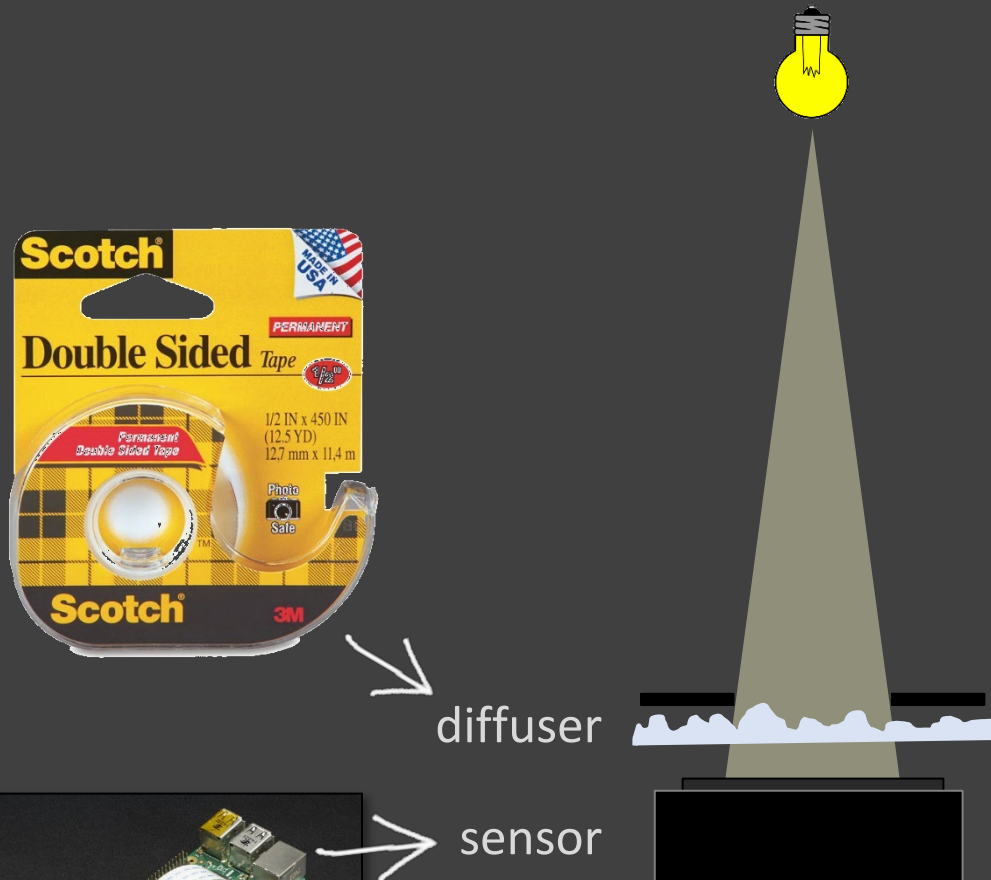
Lenses map points to points



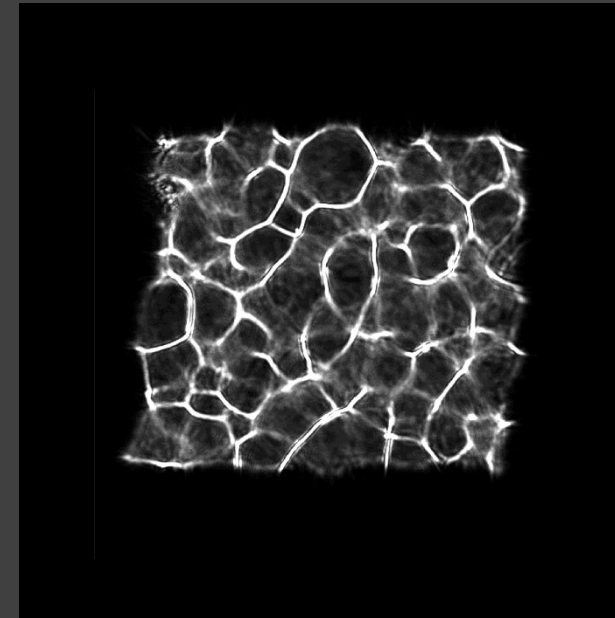
System response to point source



DiffuserCam: stick a scatterer on a sensor

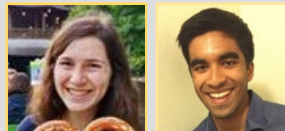


System response to point source

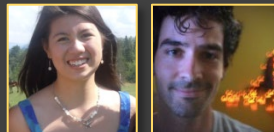
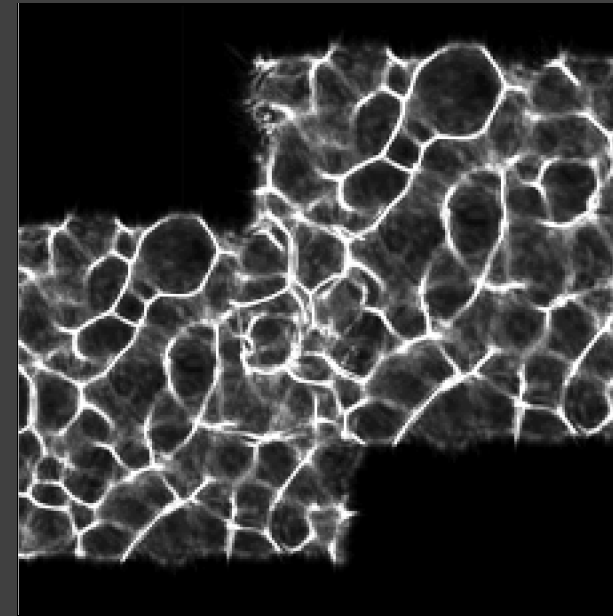
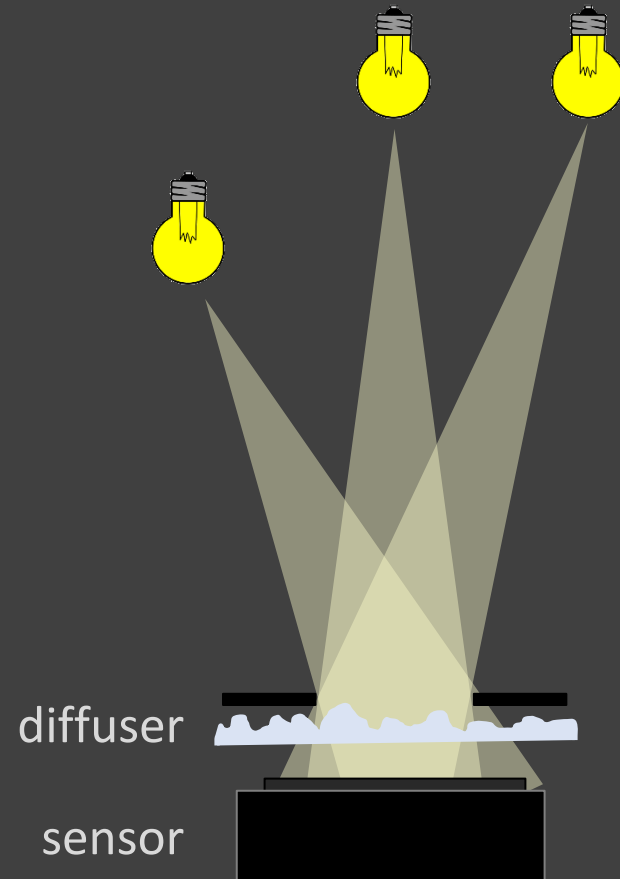


<https://laurawaller.com/opensource>

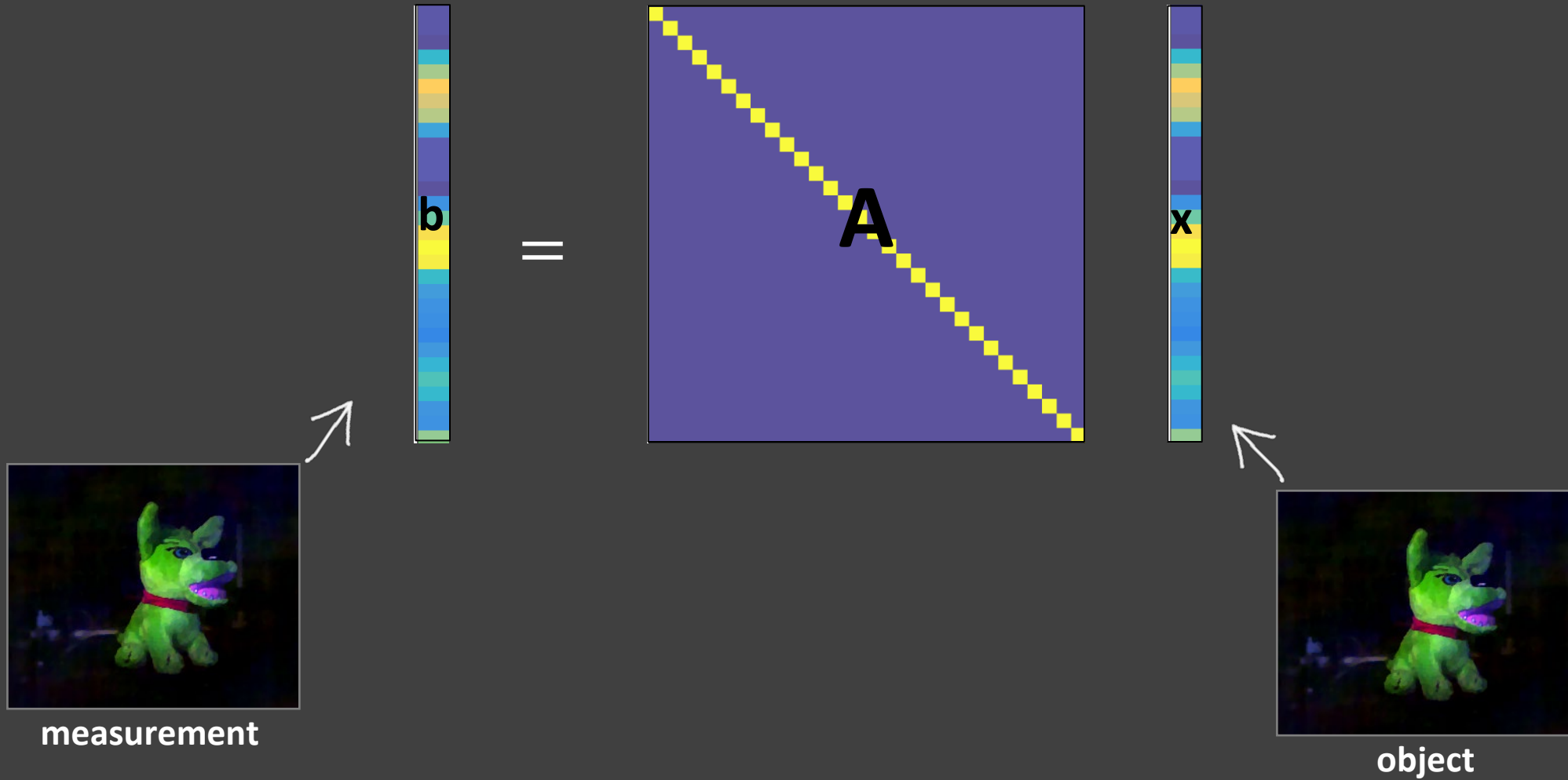
Camille Biscarrat
Shreyas Parthasarathy



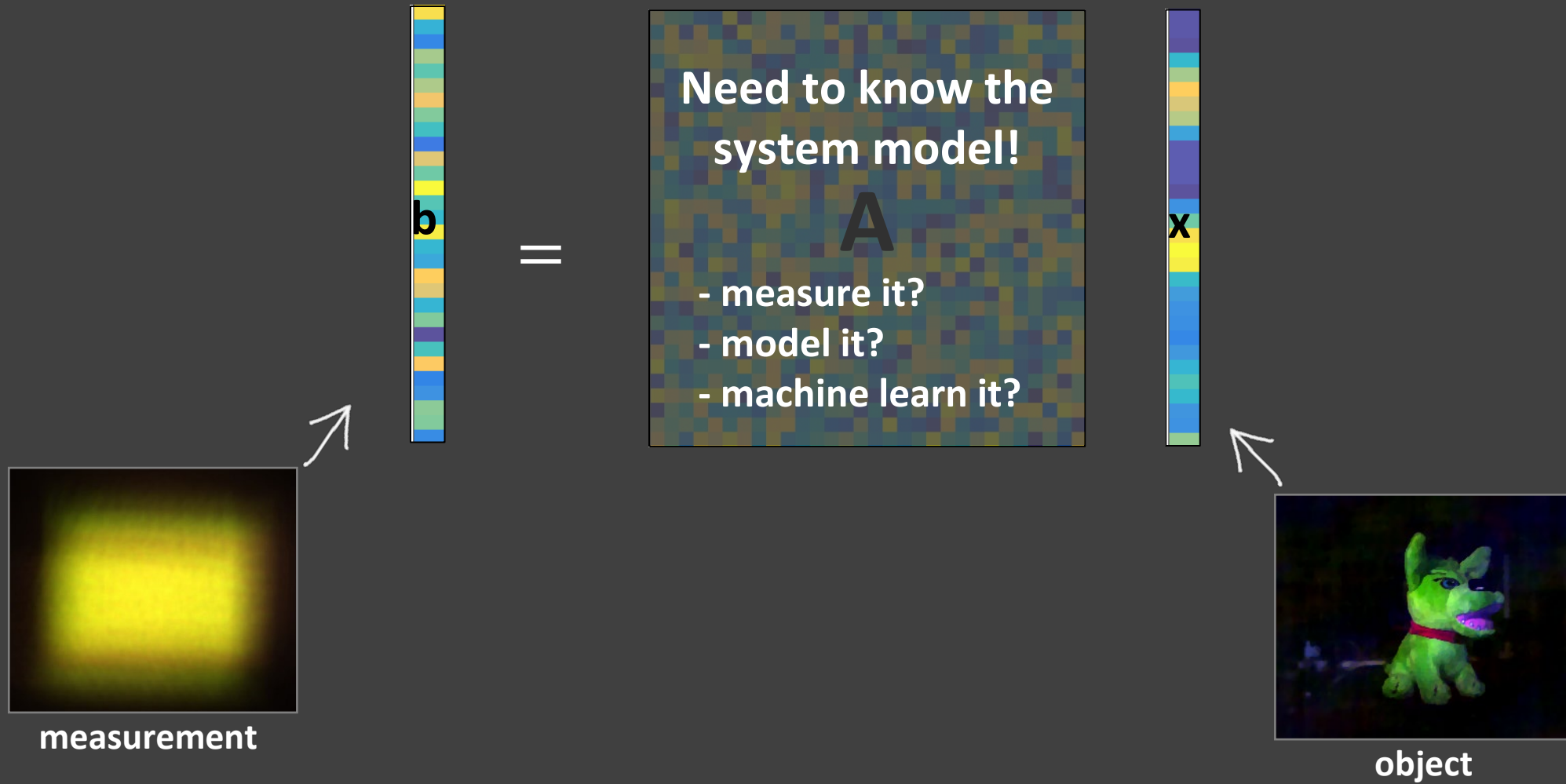
DiffuserCam: stick a scatterer on a sensor



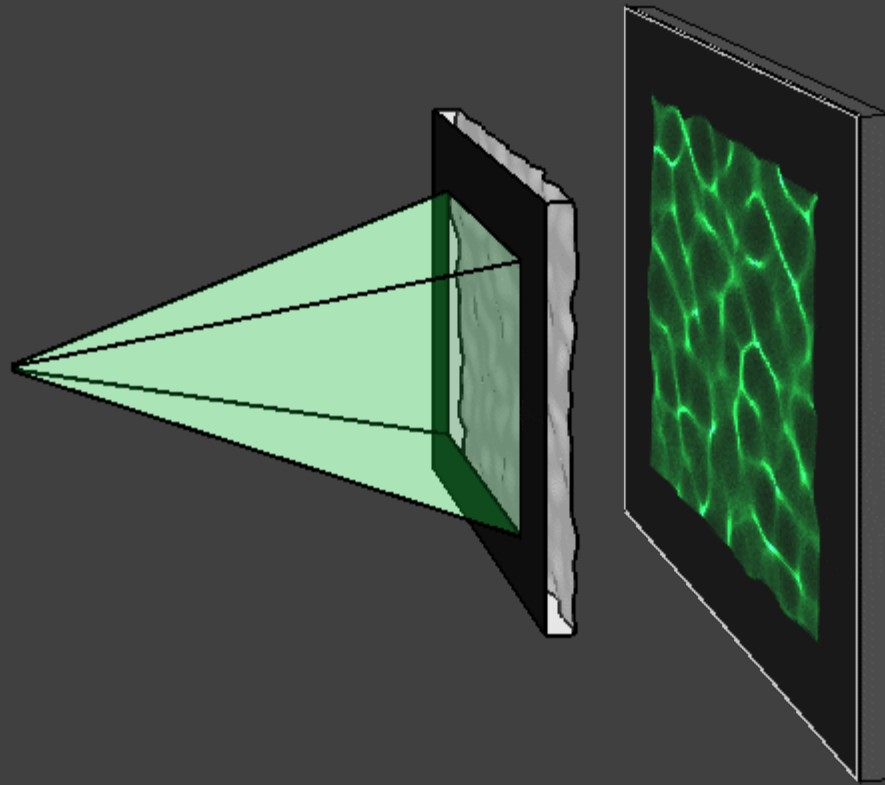
Traditional cameras take direct measurements



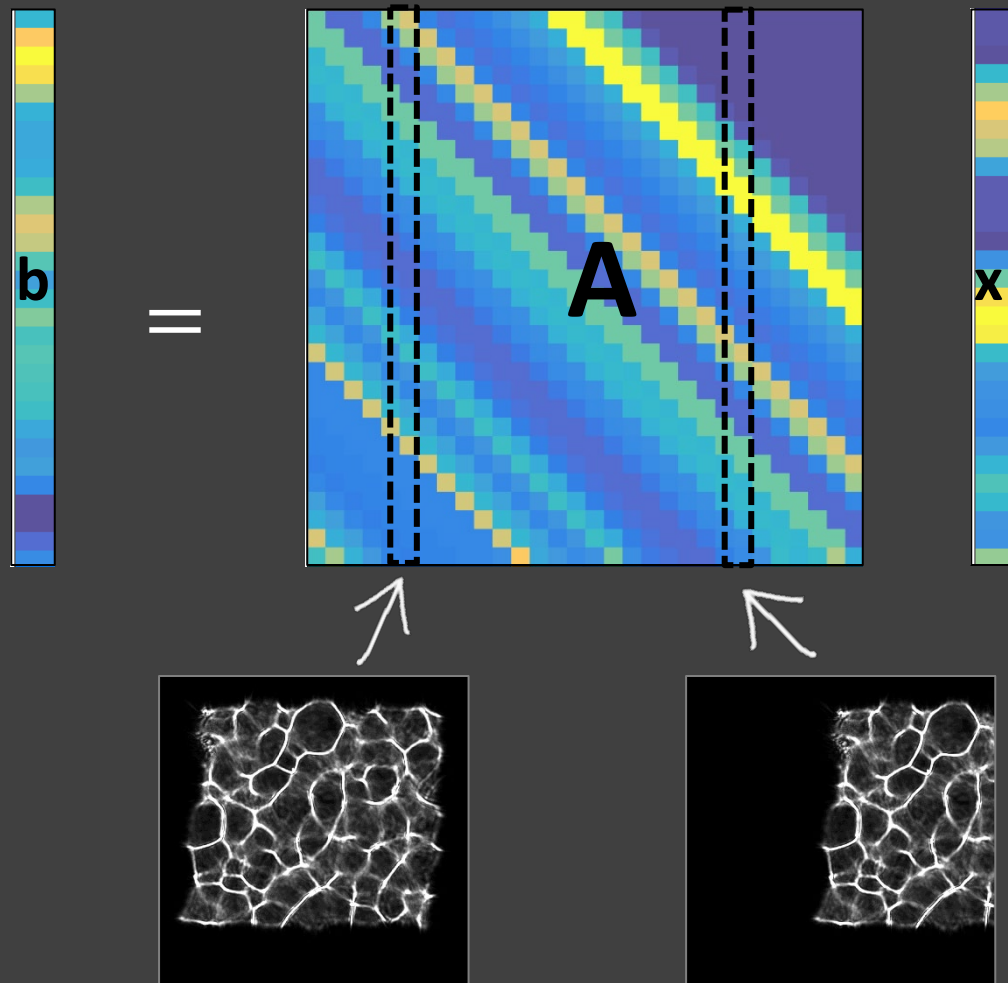
Computational cameras can multiplex



System response shifts with position

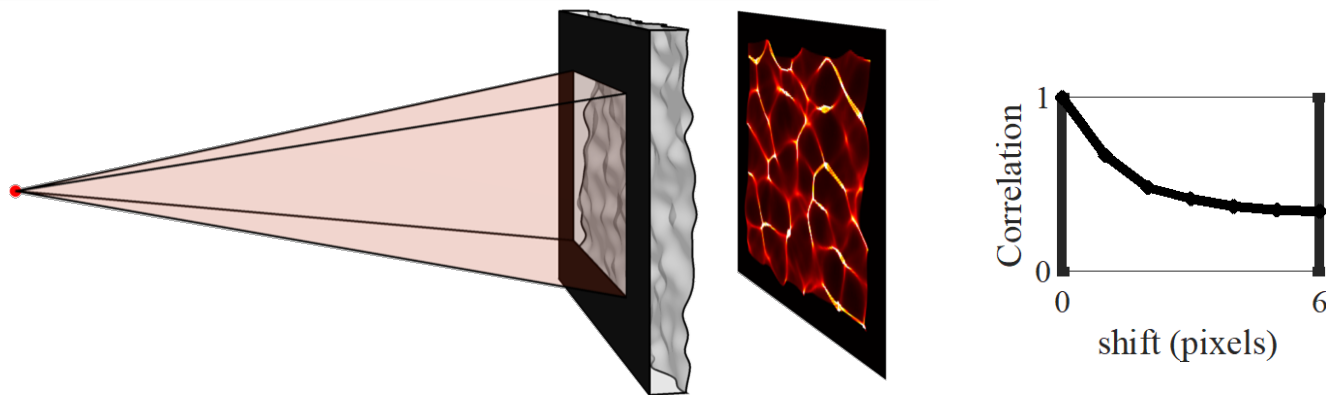


DiffuserCam system model is a 'shift-invariant'



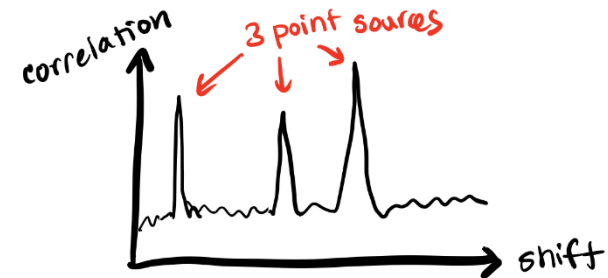
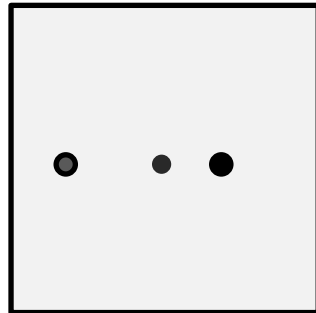
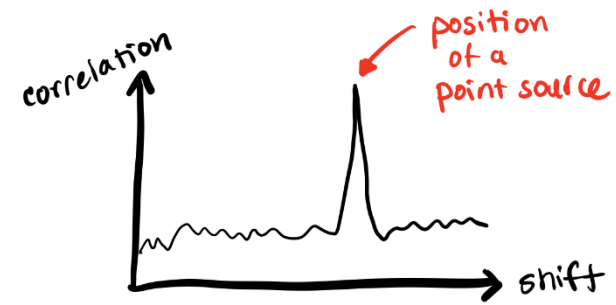
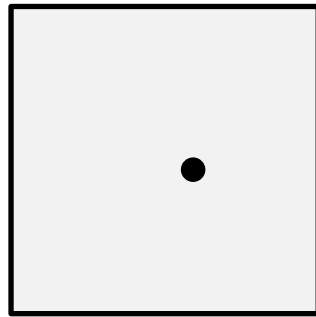
System response is same but shifted
for different image pixels

We could find location of a point by correlating image captured with shifts in system response!



Reconstruction finds strength of each 'point source':

Looks a lot like our GPS problem! (especially if image is sparse)





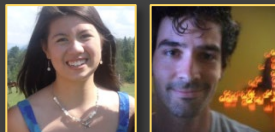
raw sensor data



recovered scene

*solver is ADMM with TV reg in Halide

Grace Kuo
Nick Antipa





raw sensor data



recovered scene

*solver is ADMM with TV reg in Halide

Grace Kuo
Nick Antipa

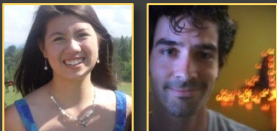


Image reconstruction is nonlinear optimization

$$\arg \min_{x \geq 0} \left\| \begin{array}{c} \text{b} \\ \text{A} \end{array} * x \right\|_2^2 + \lambda \left\| \Phi \begin{array}{c} x \\ \dots \\ x \end{array} \right\|_1$$

↑
Sparsity basis

***solved with ADMM in Halide**

S. Boyd, et al. *Foundations and Trends in Machine Learning* (2011)

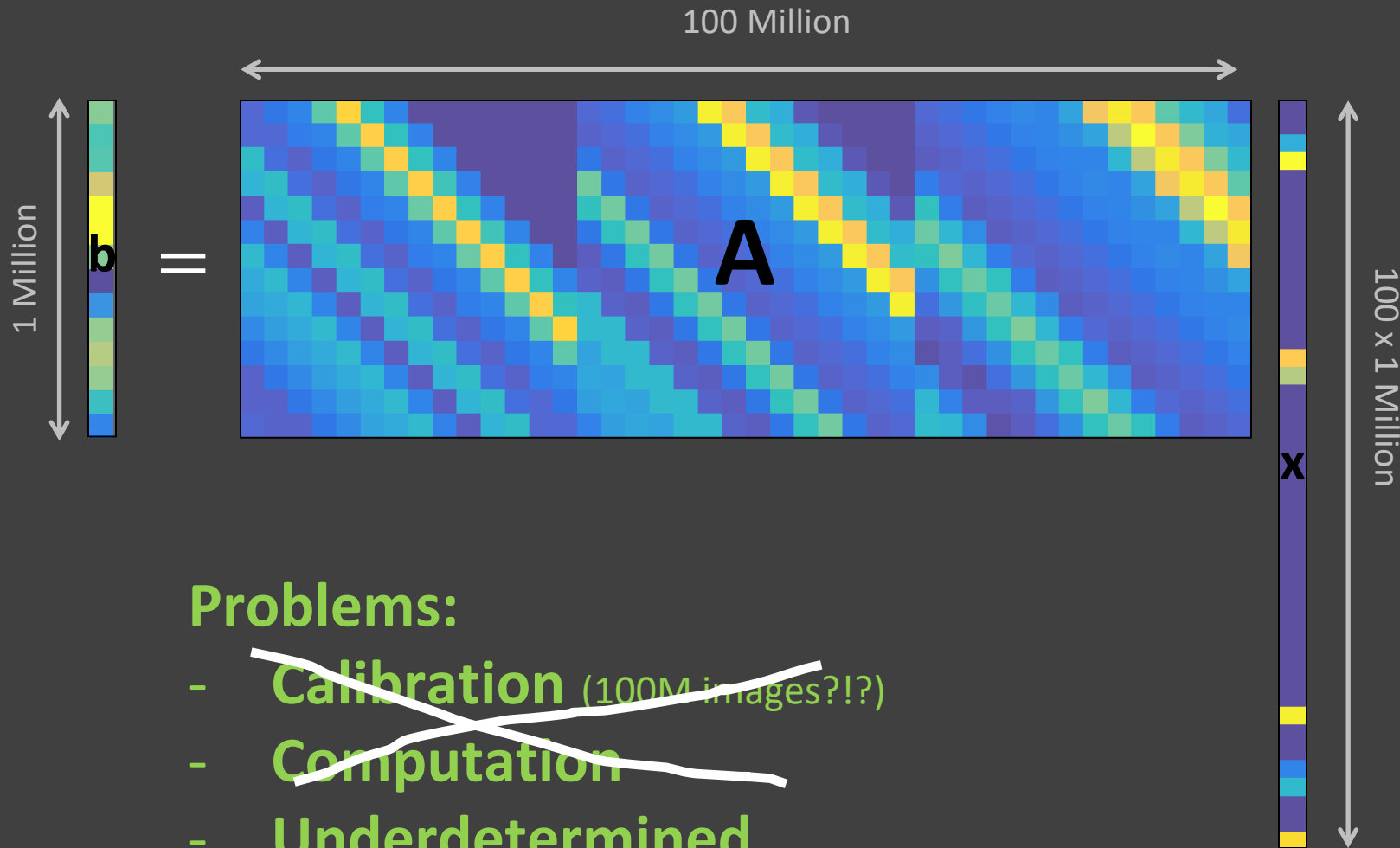
J. Ragan-Kelley, et al. *AMC SIGPLAN* (2013)



Cute! But what's it good for?

2D → 3D

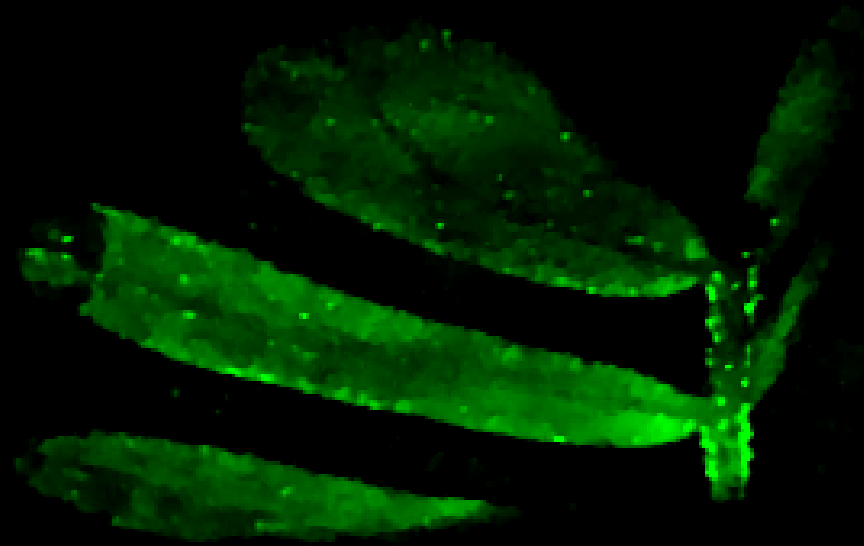
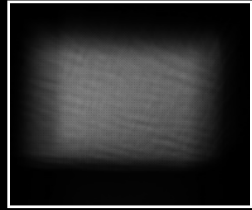
Single-shot 3D is underdetermined



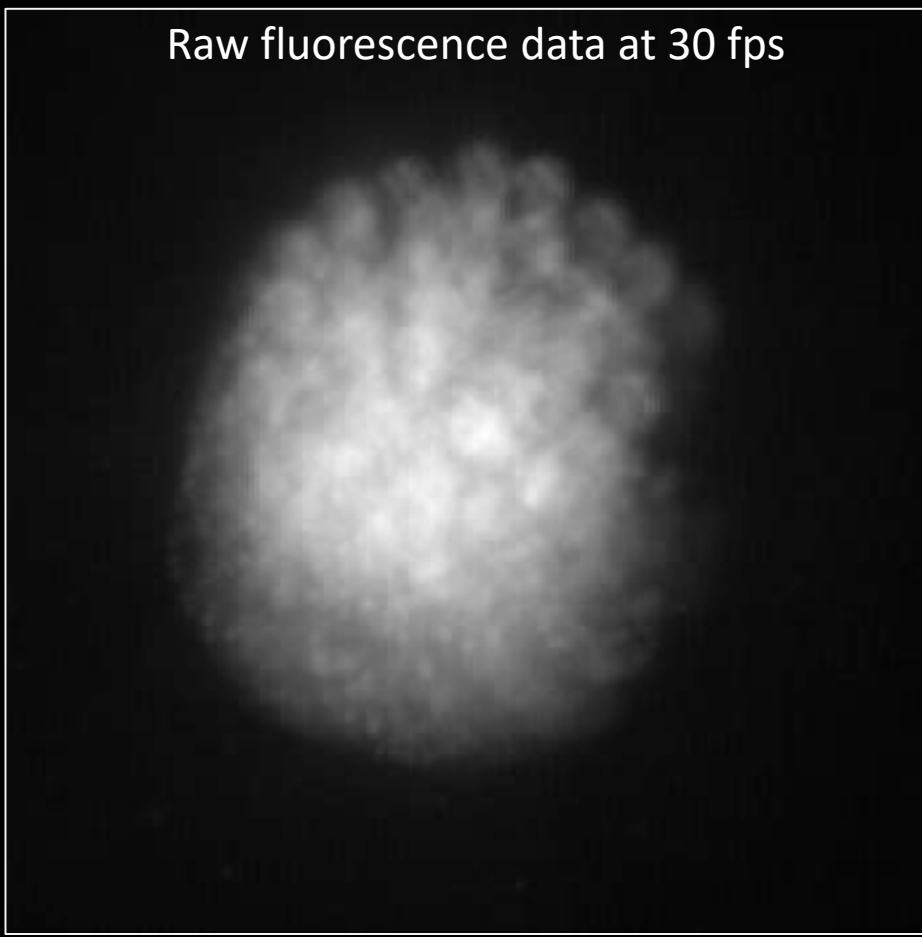
Problems:

- ~~Calibration~~ (100M images?!?)
- ~~Computation~~
- Underdetermined

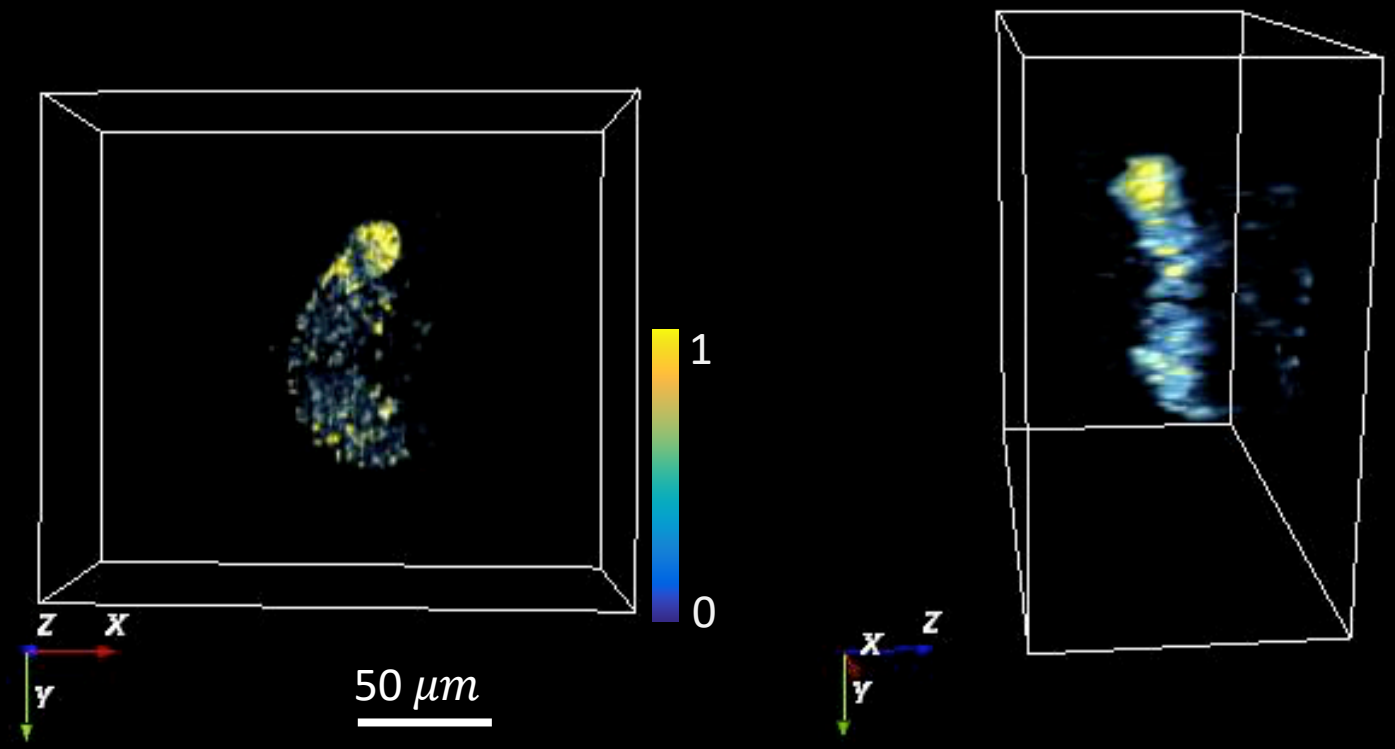
128x more voxels for **FREE!**



Raw fluorescence data at 30 fps



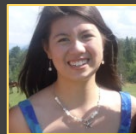
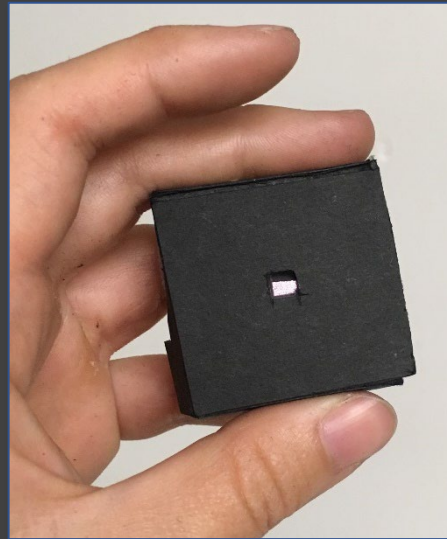
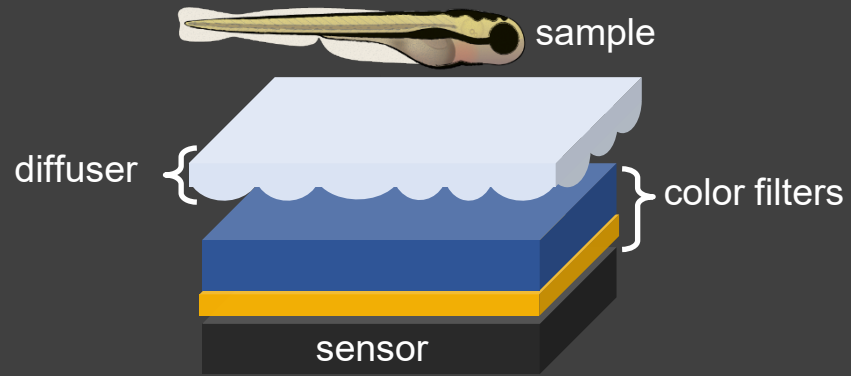
3D video reconstruction



Kyrollos Yanny
Nick Antipa



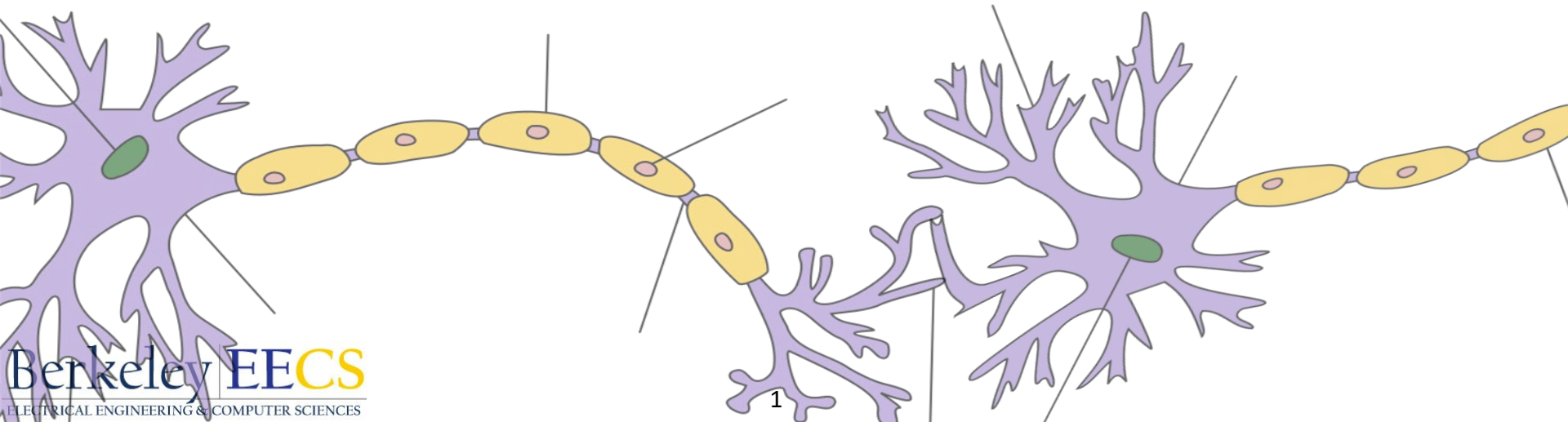
Neural activity tracking with flat DiffuserScope



Grace Kuo

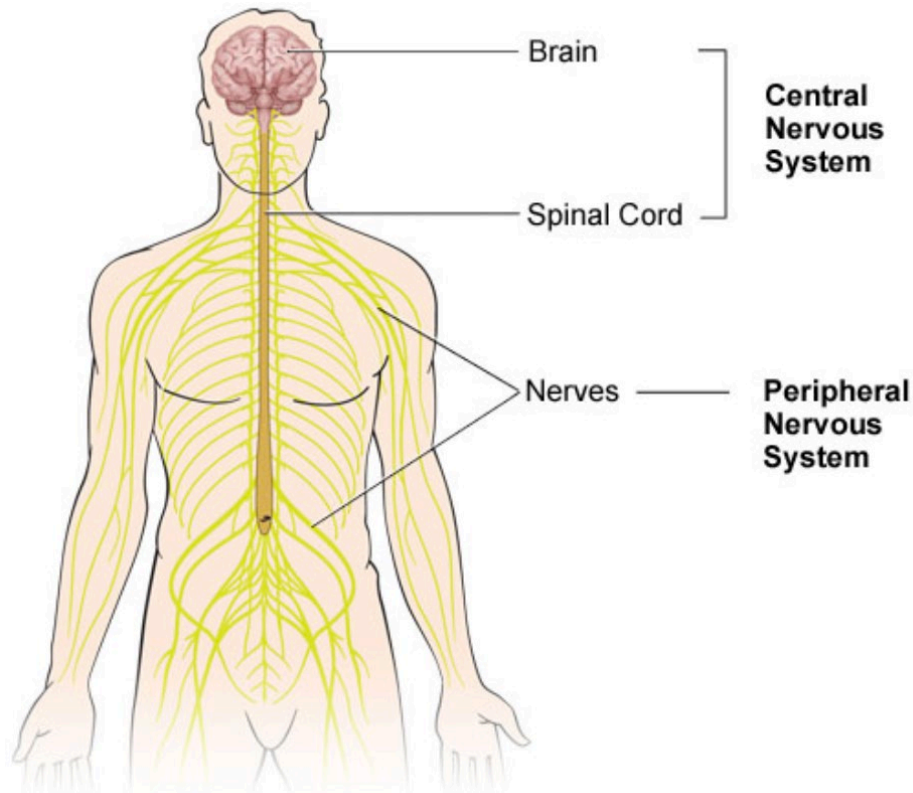
EECS 16A

Neurons are Circuits!

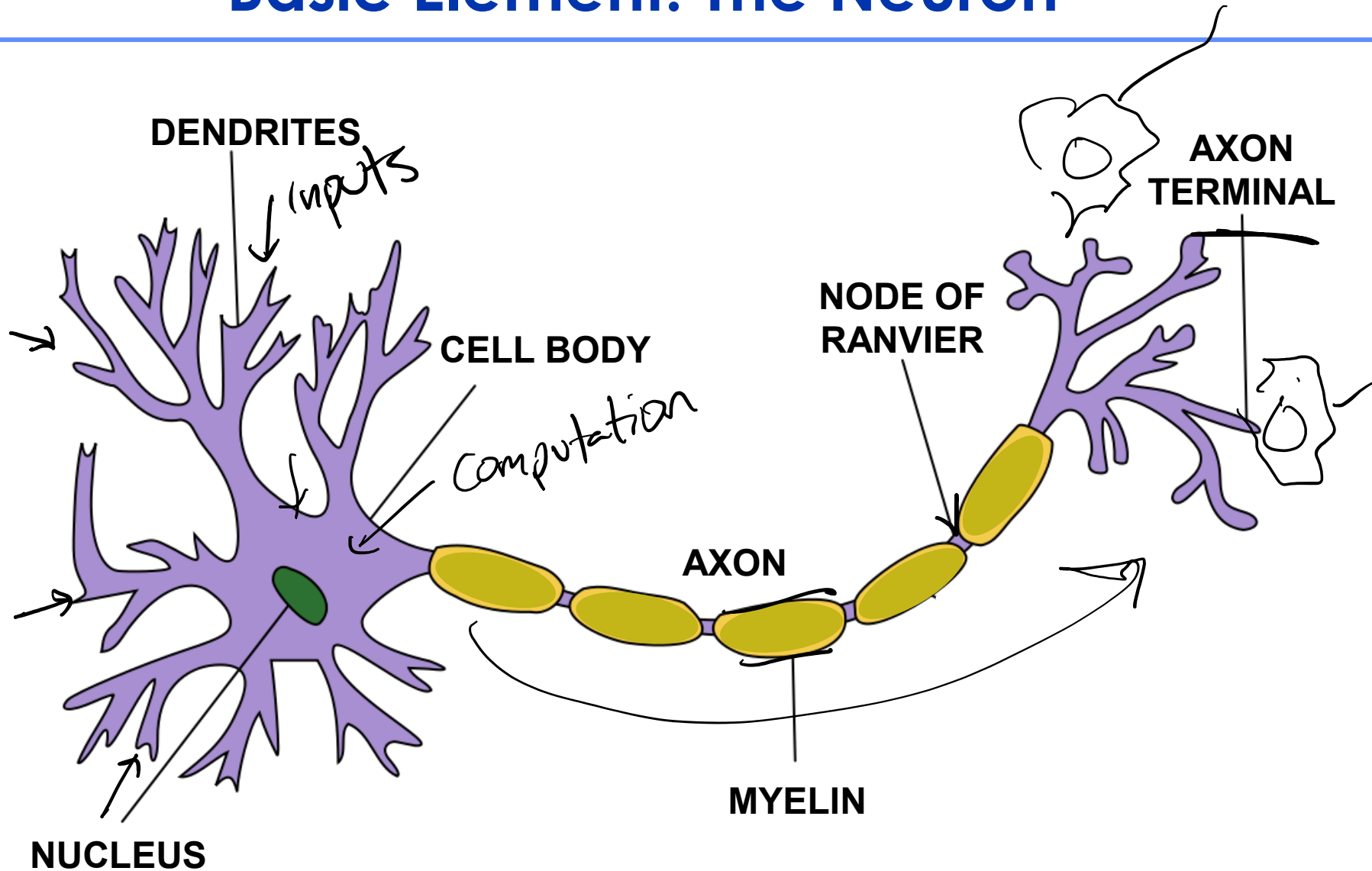


The Body Electric - Nervous System

- **There are two distinct parts of the nervous system**
 - Central Nervous system: Brain, Spinal Cord
 - Peripheral Nervous system: All other neural elements, including the peripheral nerves (motor and sensory) and the autonomic nerves (regulate internal organs)

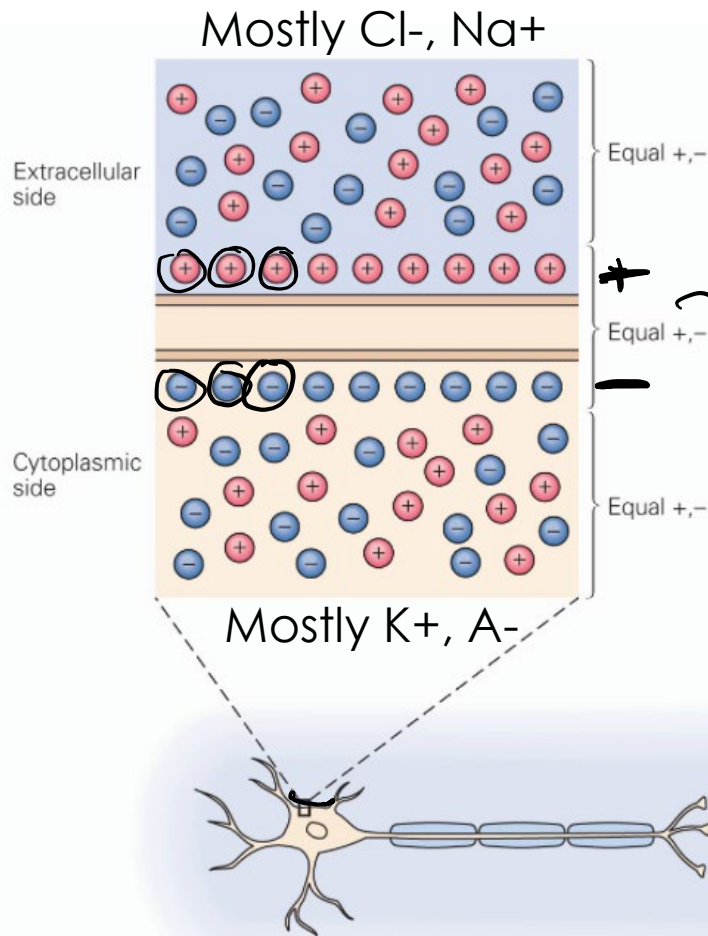


Basic Element: The Neuron



You have 100 billion of these

Resting Membrane Potential



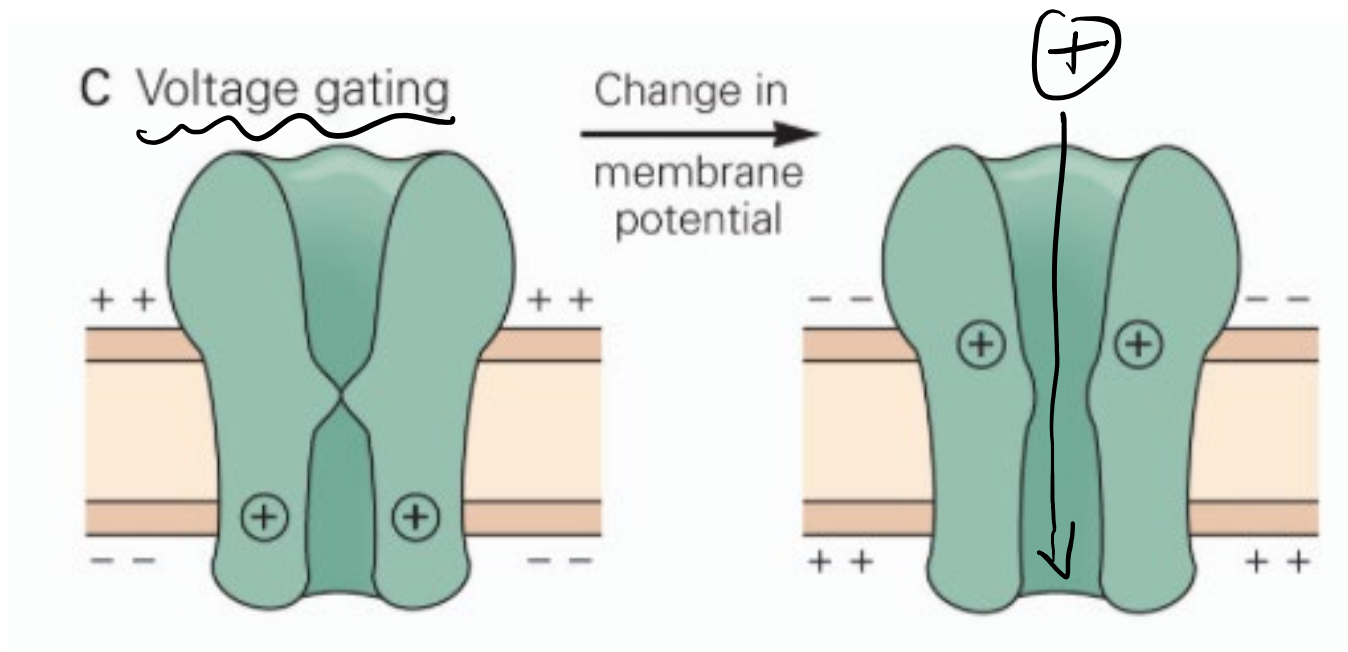
- Nernst Equation ($E \rightarrow$ potential)

$$E_X = \frac{RT}{zF} \ln \frac{[X]_o}{[X]_i} \leftarrow \text{Concentrations}$$

- R – gas constant
- T – Temperature (K)
- F – Faraday constant
- z – Valence of ion
- $RT/zF \sim 25\text{mV}$ at 25C, K⁺ – Equivalent to $kT/q!$
- E_k is typically ~ -70 to -80mV

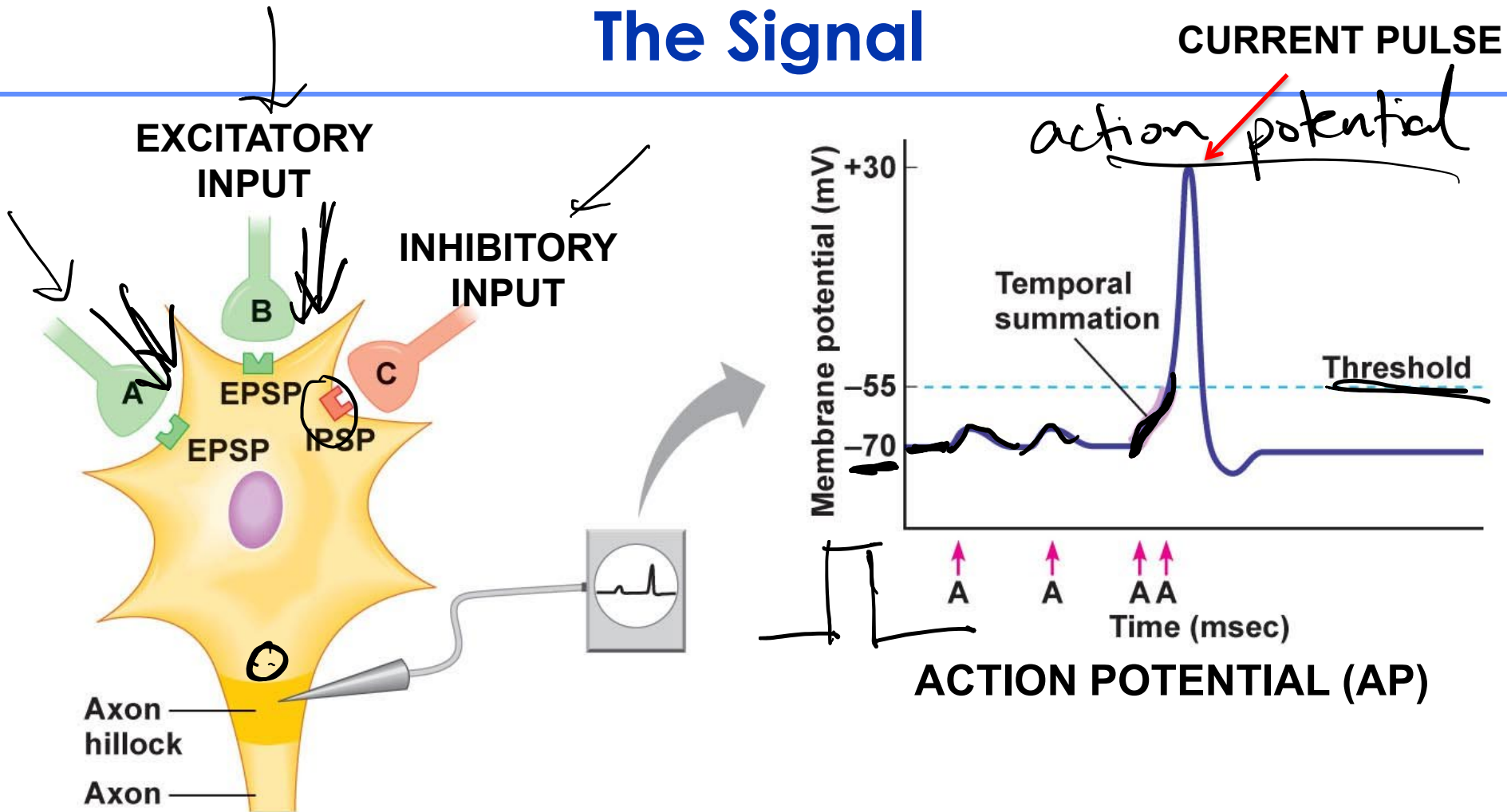
- Resting channels are permeable to K⁺ diffusing out of the cell causing (+) charges to accumulate at the cell surface and (-) charges inside
- This self limits when the electrical force negates the chemical force

Ion Channels (switches)



- **There are several types of stimuli controlling ion channels opening and closing**
 - These can be chemical, electrical or mechanical

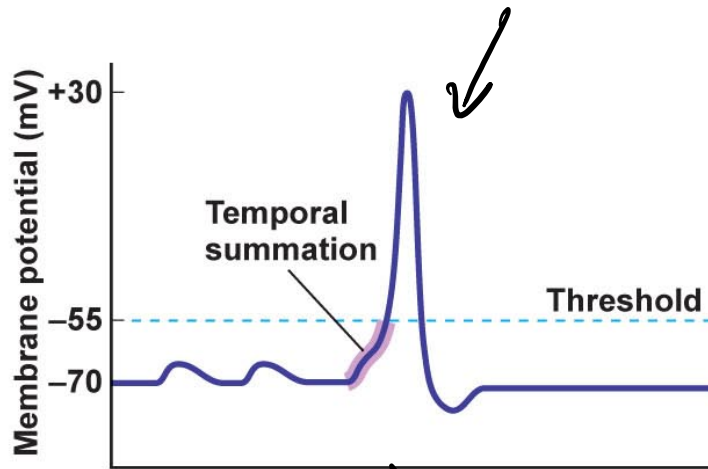
The Signal



© 2011 Pearson Education, Inc.

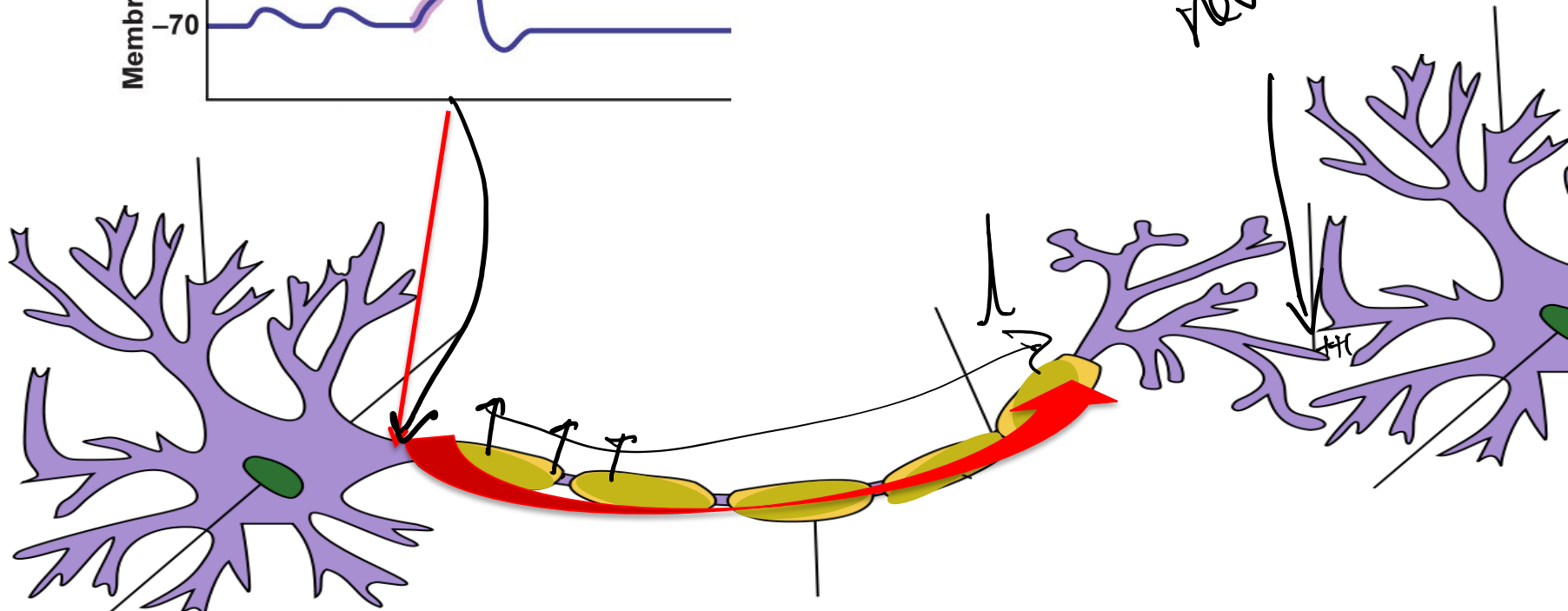
- Ion channels open in response to stimuli causing the cell to depolarize
- There is temporal and spatial summation
- Once the membrane potential goes above threshold, it starts an AP

Action Potential Propagation



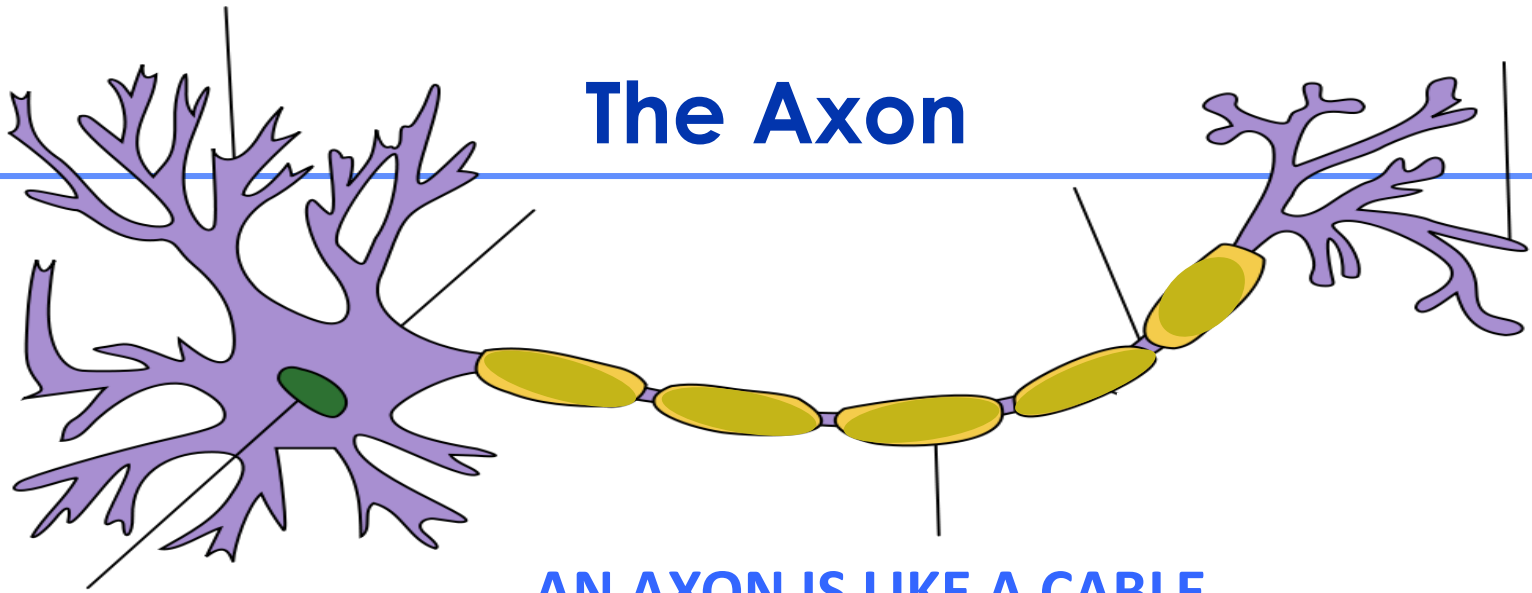
HOW FAR CAN THE SIGNAL PROPAGATE WITHOUT DYING OUT?

Neurotransmitter

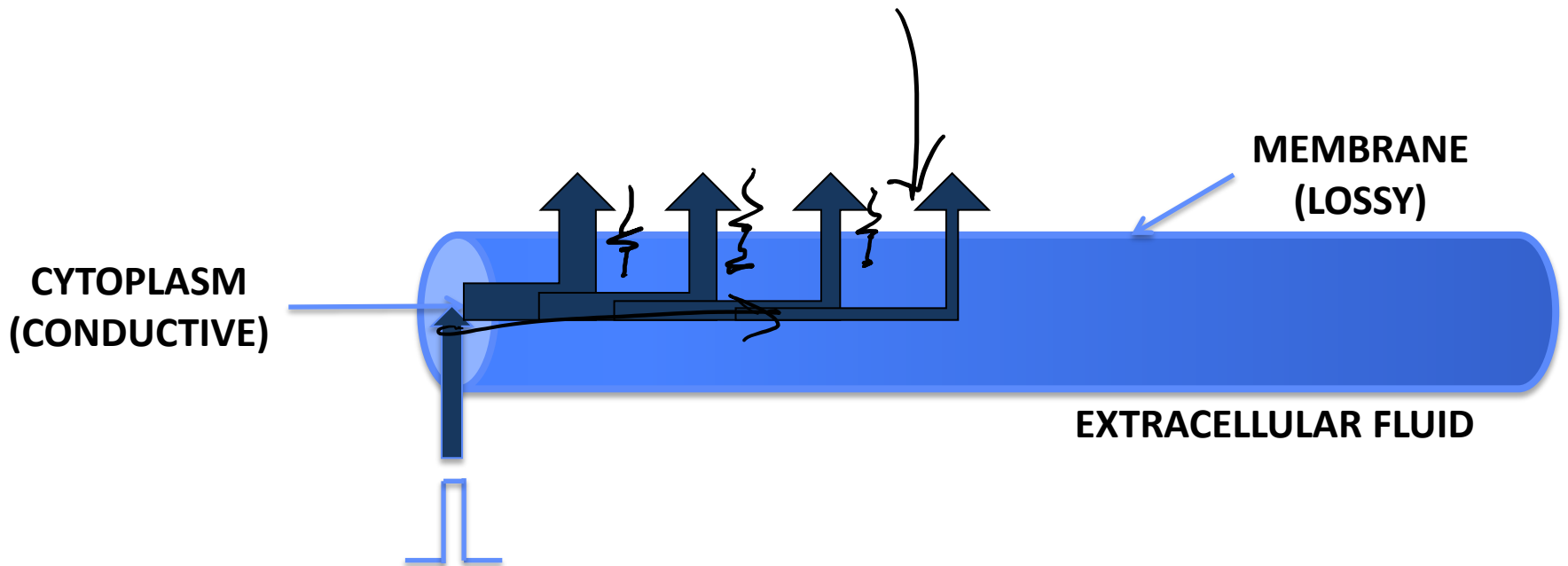


MICRONS TO METERS

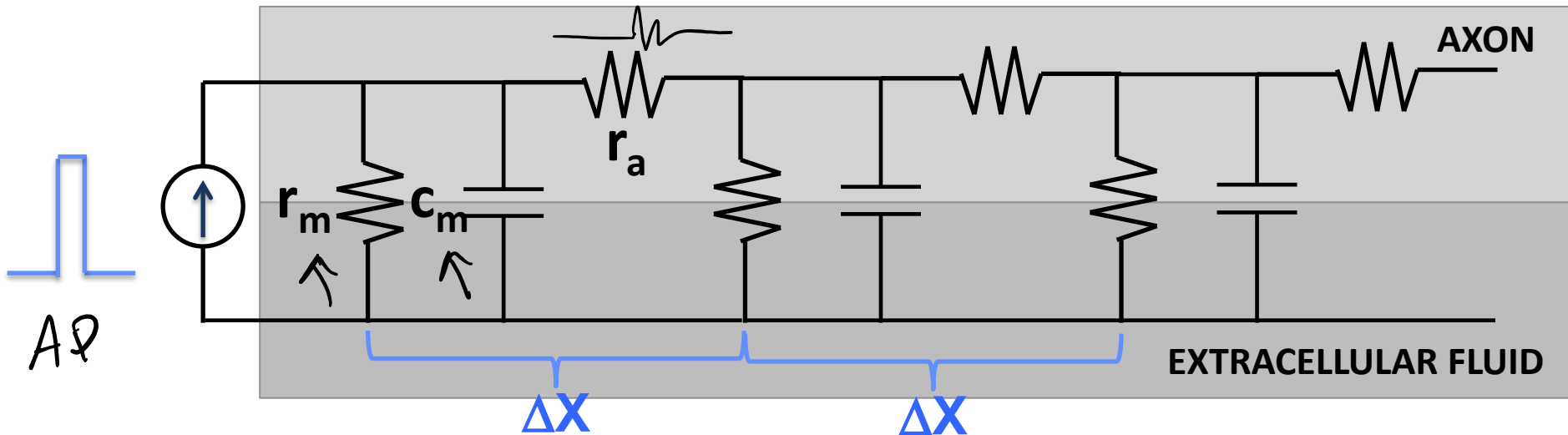
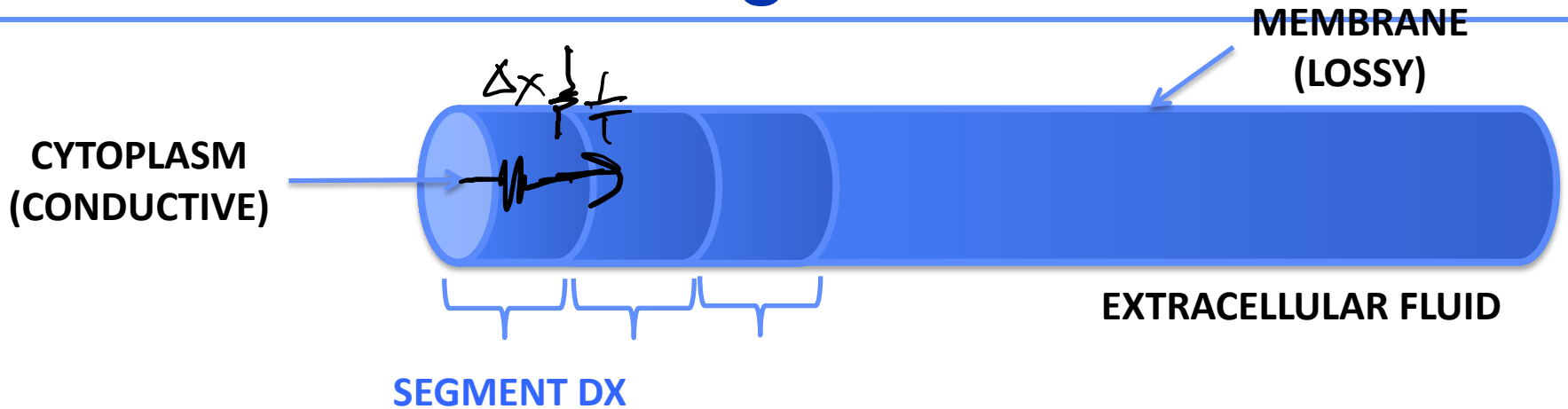
The Axon



AN AXON IS LIKE A CABLE



Modeling the Axon

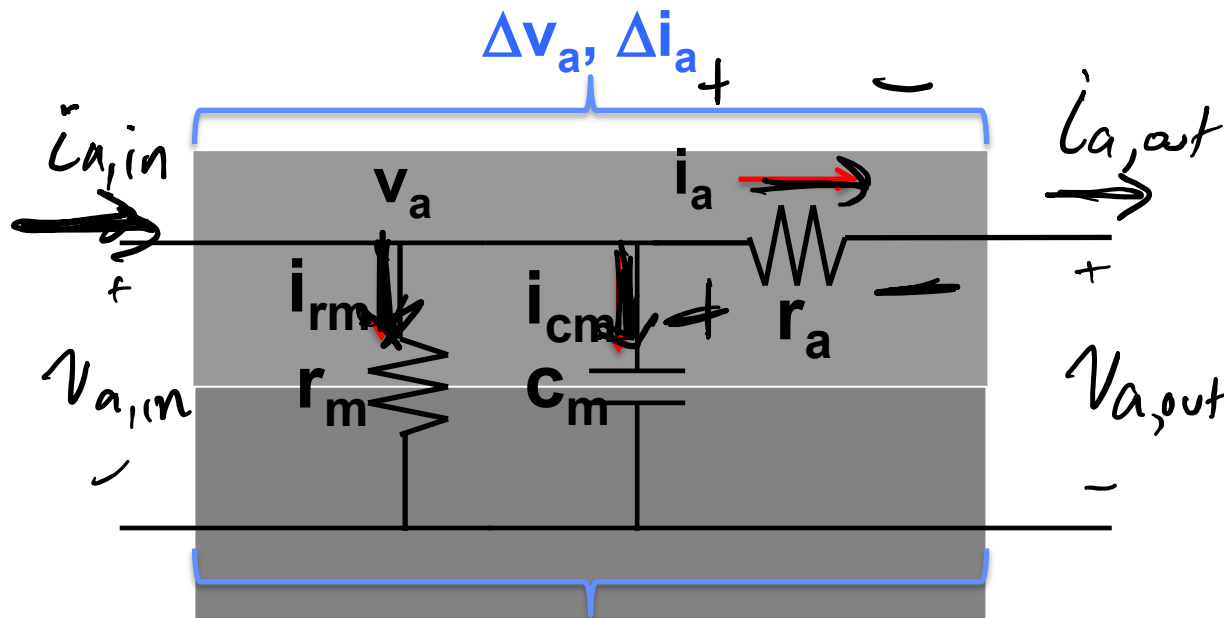


r_m = membrane resistance [$\Omega \cdot m$];

r_a = axon resistance [Ω/m]

c_m = membrane capacitance [F/m]

A Single Axon Segment



Δx

KIRCHOFF'S VOLTAGE LAW

$$\Delta v_a = -i_a r_a \Delta x$$

$$\frac{\Delta v_a}{\Delta x} = -i_a r_a$$

KIRCHOFF'S CURRENT LAW

$$\Delta i_a = (-i_{rm} - i_{cm}) \Delta x$$

$I = C \frac{dv}{dt}$

$$\frac{\Delta i_a}{\Delta x} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

The Rest is Math

→ EECS 16B
120

$$\frac{\Delta v_a}{\Delta x} = -i_a r_a$$

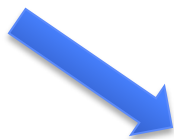
$$\underline{\Delta x} \rightarrow 0$$

$$-\frac{1}{r_a} \frac{\partial v_a}{\partial x} = i_a$$

$$\frac{\Delta i_a}{\Delta x} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

$$\frac{\partial i_a}{\partial x} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = \frac{\partial i_a}{\partial x}$$



*

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$



The Equation

$$-\frac{1}{r_a} \frac{\partial^2 v_a}{\partial x^2} = -\frac{v_a}{r_m} - c_m \frac{\partial v_a}{\partial t}$$

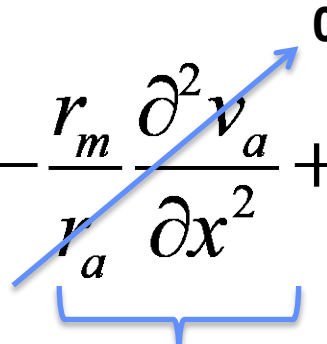
REARRANGE
TERMS

$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0 \quad \left. \vphantom{\frac{\partial^2 v_a}{\partial x^2}} \right\} \text{final equation}$$

SPACE DEPENDENT TIME DEPENDENT

LET'S ANALYZE ONE AT A TIME

Time Dependence

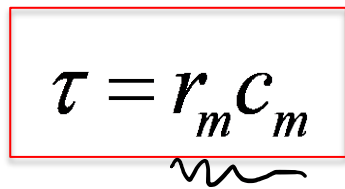
$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0$$


HOLD SPACE
CONSTANT

$$c_m r_m \frac{\partial v_a}{\partial t} + v_a = 0$$

ODE WITH STANDARD SOLUTION

$$\rightarrow v_a(t) = v_o e^{-t/\tau}$$

$$\tau = r_m c_m$$


TIME CONSTANT

Space Dependence

$$\underbrace{-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + c_m r_m \frac{\partial v_a}{\partial t}}_0 + v_a = 0$$

HOLD TIME
CONSTANT

$$-\frac{r_m}{r_a} \frac{\partial^2 v_a}{\partial x^2} + v_a = 0$$

ODE WITH STANDARD SOLUTION

$$v_a(x) = v_o e^{-x/\lambda}$$

$$\lambda = \sqrt{\frac{r_m}{r_a}}$$

SPACE CONSTANT

What does it mean?

$$\lambda = \sqrt{\frac{r_m}{r_a}}$$

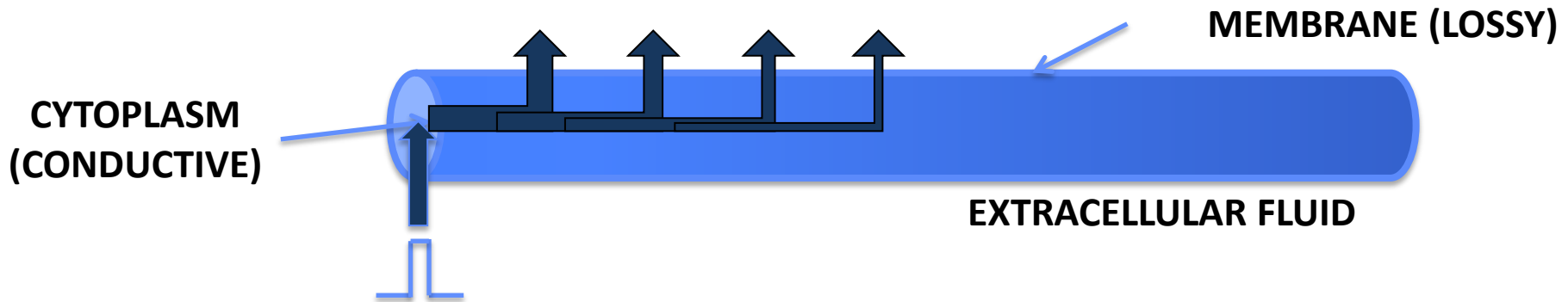
* **SPACE CONSTANT**

The distance it takes for a signal to decay away

$$\tau = r_m c_m$$

* **TIME CONSTANT**

The amount of time it takes for a signal to decay away



HOW CAN IT BE IMPROVED?

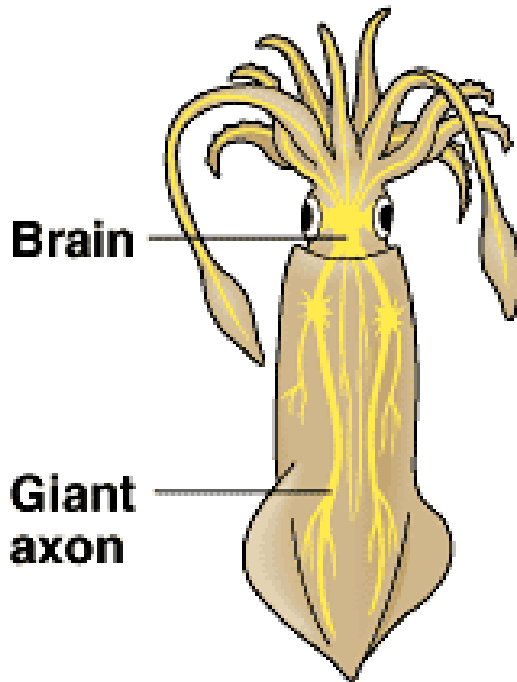
Evolution

$$\uparrow \lambda = \sqrt{\frac{r_m \uparrow}{r_a \downarrow}}$$

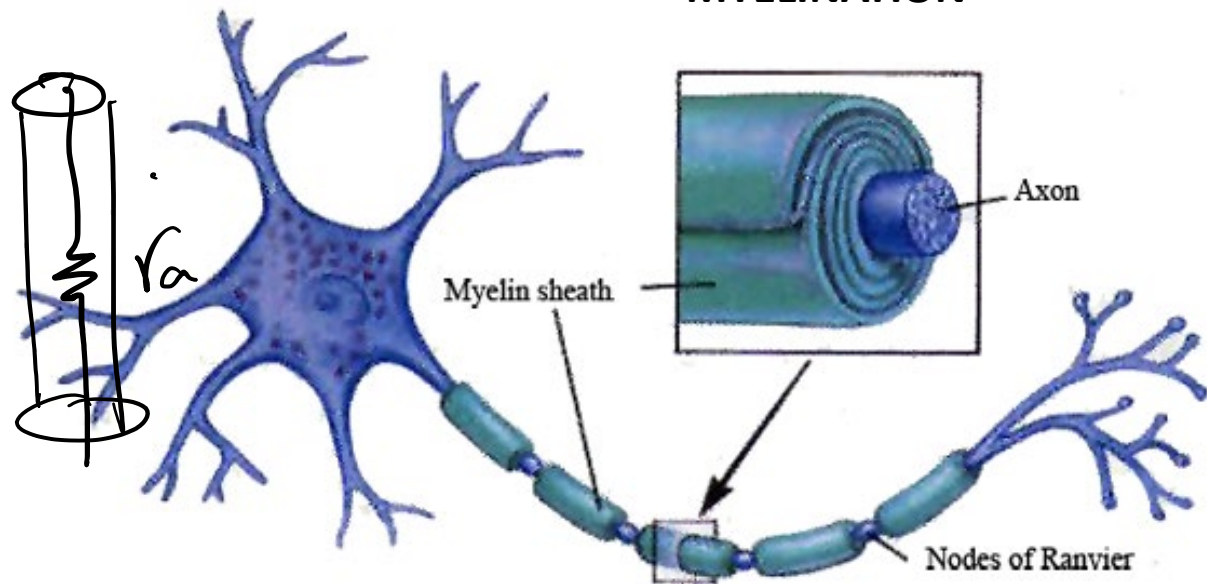
SPACE CONSTANT

$$\uparrow \tau = r_m c_m$$

TIME CONSTANT



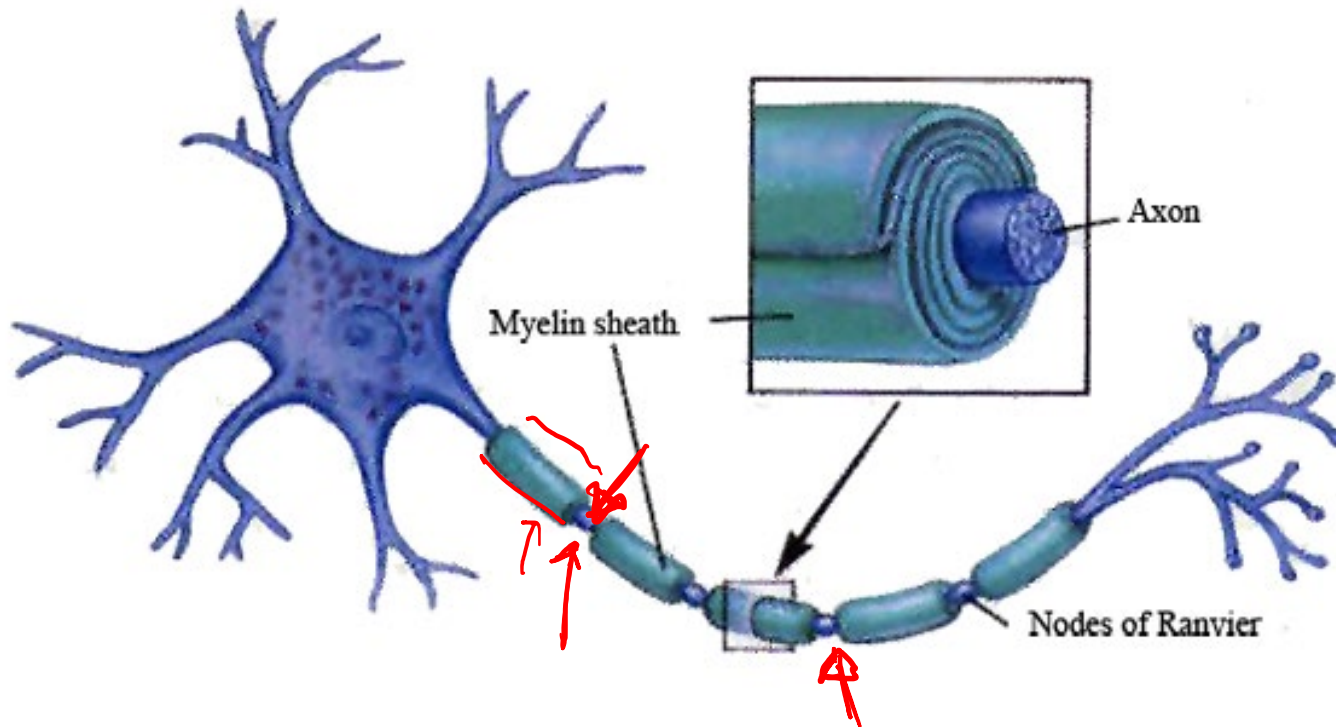
$$r_a \downarrow, \lambda \uparrow$$



MYELINATION

$$r_m \uparrow, c_m \downarrow, \lambda \uparrow$$

Nodes of Ranvier



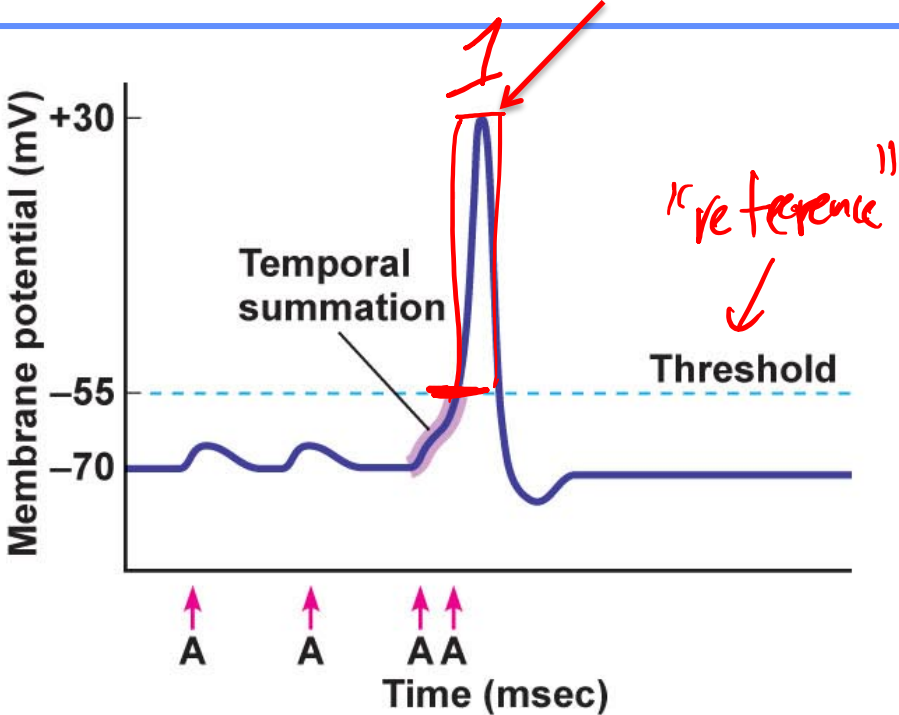
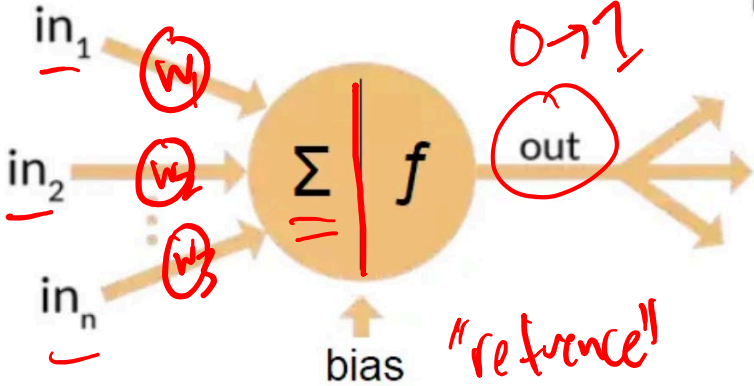
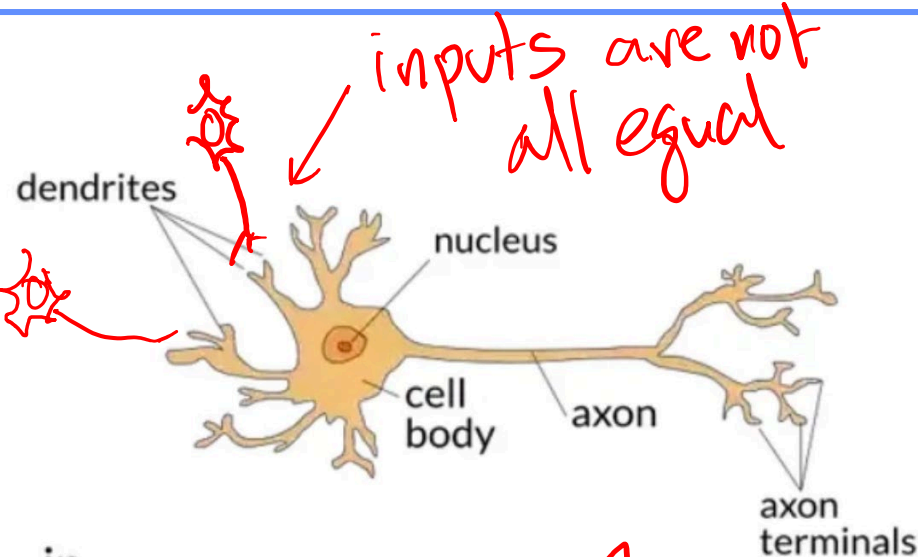
What do you think Nodes of Ranvier are for?

Hint: think about a digital signal propagating across a very long wire..

*digital repeater
EECS 151*

Neural Networks

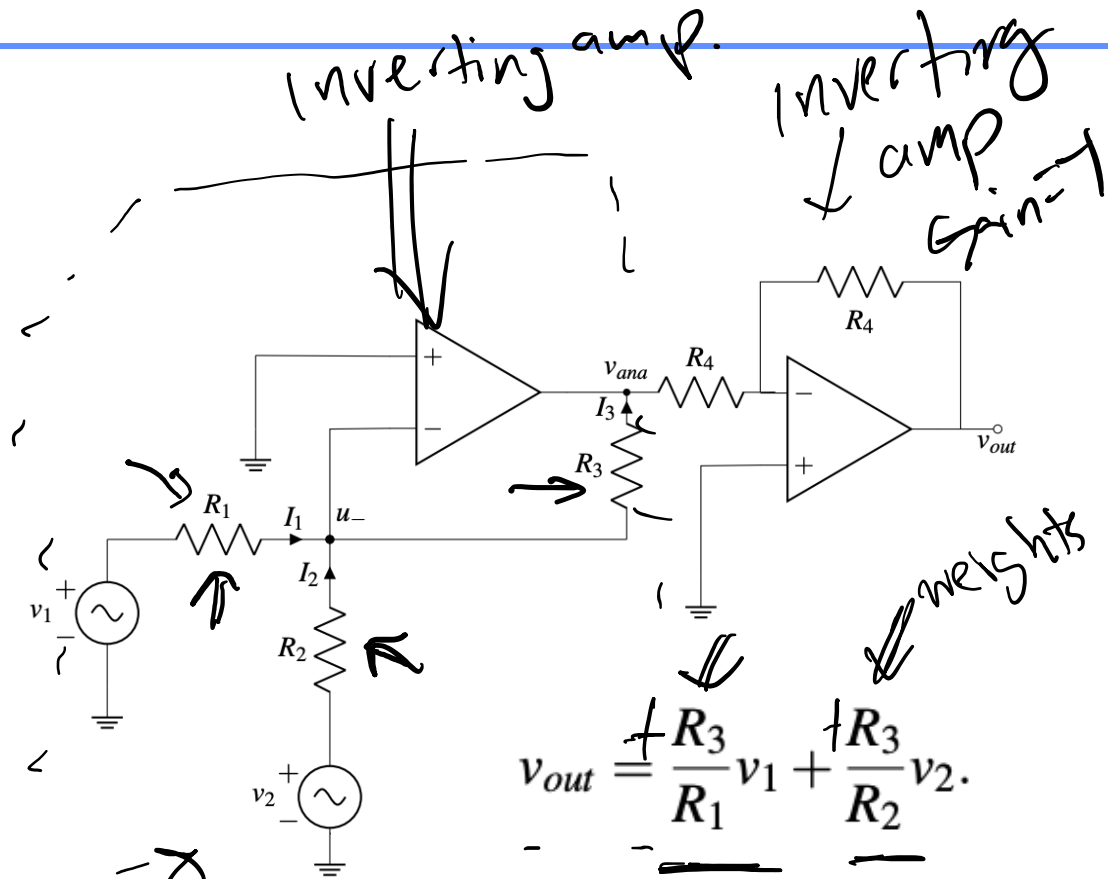
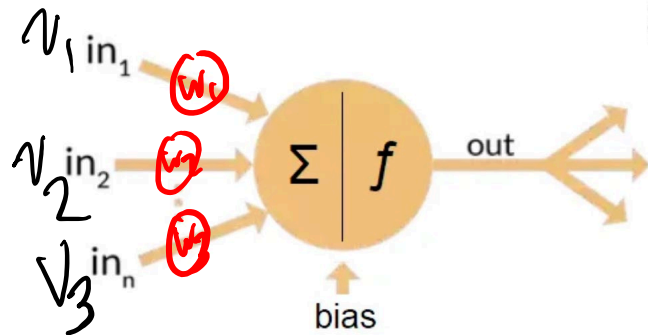
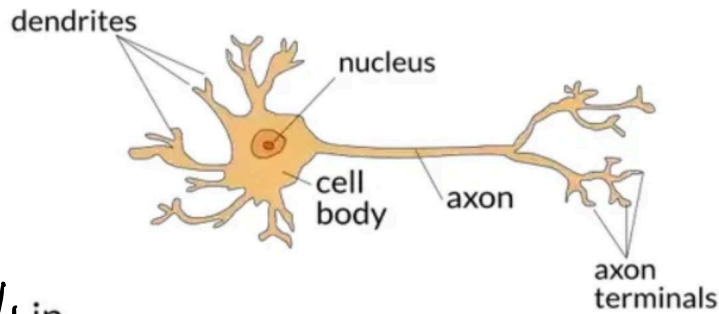
CURRENT PULSE



ACTION POTENTIAL (AP)

circuits!

Artificial Neuron

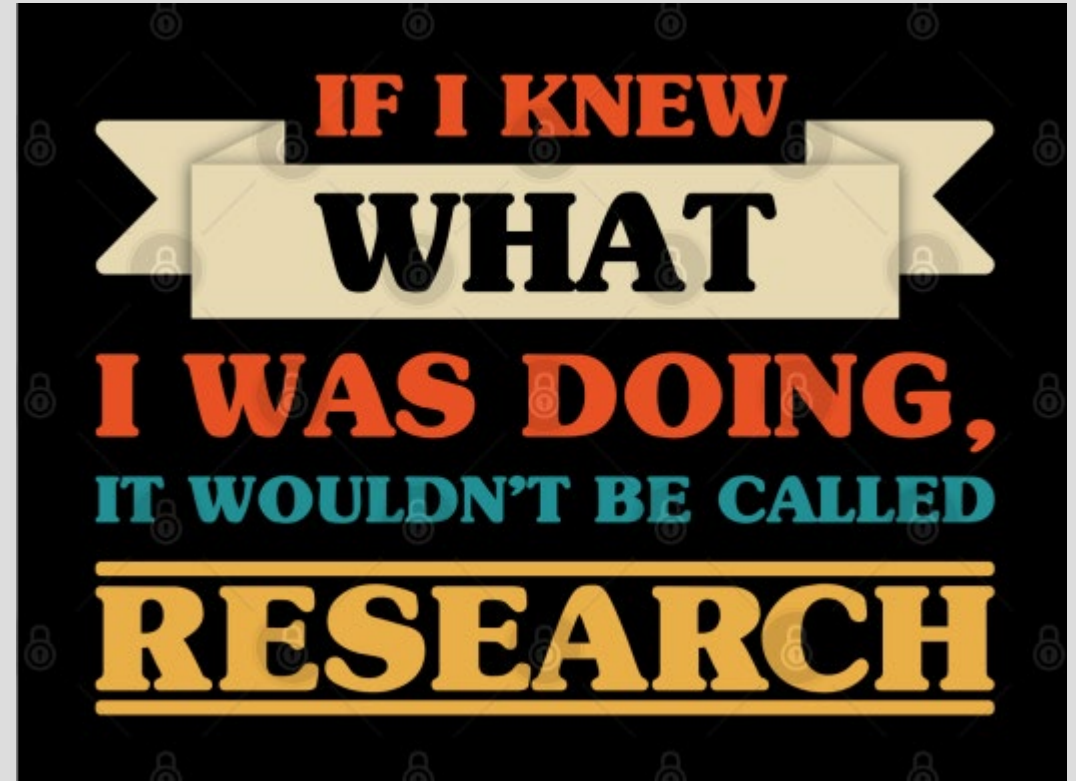


Note 19

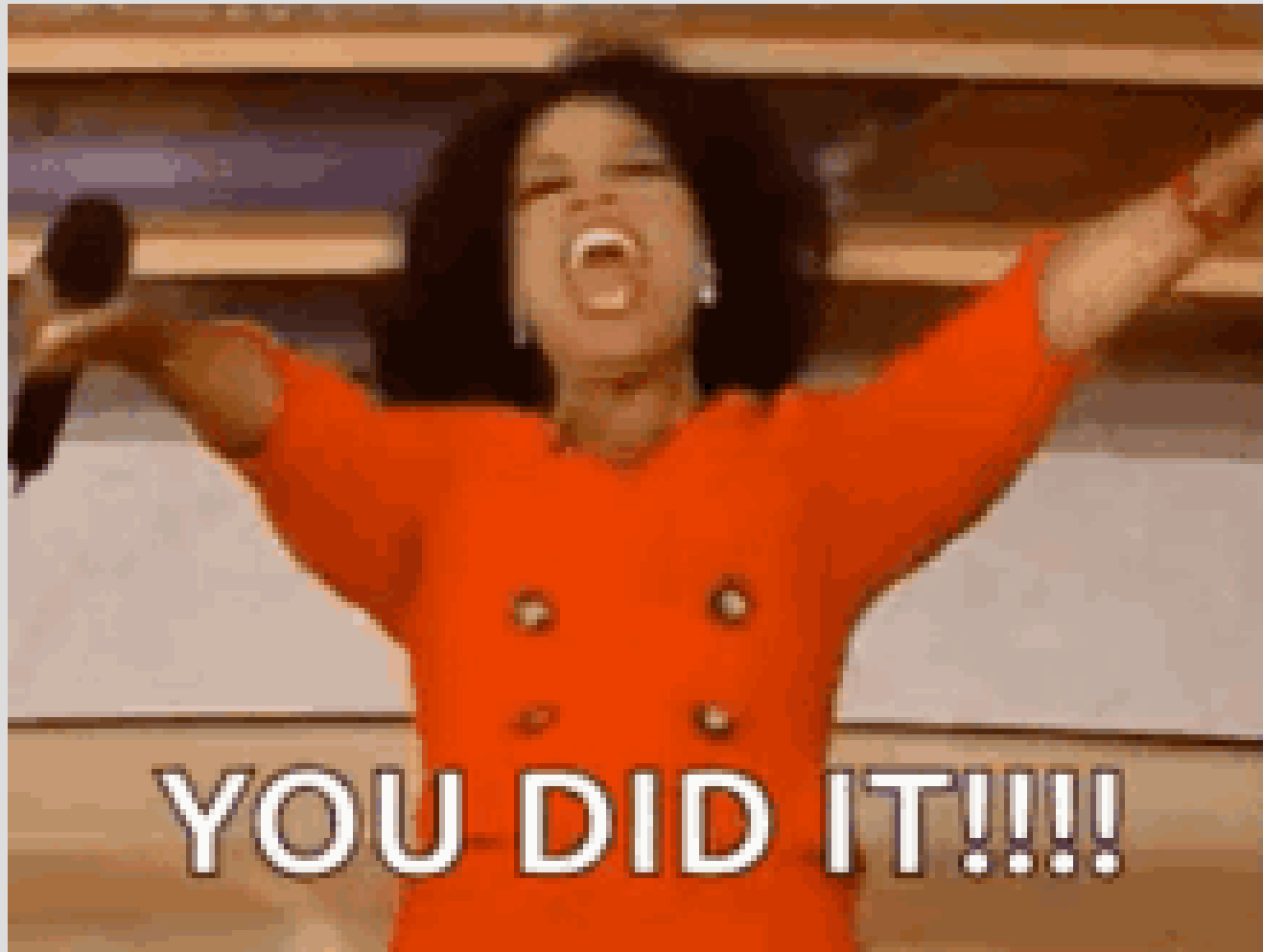
"artificial neuron"

How to get involved in research

- If you're interested in research:
 - Talk to your TAs
 - Talk to professors
 - Look for openings on Beehive/Dare/URAP websites



Enough about me...



- Congrats!
- What you have accomplished this semester:
 - Built a camera
 - Built two types of touchscreens
 - Built your own GPS system
- If you liked the class, please:
 - Thank your TAs!
 - apply to become one!

Learning Goals

Stuff We did:

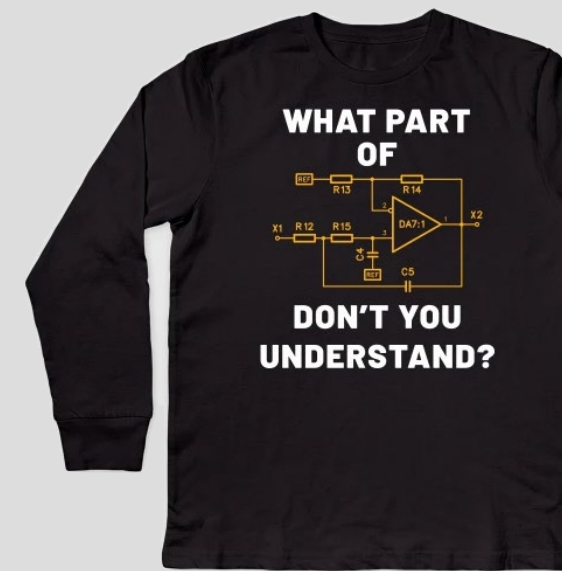
EECS 16A

- Module 1: Introduction to systems
 - How do we collect data? build a model?
- Module 2: Introduction to circuits and design
 - How do we use a model to solve a problem
- Module 3: Introduction Signal Processing and Machine Learning
 - How do we “learn” models from data, and make predictions?

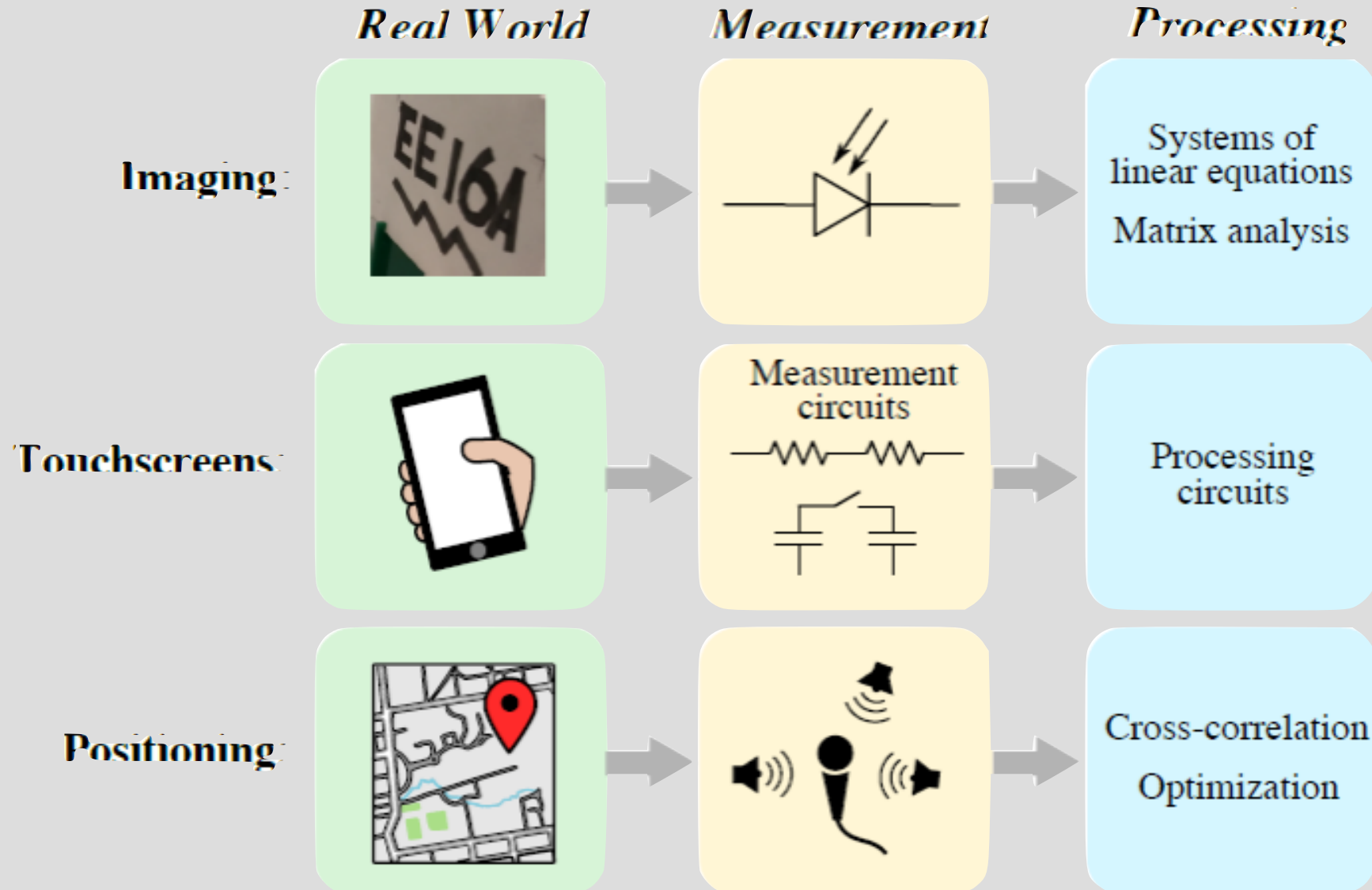
Stuff you will do next

EECS 16B

- Module 4: Advanced circuit design / analysis
- Module 5: Introduction to control and robotics
- Module 6: Introduction to data analysis and signal processing

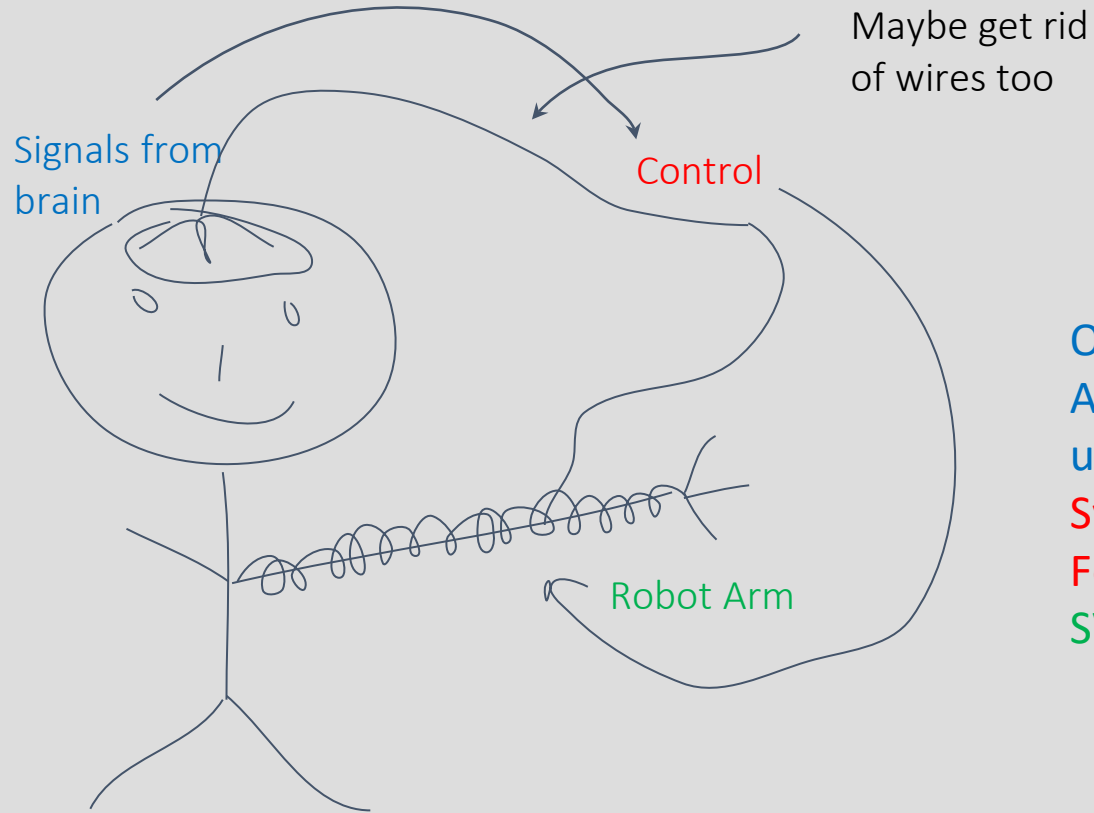


What you built:



EECS16B: Designing Information Devices and Systems II

Big goal: Get signals from brain and interpret them



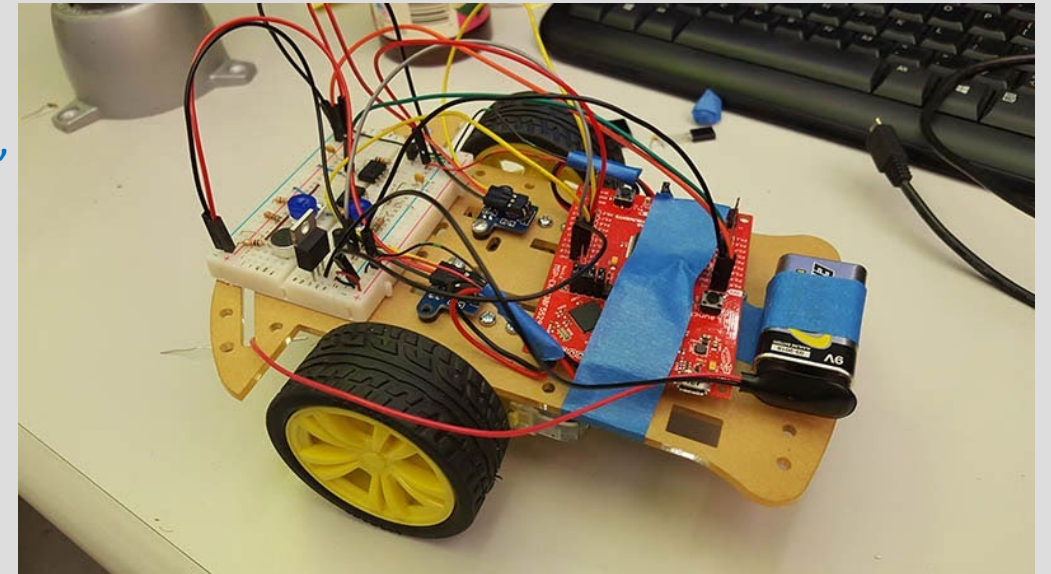
Module 1 – Circuits: Interfaces (brain, voice)

Module 2 – Control: Controls (feedback, stability)

Module 3 - Classification: Figuring out the intention

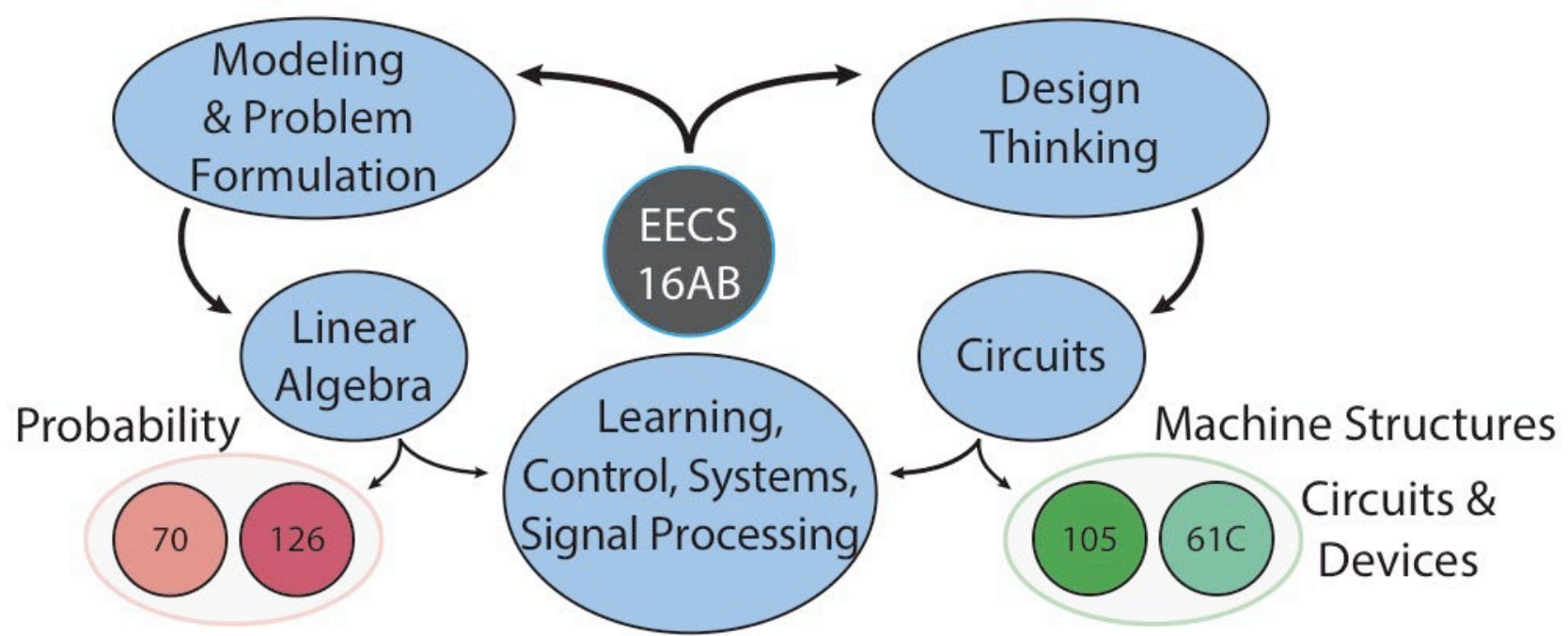
OpAmp Filters,
ADCs/DACs,
uController,
SysID,
Feedback,
SVD, PCA

Voice controlled robo car lab project – from scratch!



[Demo video](#)

**Design Contest
(make our SIXT33N better!)**

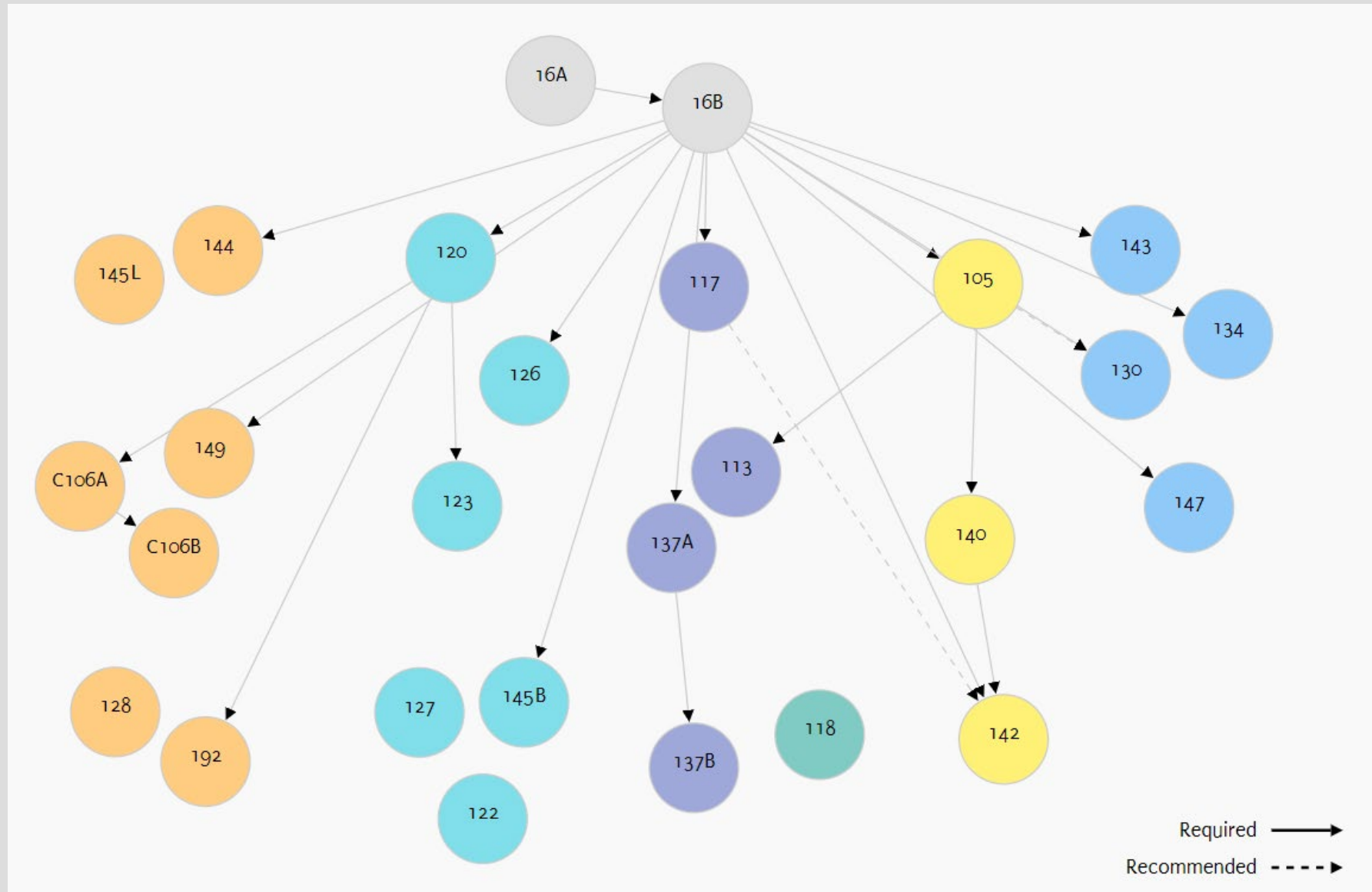


How to approach something unfamiliar
and systematically build understanding

Linear Algebra: conceptual tools to model
Circuits: How to go from model to design, grounded in physical world

Intro to foundational concepts in Machine Learning

EECS course map



CS COURSE MAP



core



hardware



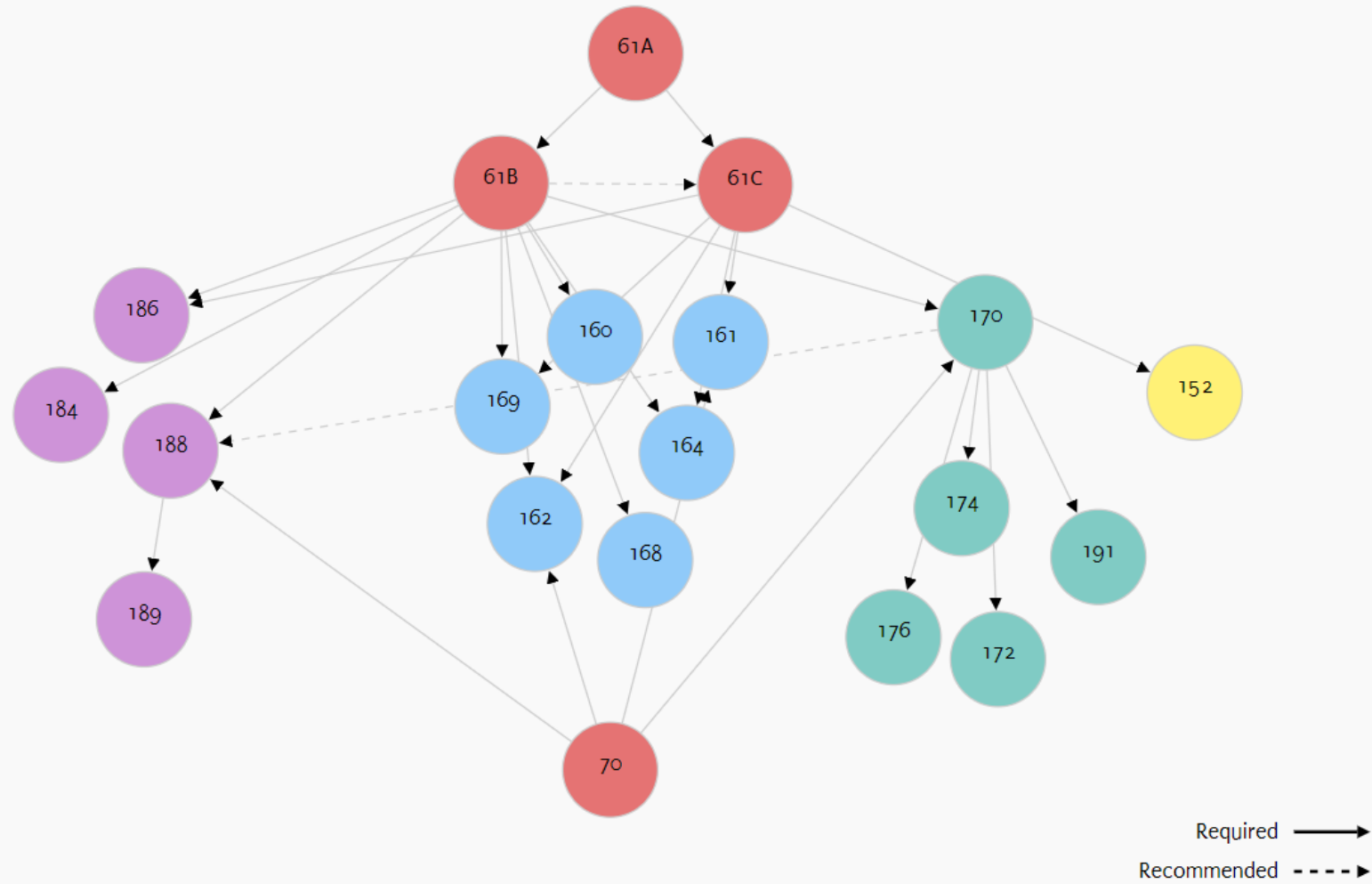
software



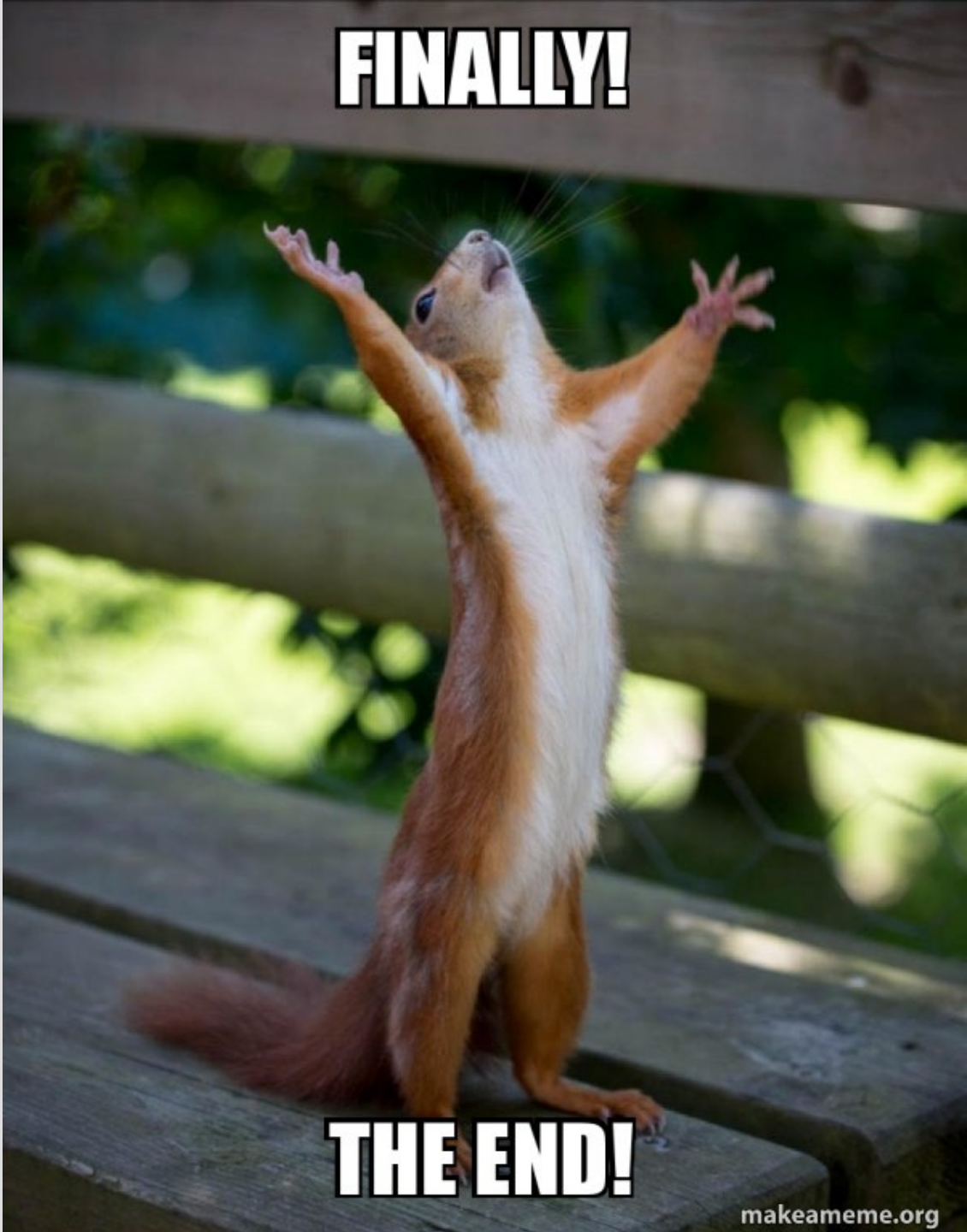
theory



applications



FINALLY!



THE END!

The End

Oh, except for the final exam...

