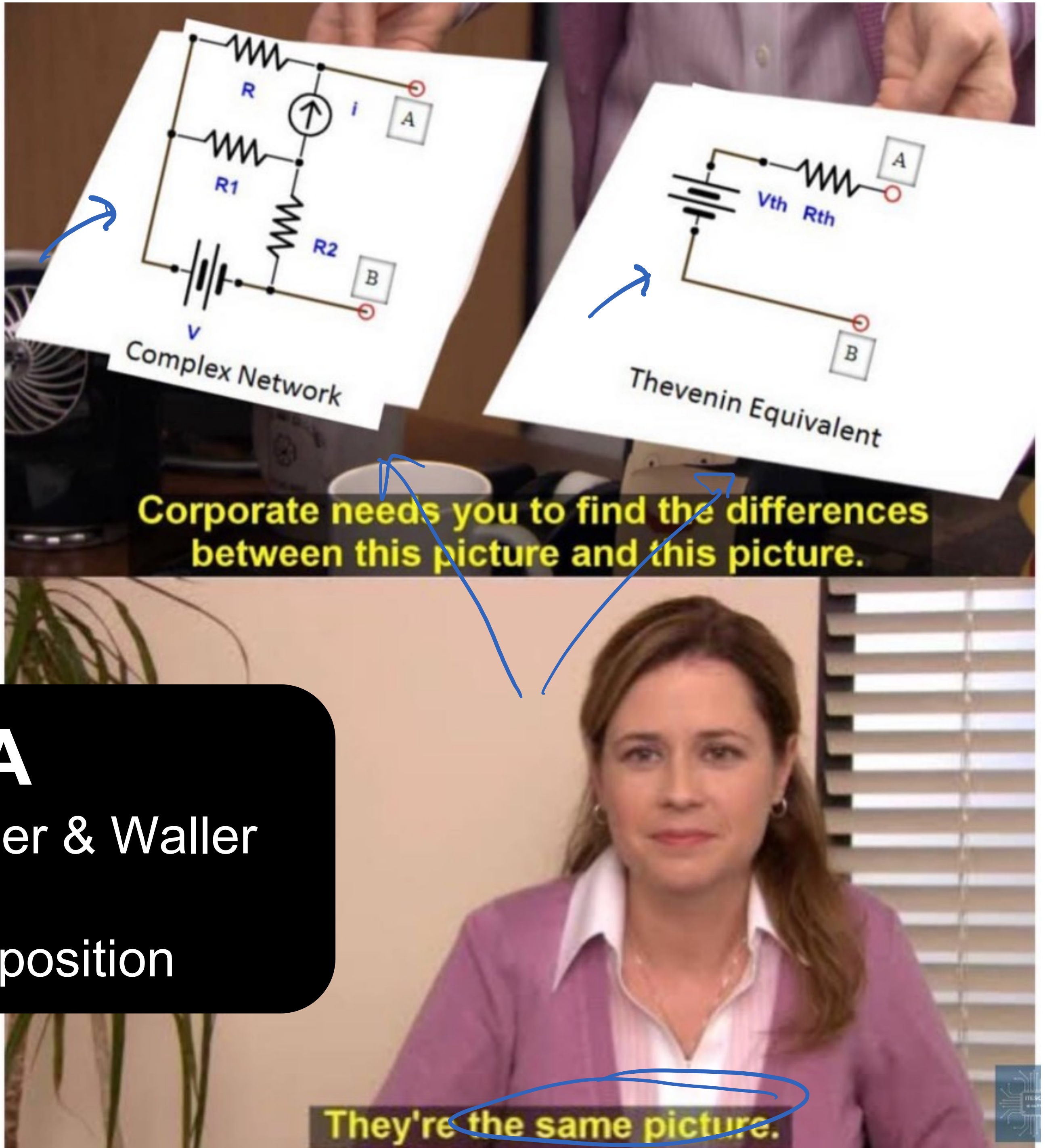



# EECS 16A

Spring 2023 - Profs. Muller & Waller

Lecture 7B

Equivalence & Superposition

# Admin

We’re almost done grading midterms!
Let’s discuss discussion…

# Toolbox

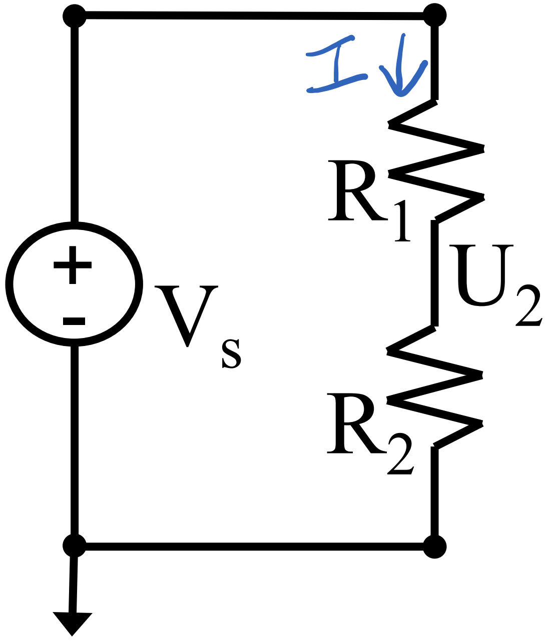
*KVL: Voltage drops around a loop sum to 0*

*KCL: All currents coming out of a node sum to 0*

$V = IR$

$P = IV$

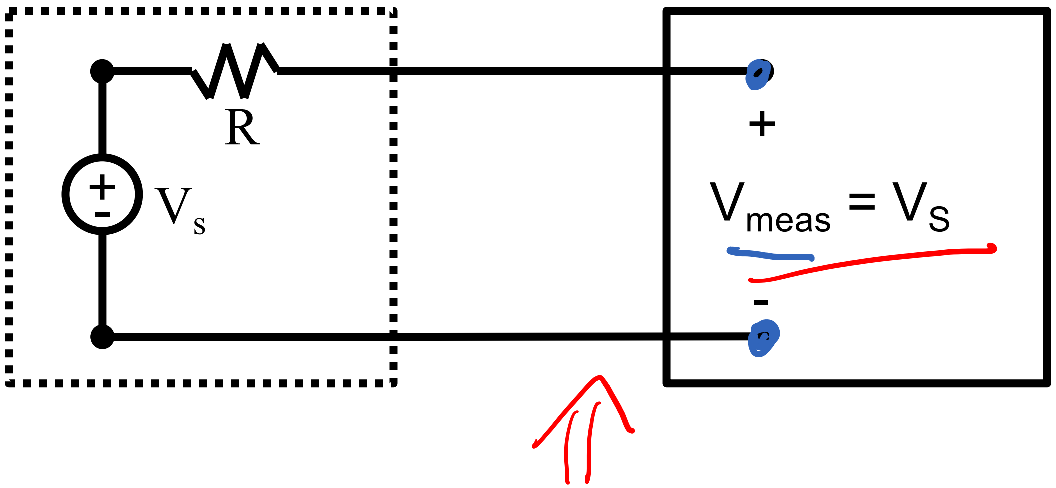
$$R = \frac{\rho L}{A}$$



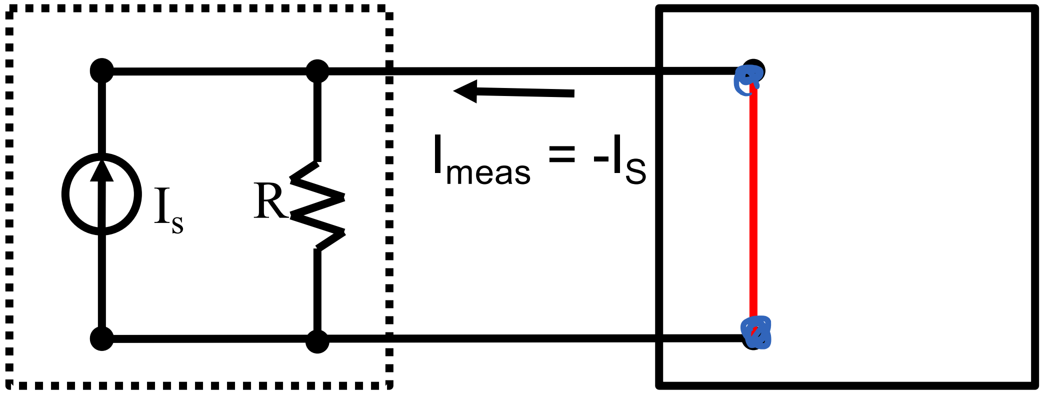

$$I = \frac{V_S}{R_1 + R_2}$$

$$U_2 = \frac{V_S R_2}{R_1 + R_2}$$

V Measurement Circuit

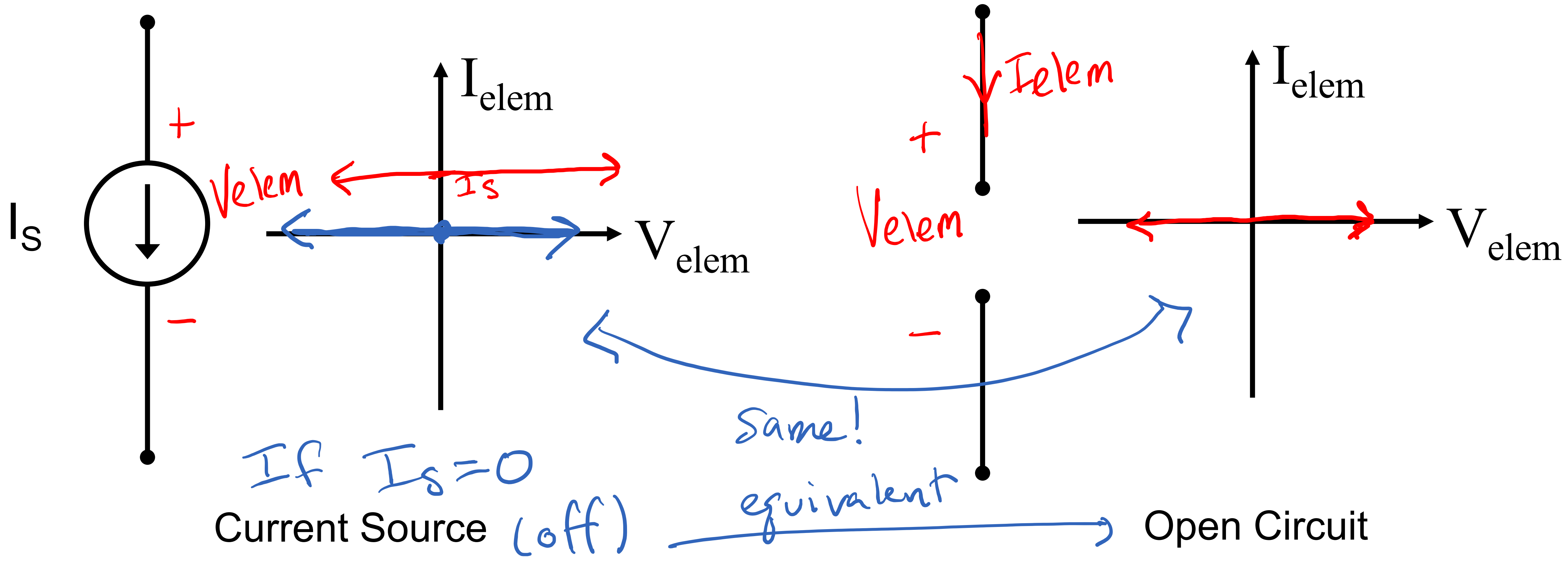



I Measurement Circuit



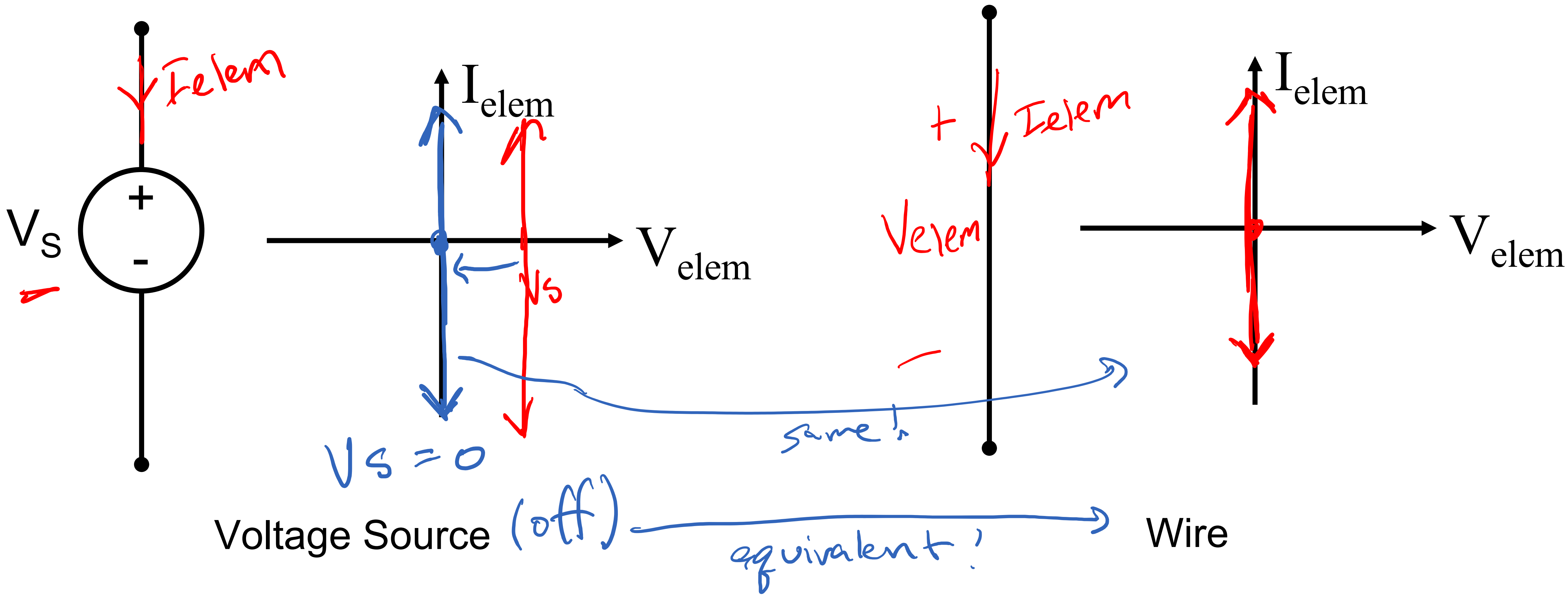
# Recap: Equivalence

Two circuits are equivalent if they have the same IV relationship



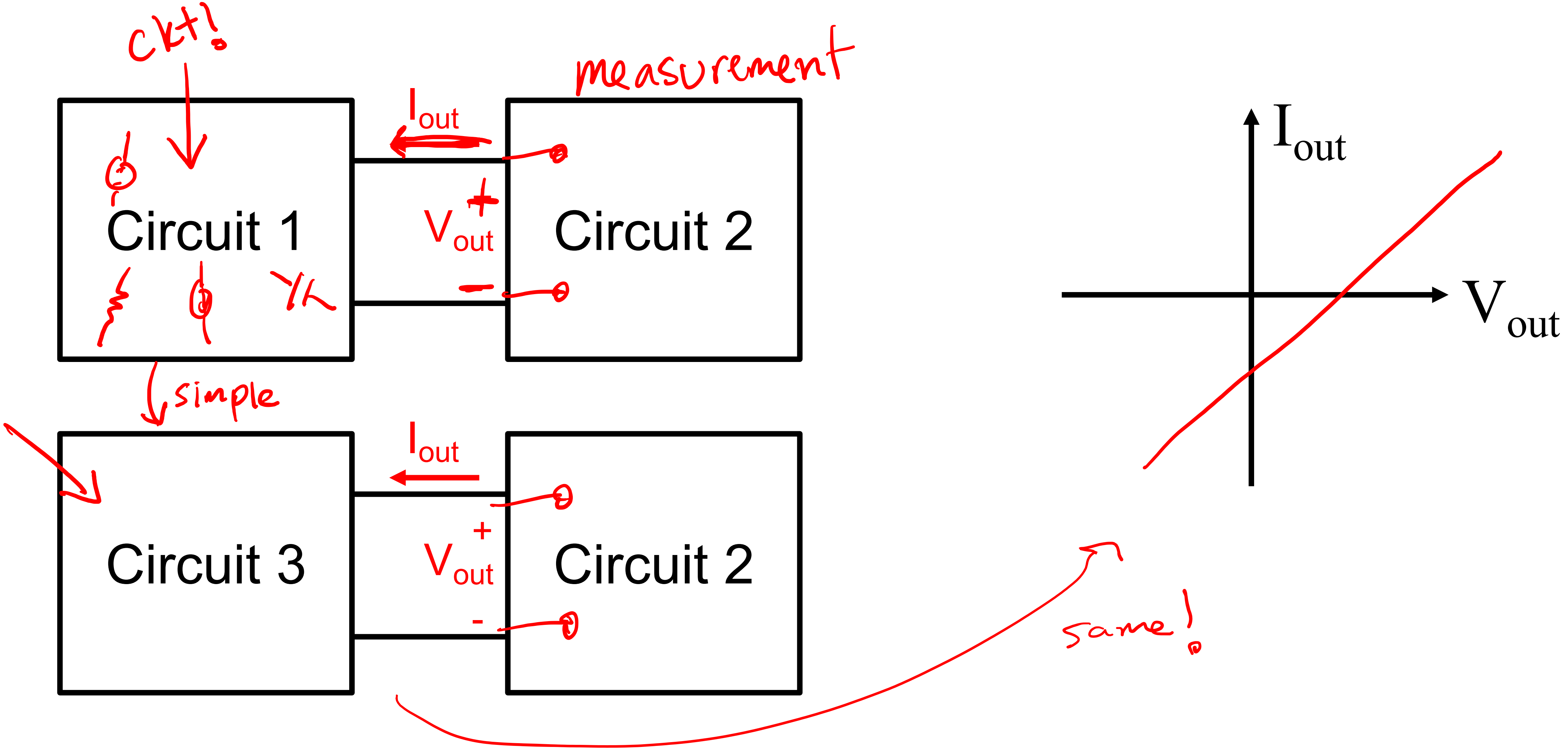
# Recap: Equivalence

Two circuits are equivalent if they have the same IV relationship



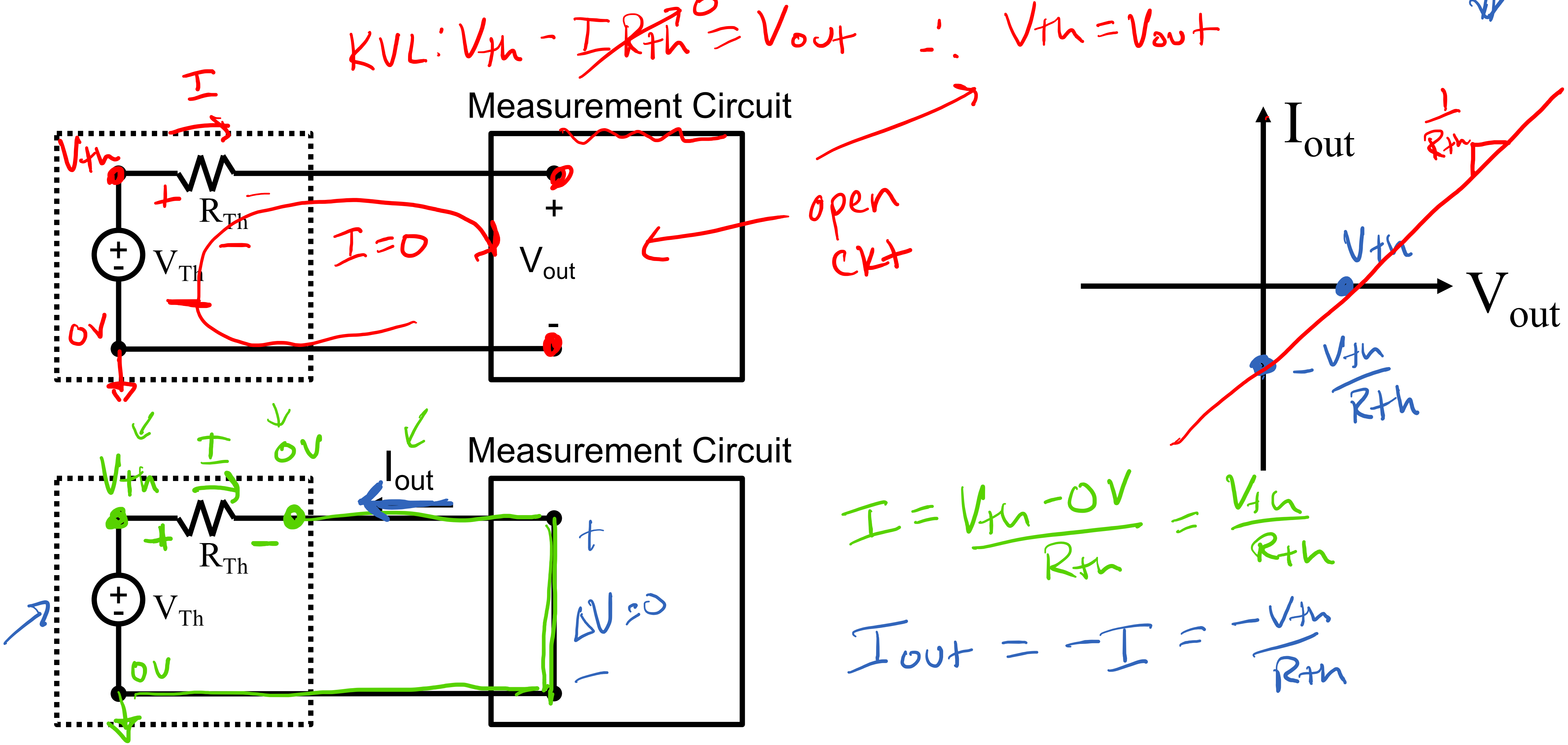
# Equivalence

* Two circuits are equivalent if they have the same IV relationship



# Equivalence Example

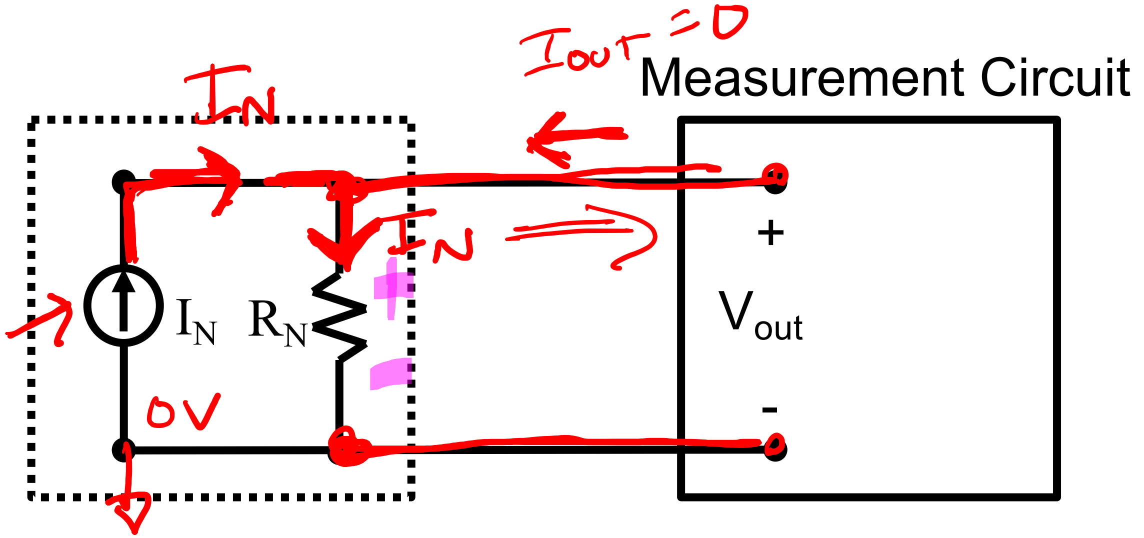
Two circuits are equivalent if they have the same IV relationship



# Equivalence Example 2

$*V = IR$

Two circuits are equivalent if they have the same IV relationship



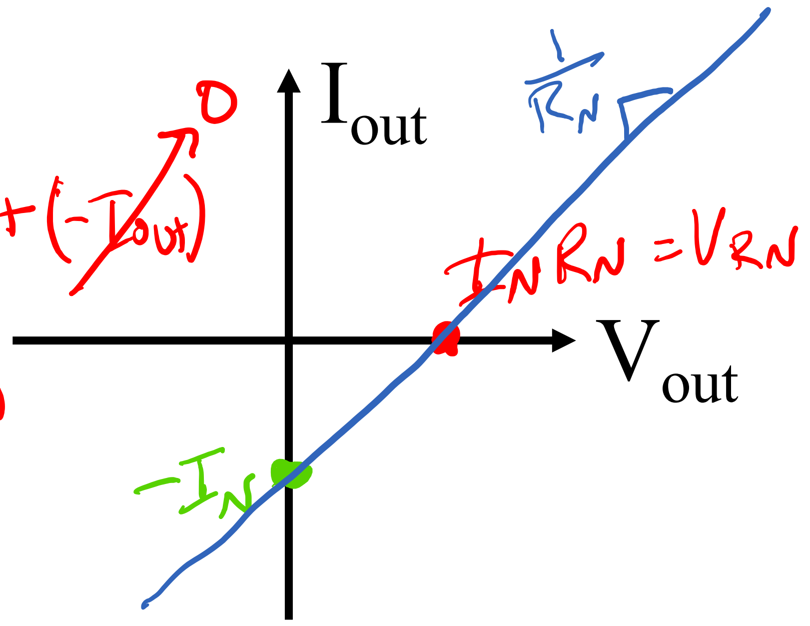

$I_{out} = 0$ ✓

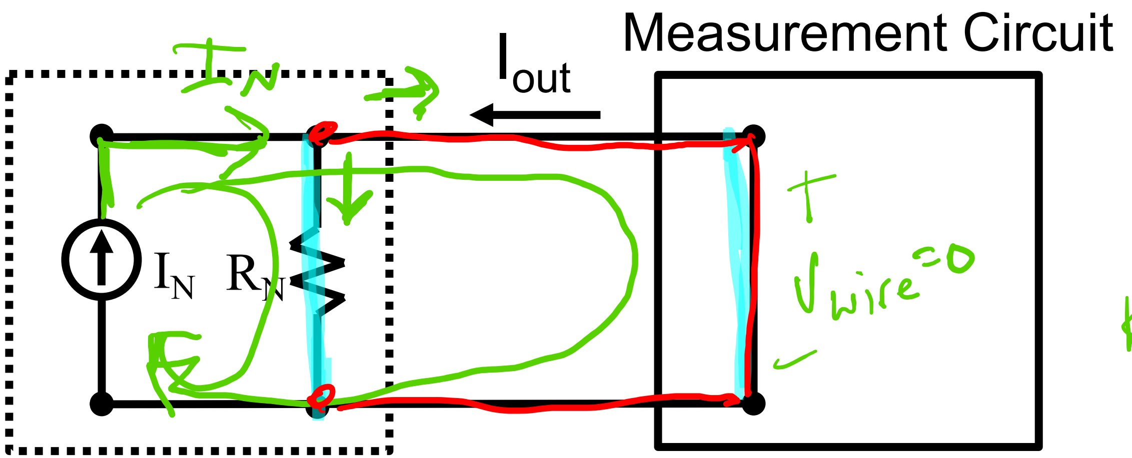
$V_{OUT} = ?$

KCL: $I_N = I_{RN} + (-I_{out})$

$V_{RN} = I_N \cdot R_N$

$V_{OUT} = V_{RN}$




$V_{RN} = 0 \quad I_{RN} = 0$

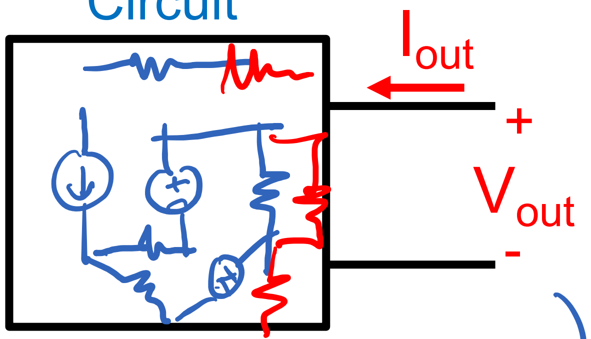
KCL: $I_N = I_{RN} + (-I_{OUT})$

$I_N = -I_{OUT}$

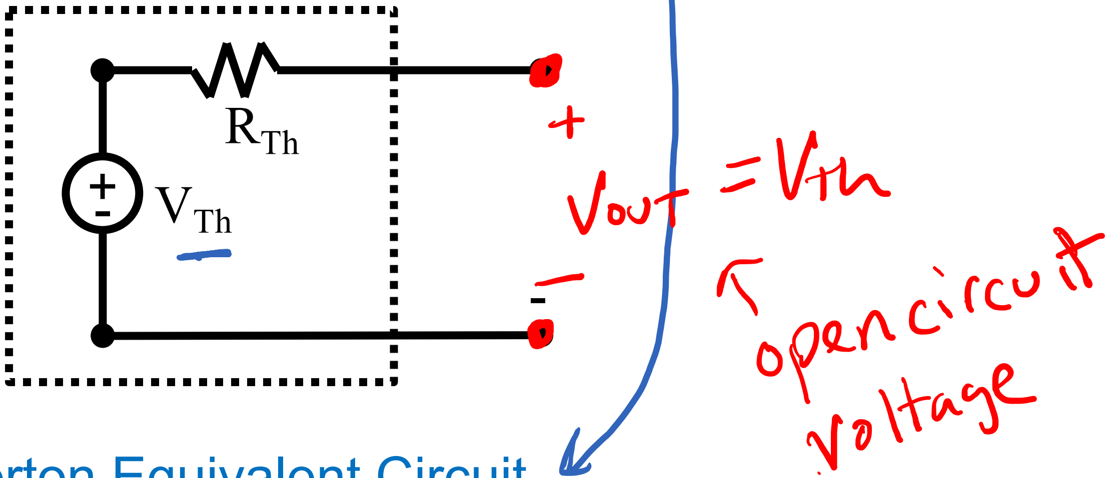
# Thevenin and Norton Equivalents

Any linear circuit (network of linear elements and sources) can be represented by a Thevenin or Norton equivalent circuit

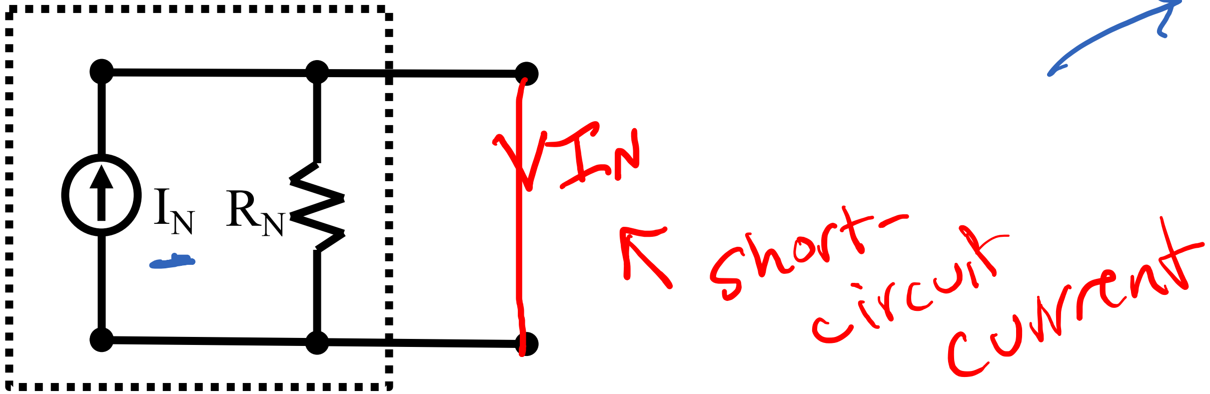
Circuit

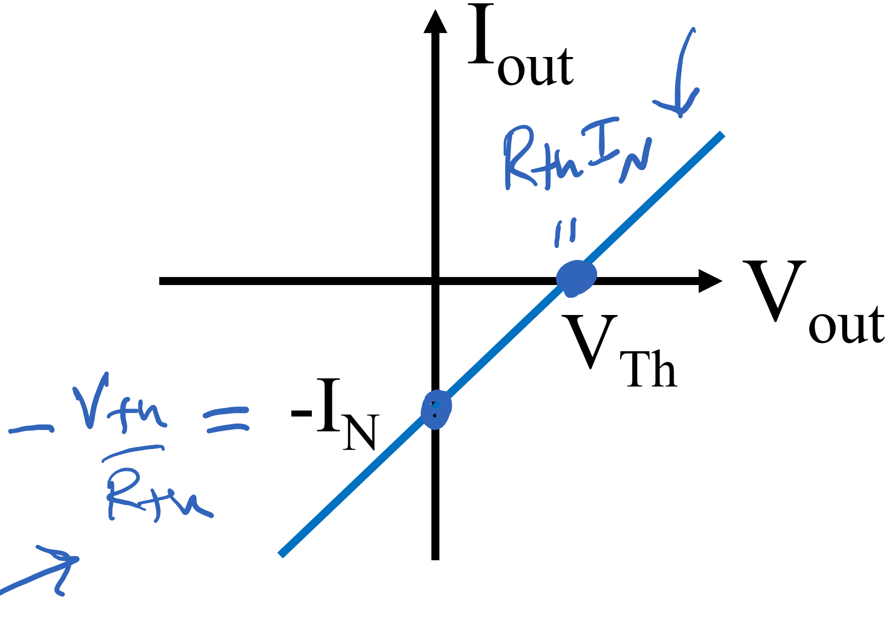



Thevenin Equivalent Circuit




Norton Equivalent Circuit

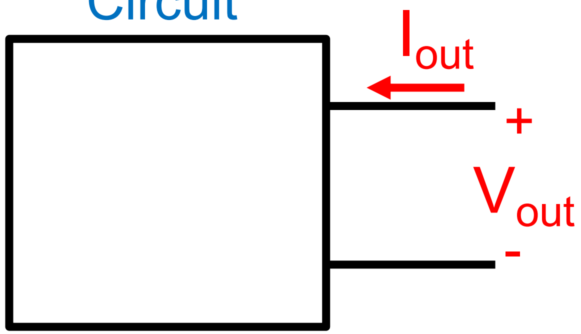


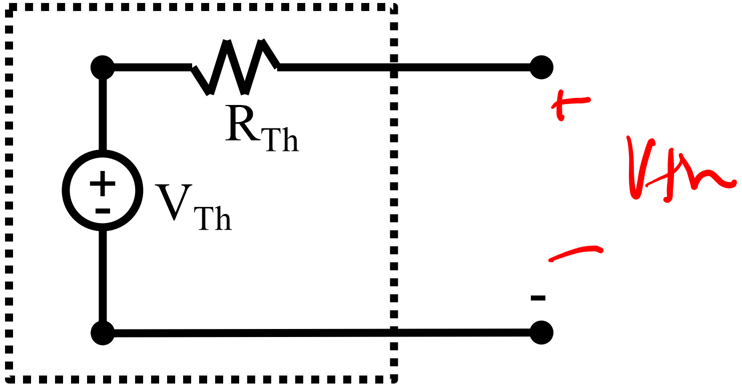

$$R_{Th} = V_{Th}/I_N$$

$* \; R_{Th} = R_N$

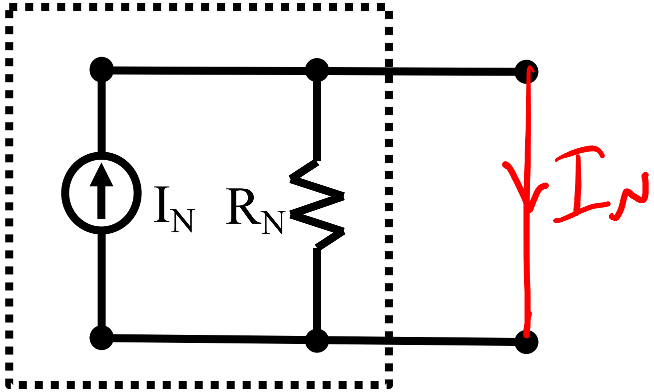
Circuit




Thevenin Equivalent Circuit




Norton Equivalent Circuit



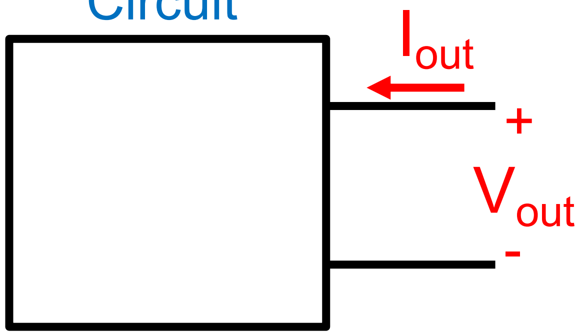

# Thevenin and Norton Equivalents

Thevenin Equivalent Voltage -

1. Find $V_{Th}$ by <u>opening</u> the output terminal and measuring $V_{out}$

Norton Equivalent Current –

2. Find $I_N$ by <u>shorting</u> the output terminal and measuring $-I_{out}$




# Thevenin and Norton Equivalent Resistance

Direct Measurement

Thevenin or Norton Resistance -

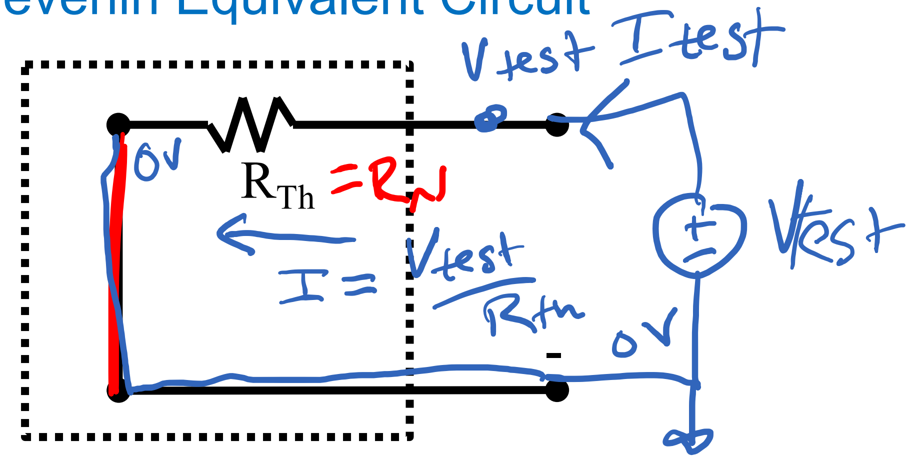
1. Turn off all sources
2. Apply test voltage and measure current

OR

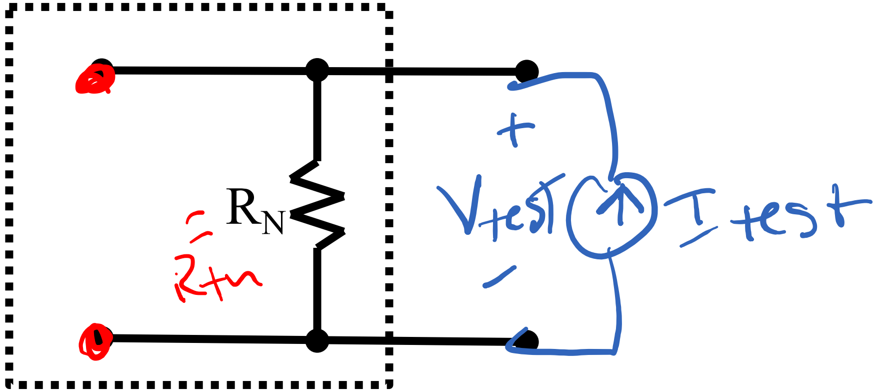
2. Apply test current and measure voltage

$$I_{test} = \frac{V_{test}}{R} \qquad R_{th} = \frac{V_{test}}{I_{test}}$$

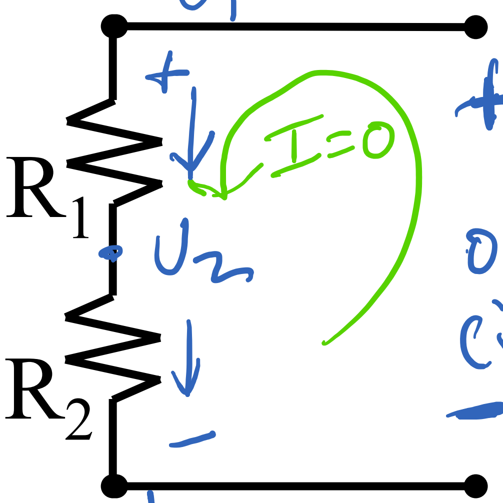
Thevenin Equivalent Circuit



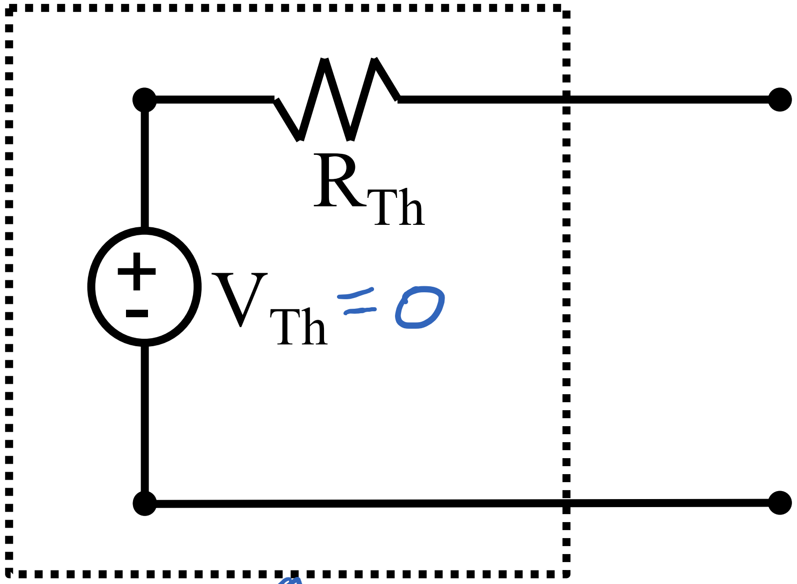

Norton Equivalent Circuit



# Example 1: Thevenin Equivalent of Series Resistors



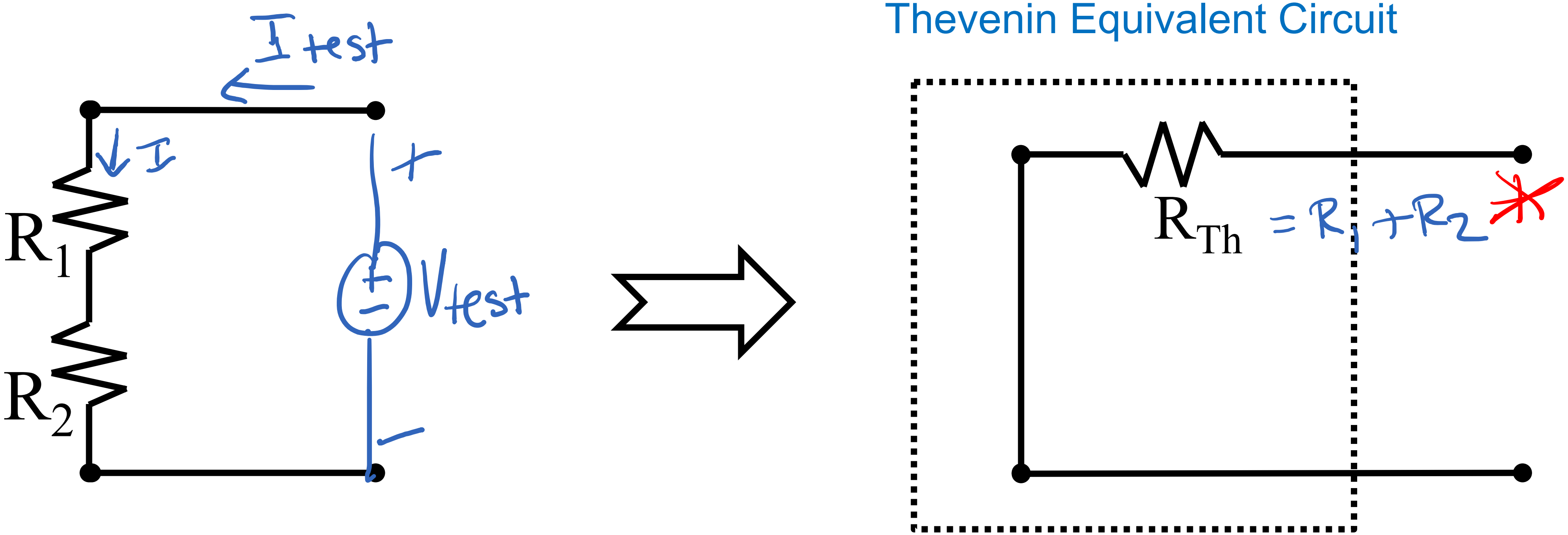

Thevenin Equivalent Circuit




Step 1: Find $V_{Th}$ by opening the output terminal and measuring $V_{out}$

# Example 1: Thevenin Equivalent of Series Resistors

Thevenin Equivalent Circuit




Step 2: Turn off all sources. Apply $V_{test}$ and measure $I_{test}$

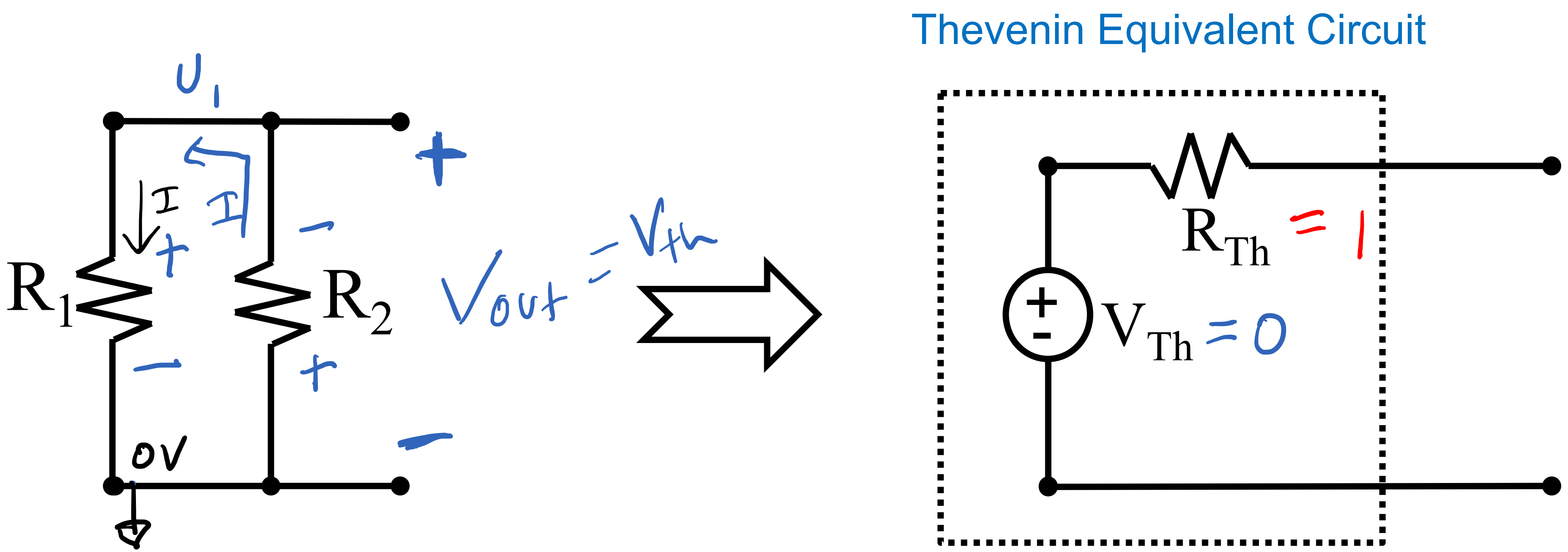
* Voltage divider ($V_s = V_{test}$)

$$I_{test} = \frac{V_{test}}{R_1 + R_2}$$

*

$$R_{th} = \frac{V_{test}}{I_{test}} = \boxed{R_1 + R_2}$$

# Example 2: Thevenin Equivalent of Parallel Resistors




Step 1: Find $V_{Th}$ by opening the output terminal and measuring $V_{out}$

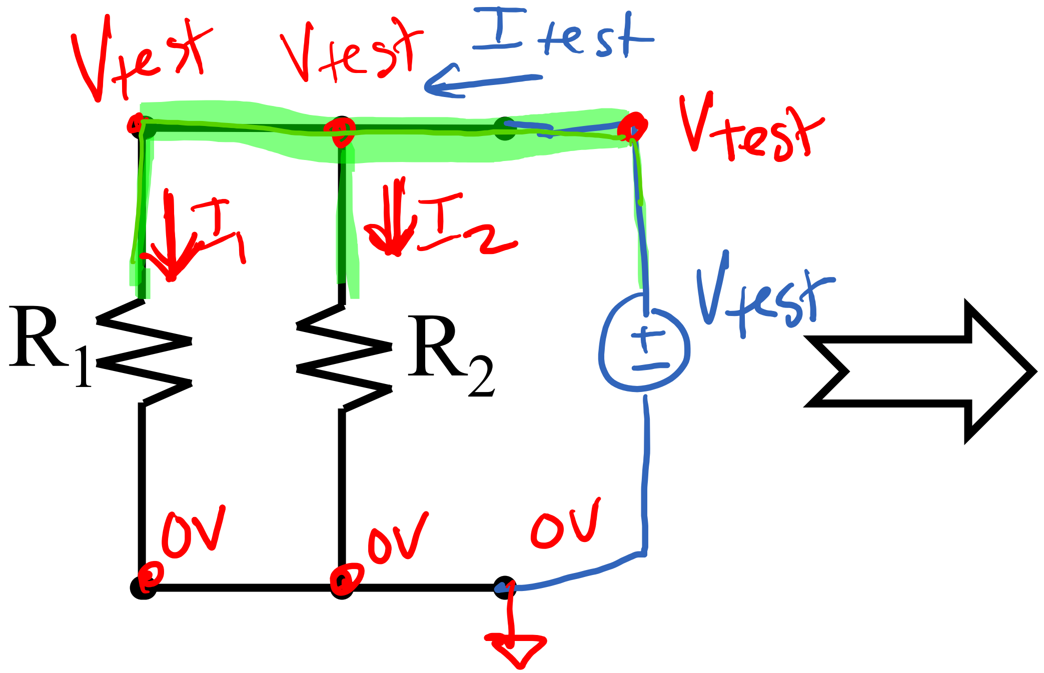
KCL: $I_1 = -I_2$

KVL: $0 - U_1 = IR_2$

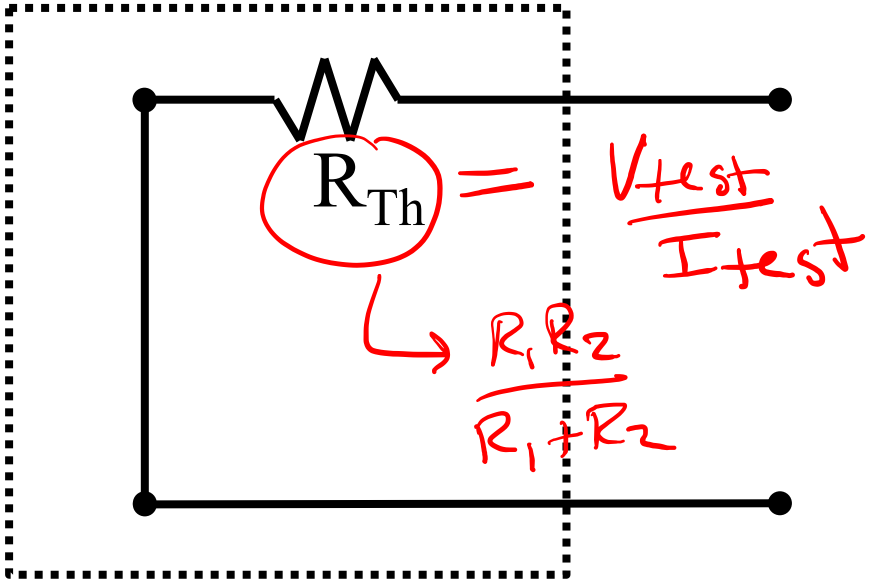
$U_1 - 0 = IR_1$

$U_1 = 0$

# Example 2: Thevenin Equivalent of Parallel Resistors




Thevenin Equivalent Circuit




Step 2: Turn off all sources. Apply $V_{test}$ and measure $I_{test}$

$V = IR$

$I_1 = \frac{V_{test} - 0}{R_1} = \frac{V_{test}}{R_1}$

$I_2 = \frac{V_{test} - 0}{R_2} = \frac{V_{test}}{R_2}$

KCL: $I_{test} = I_1 + I_2$

$I_{test} = \frac{V_{test}}{R_1} + \frac{V_{test}}{R_2}$

# Example 2: Thevenin Equivalent of Parallel Resistors

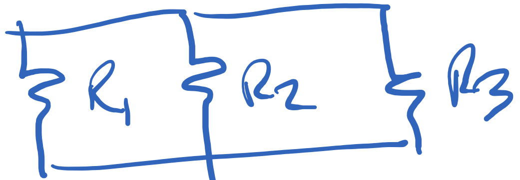
$$I_{test} = \frac{V_{test}}{R_1} + \frac{V_{test}}{R_2}$$

$$R_{th} = \frac{V_{test}}{I_{test}}$$

$$I_{test} = V_{test}\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$$

$$R_{th} = \frac{V_{test}}{I_{test}} = \left[\frac{1}{R_1}+\frac{1}{R_2}\right]^{-1} = \frac{R_1 R_2}{R_1+R_2} = R_1 // R_2$$

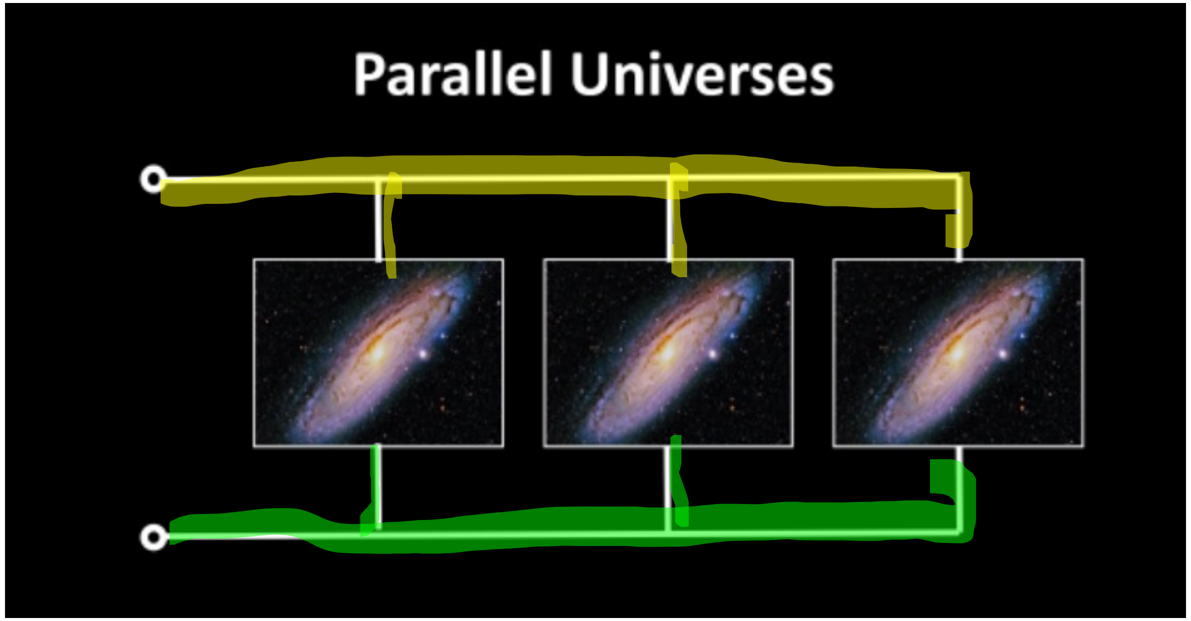

$$= R_1 + R_2 + R_3$$



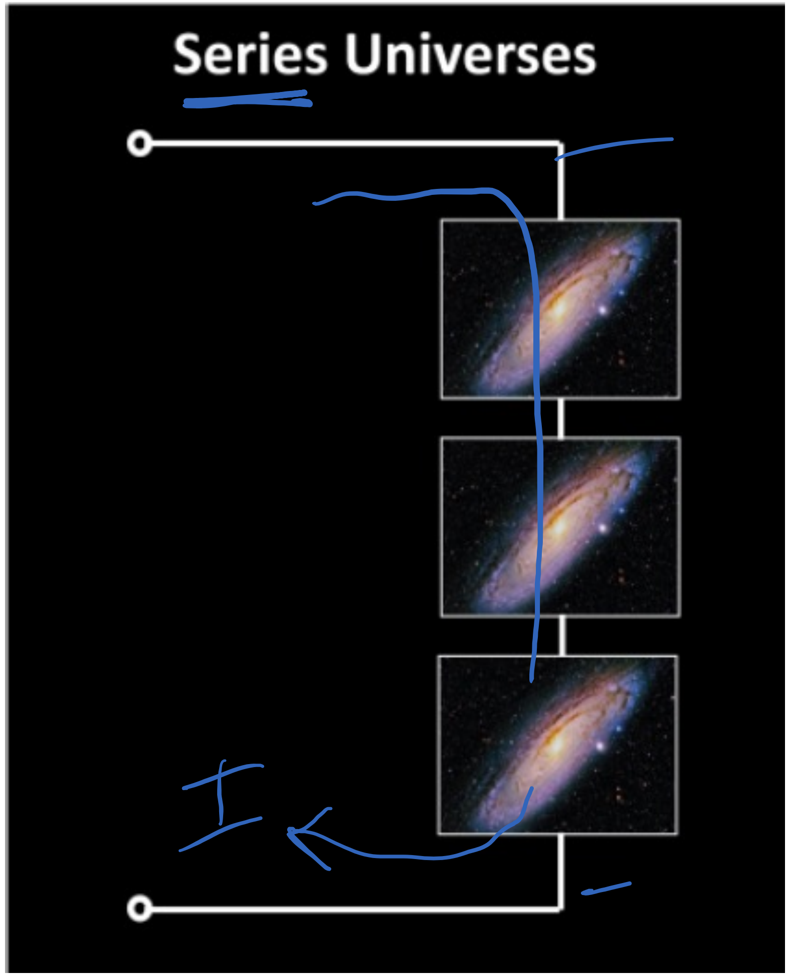

$$= \left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]^{-1}$$

# Laws of the Universes

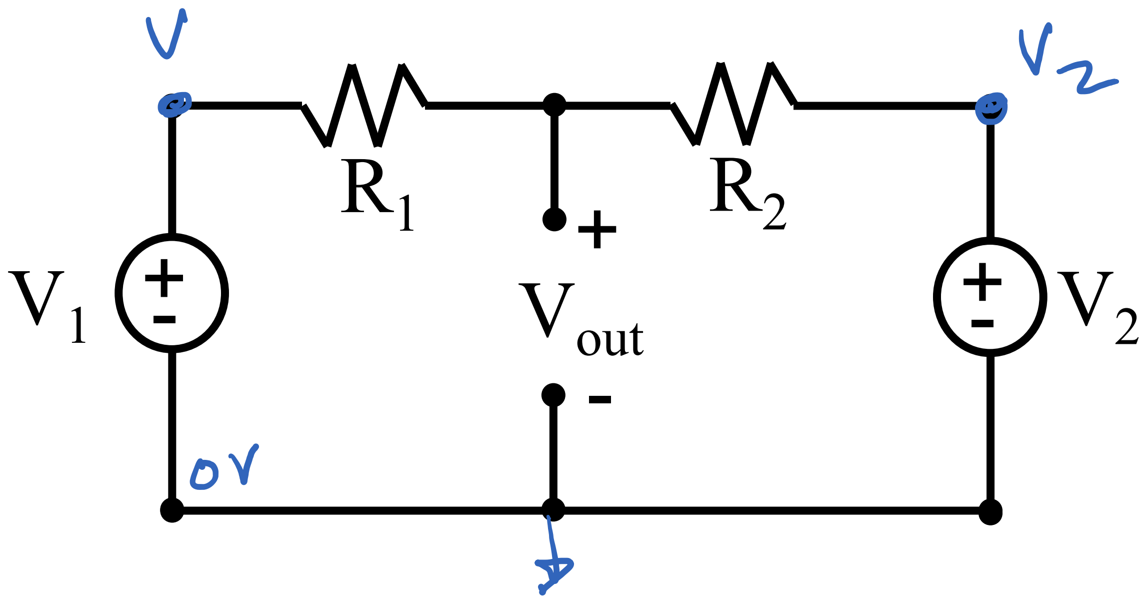
Series elements will have the same current through them due to KCL

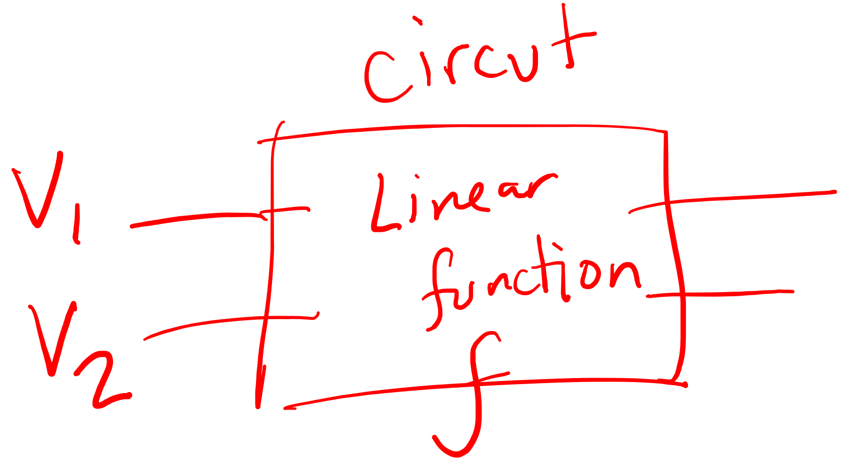



Parallel elements will have the same voltage across them due to KVL

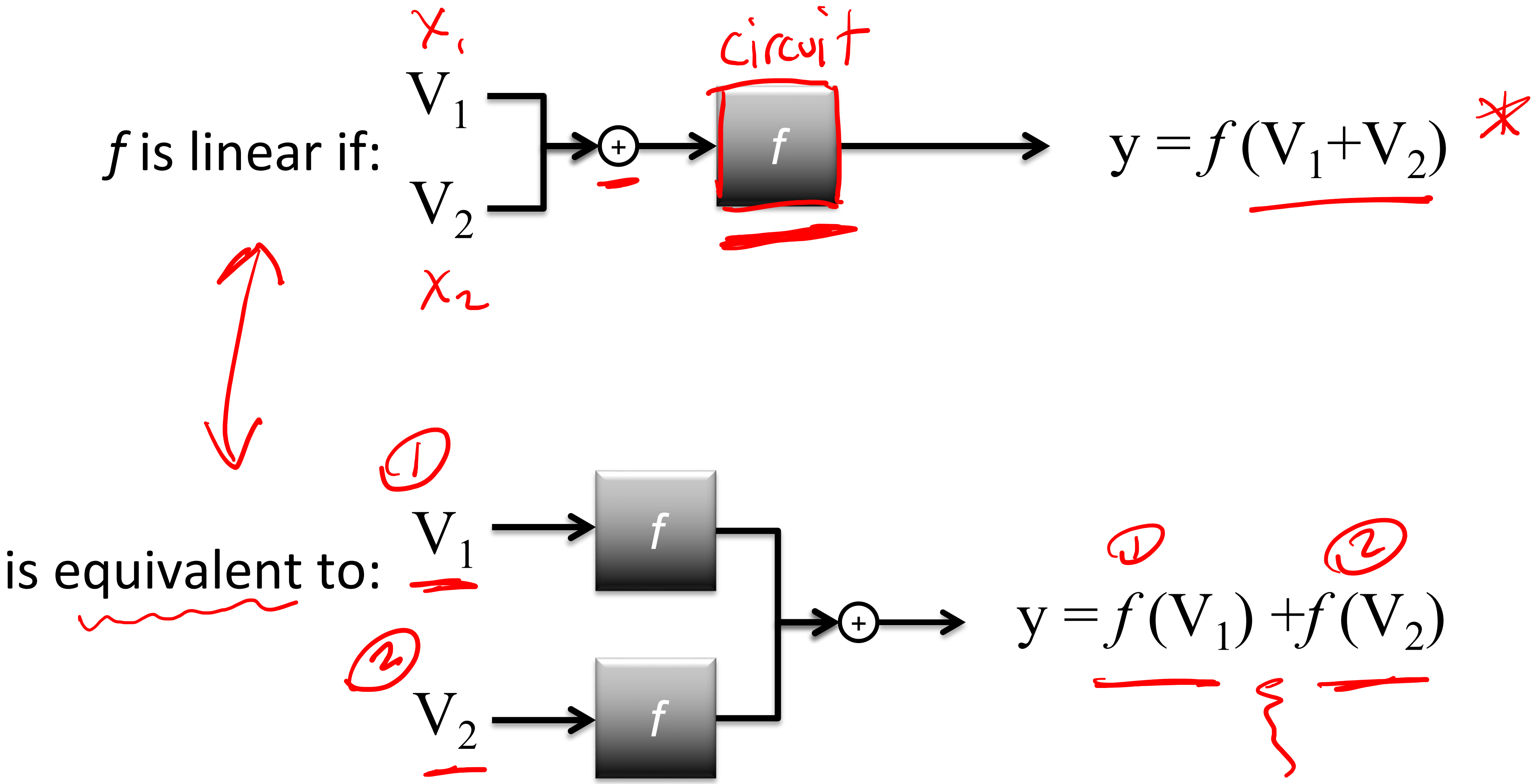


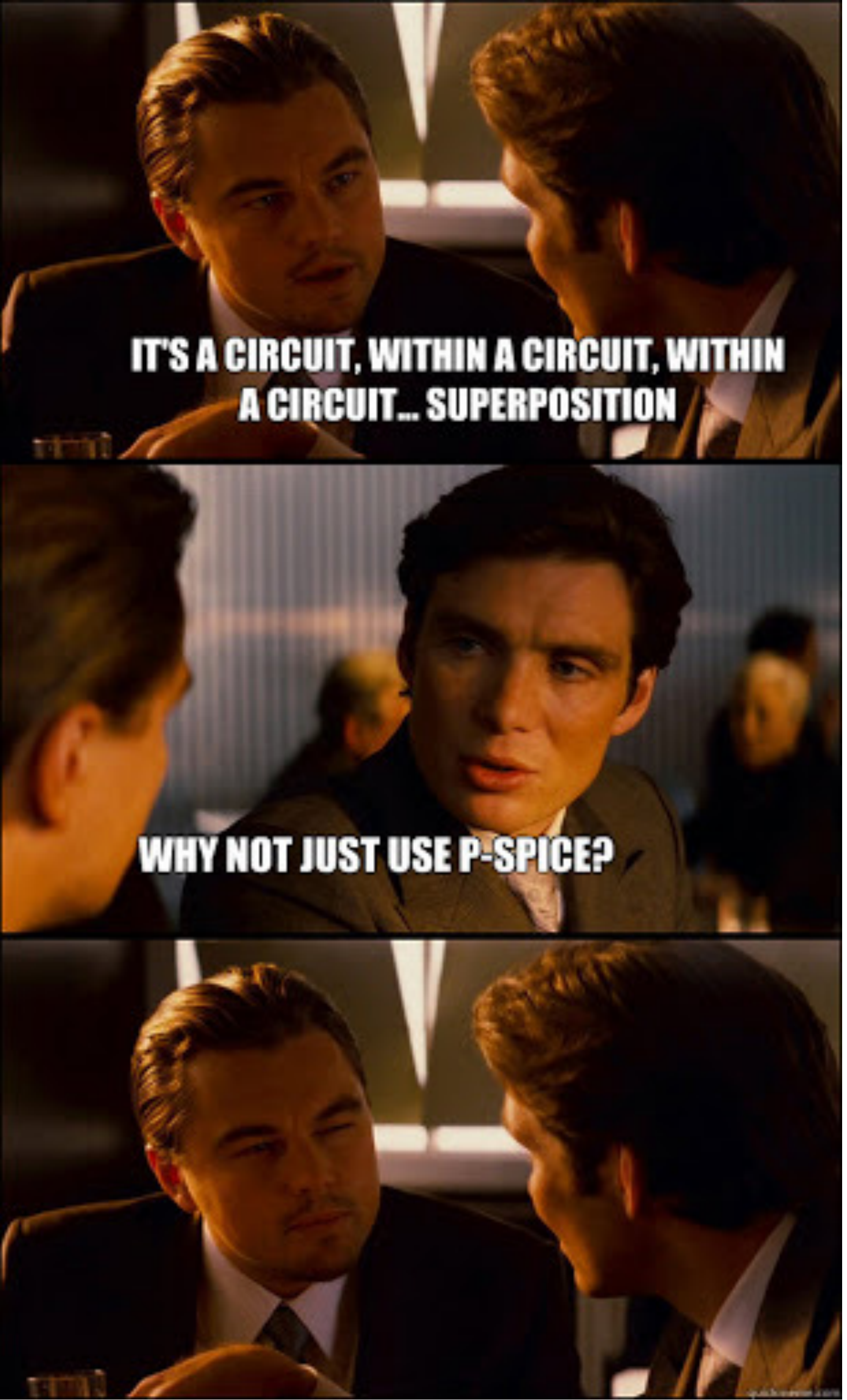
# What if there are Multiple Sources?



# We can test for linearity (from Lecture 0B)



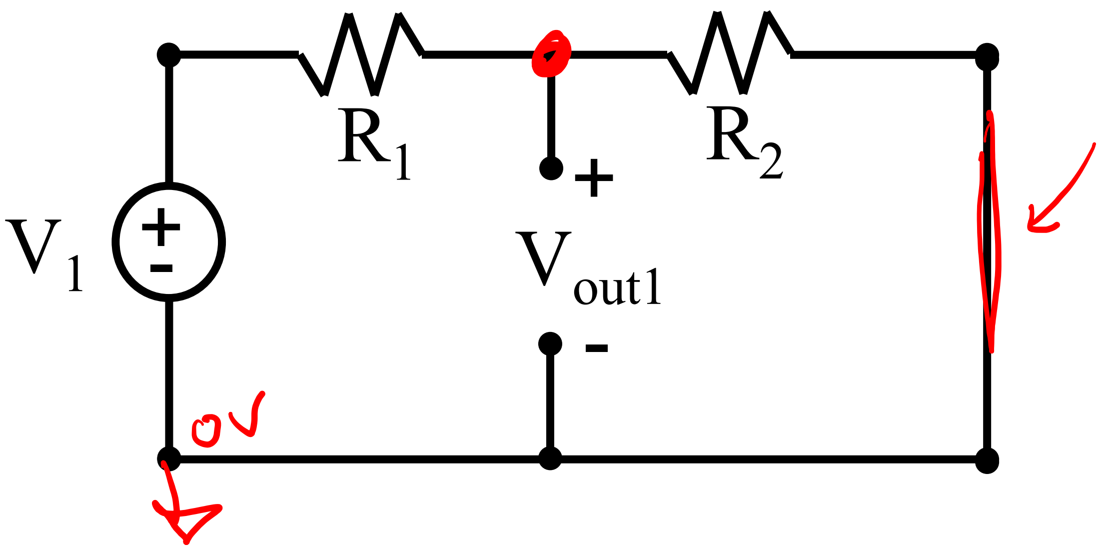



# Superposition

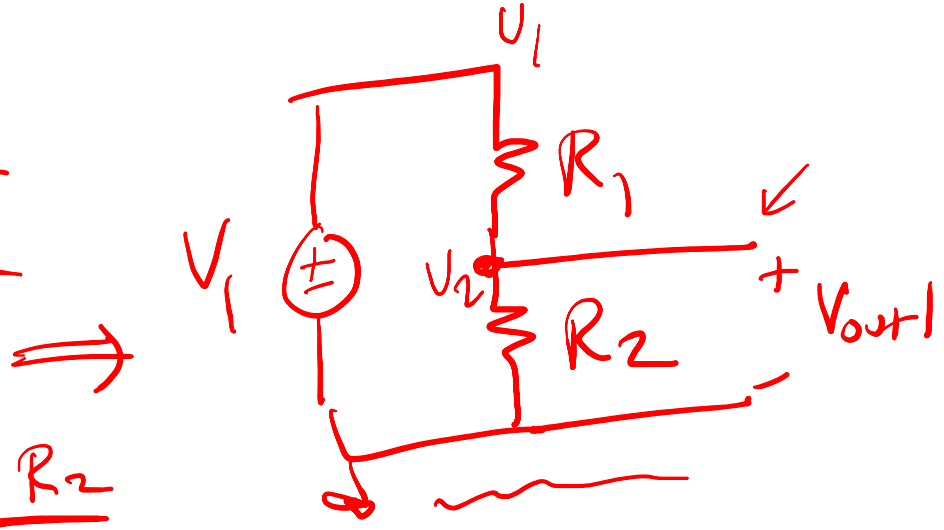
For each independent source k (voltage or current source):

- Set all other independent sources to 0 (off)
- Voltage source: replace with a wire
- Current source: replace with an open circuit
- Compute the output voltage or current due to this source k
- Compute output by summing the output for all k

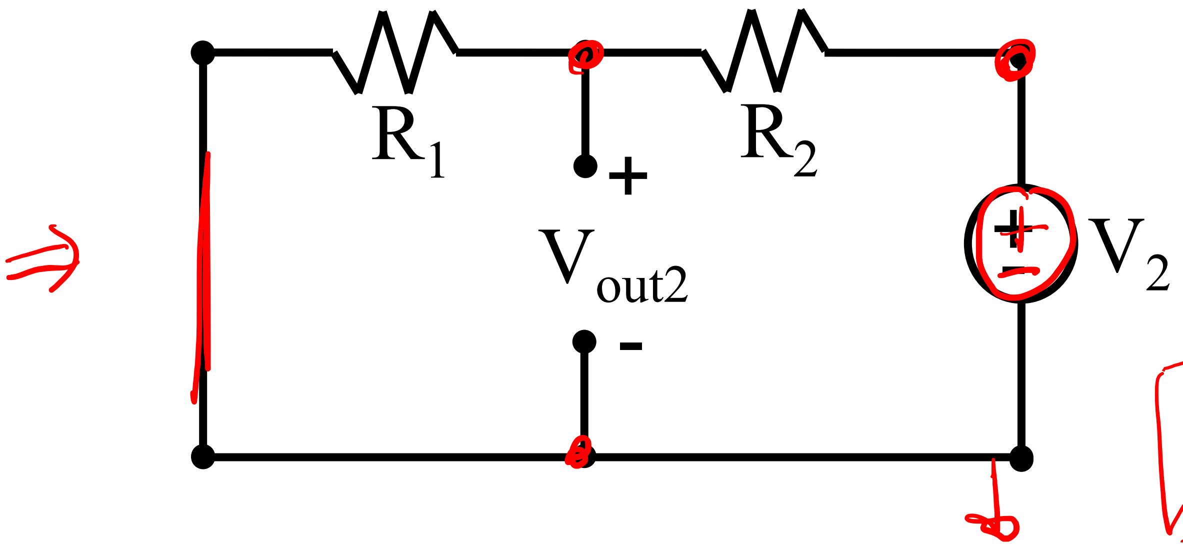
# Solve with One Source at a Time – Sum at the End

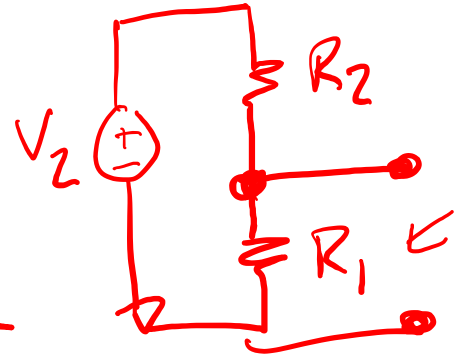



$V_1$ is off




$$V_{out1} = \frac{V_1 R_2}{R_1 + R_2}$$




$$V_{out2} = \frac{V_2 \cdot R_1}{R_1 + R_2}$$

$$V_{out} = V_{out1} + V_{out2} = \frac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$$