EECS 16A

Spring 2023 - Profs. Muller & Waller

Lecture 8B –

Capacitors & Capacitive Touchscreens

Toolbox

KVL: Voltage drops around a loop sum to 0

KCL: All currents coming out of a node sum to 0

$V = IR$

$P = IV$

$$R = \frac{\rho L}{A}$$

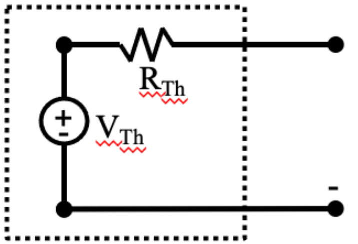
V_{source}(off) → short

I_{source}(off) → open

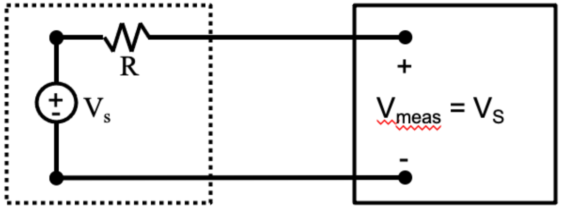
$$R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

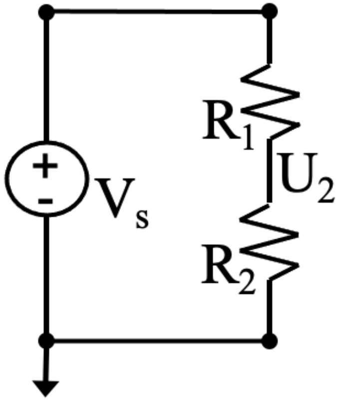
$$R_{Th} = V_{Th}/I_N$$

Thevenin Equivalent Circuit



V Measurement Circuit

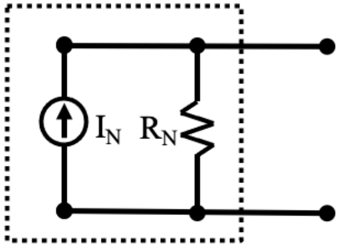




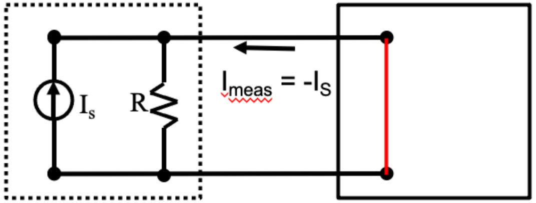
$$I = \frac{V_S}{R_1 + R_2}$$

$$U_2 = \frac{V_S R_2}{R_1 + R_2}$$

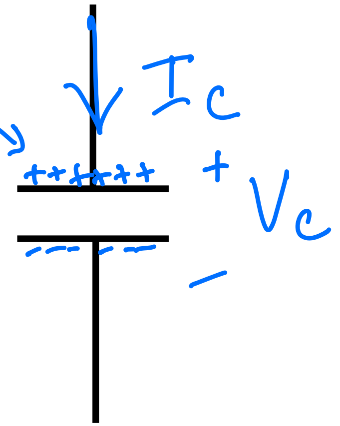
Norton Equivalent Circuit



I Measurement Circuit



Last Time

C 

Capacitance C in [Farads] or [F]

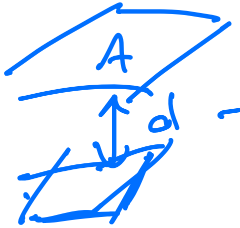
① $$Q = CV$$

$\frac{\partial}{\partial t}$

② $$I = C\frac{dV}{dt}$$ *

I-V relation

③ $$C = \frac{\epsilon A}{d}$$



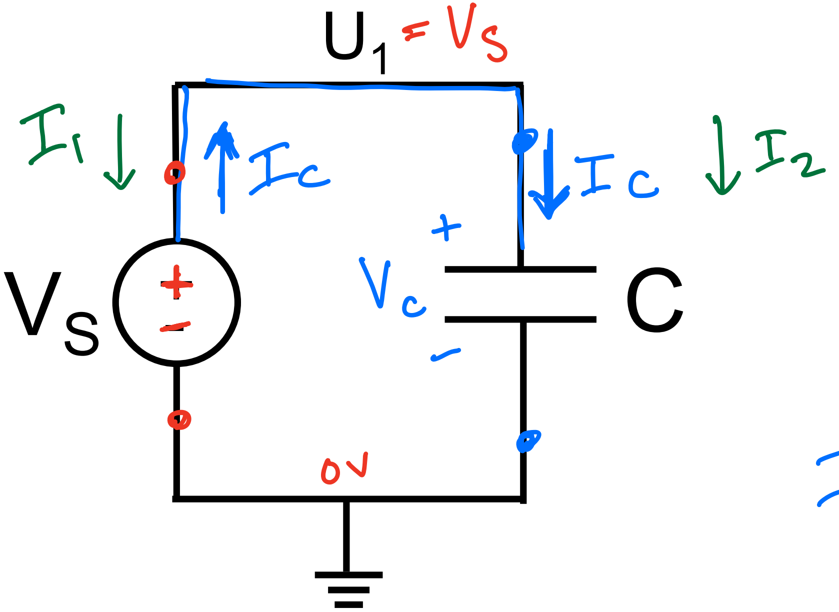
→ ε

permittivity

Circuit Example 1

Find the current in the capacitor I_C.

→ V_c hasn't changed in a loong time. (steady state)



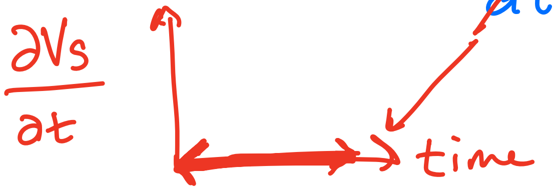
KCL: $I_1 + I_2 = 0$

$I_1 = -I_2$

Unknowns: I_c

I-V relation for C:

* $I_c = C\frac{dV_c}{dt} = 0$ $\quad V_c = U_1 - 0 = V_s$



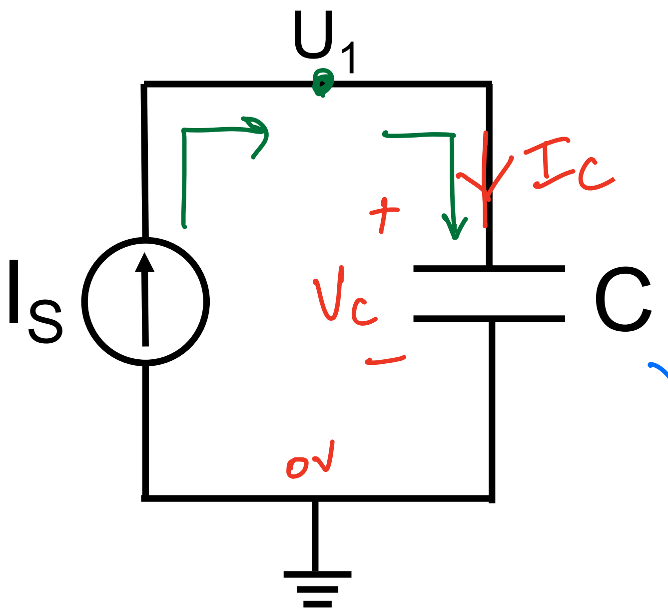
* $I_c = 0$

Circuit Example 2

At time $\underline{t = 0}$, $U_1 = \cancel{U_1(0)}\ 0$ Volts

Plot U_1 vs. time

cap is discharged



$U_1(0) = 0$

KVL: $V_c = U_1 - 0 = U_1 \Rightarrow V_c(t=0) = 0$

KCL: $I_S = I_C$

Unknown: $V_c = U_1$

I-V

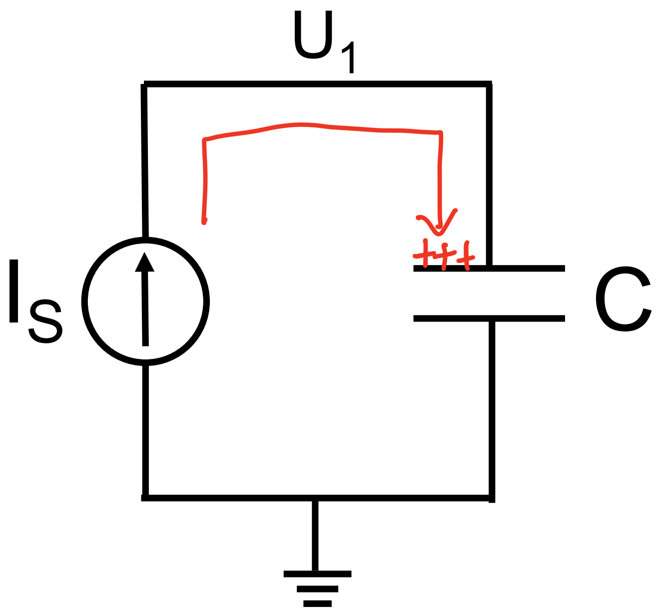
$I_c = C\frac{dV_c}{dt} \Rightarrow I_s^{\times dt} = C\frac{dU_1}{dt}^{\times dt}$

$\frac{I_s\,dt}{C} = \frac{\cancel{C}\,dU_1}{\cancel{C}} \quad \int dU_1 = \int_0^t \frac{I_s}{C}\,dt$

Circuit Example 2

At time $t = 0$, $U_1 = U_1(0)$ Volts

Plot U_1 vs. time



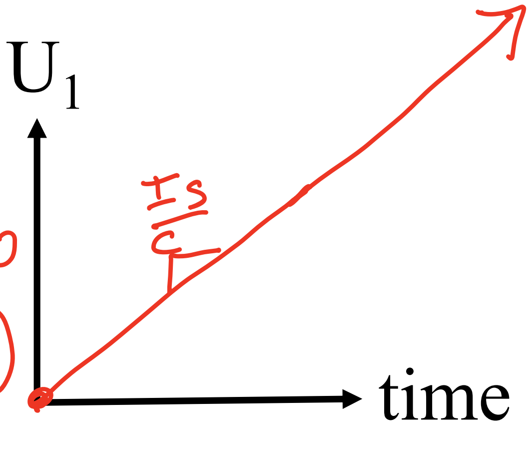
$$\int_{U_1(0)}^{U_1(t)} dU_1 = \int_0^t \frac{I_s}{C}\, dt$$

$$U_1 \Big|_{U_1(0)}^{U_1(t)} = \frac{I_s}{C}\, t \Big|_0^t$$

$$U_1(t) - U_1(0) = \frac{I_s}{C}\, t - \overset{0}{\cancel{\frac{I_s}{C}(0)}}$$

$$U_1(0) = 0$$

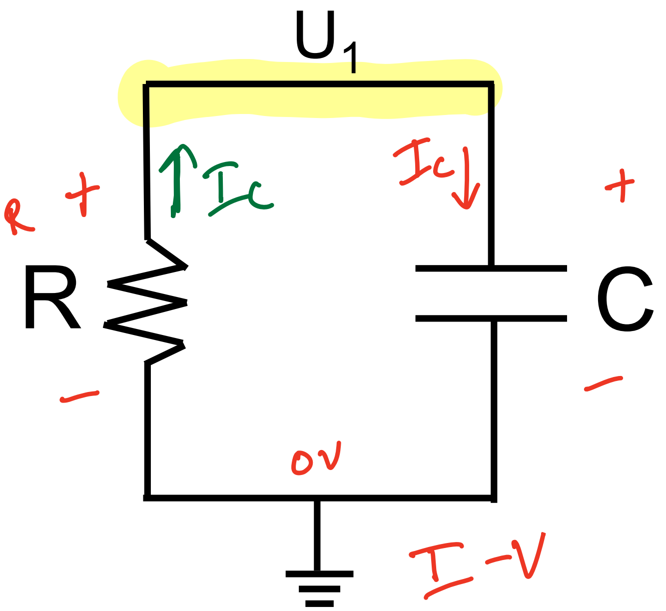
$$U_1(t) = \frac{I_s}{C}\, t$$



Circuit Example 3

* What is the steady-state potential U_1?

waited forever



V_R + −

I_c

$V_c = U_1 - 0$

I-V

KVL: $V_c = U_1$

KCL: $I_R + I_c = 0$

$I_R = -I_c$

Unknowns: U_1, I_c

I-V relations:

$V_R = I_R R$ $\quad V_R = U_1 - 0$

$V_R = -I_c R$

① * $U_1 = -I_c R$

I-V relation!

② $\underline{I_c} = C\frac{dV_c}{dt} = C\frac{\cancel{dU_1}}{\cancel{dt}}$ 0

know U_1 doesn't change

$I_c = 0$

$\therefore V_R = 0 \Rightarrow U_1 = 0$

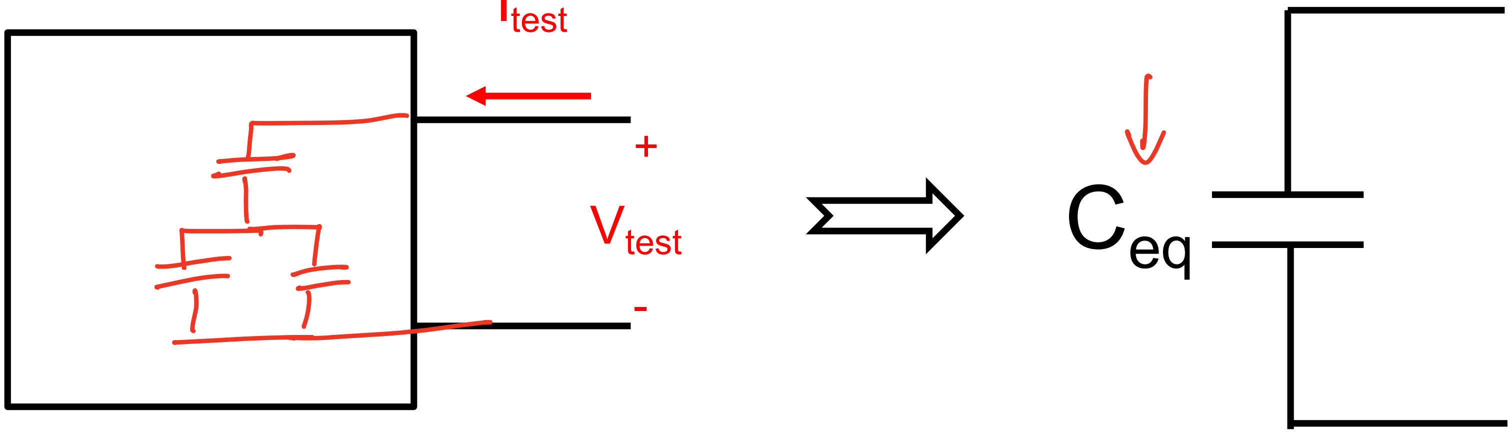
If you have a V_c that doesn't change there's no current

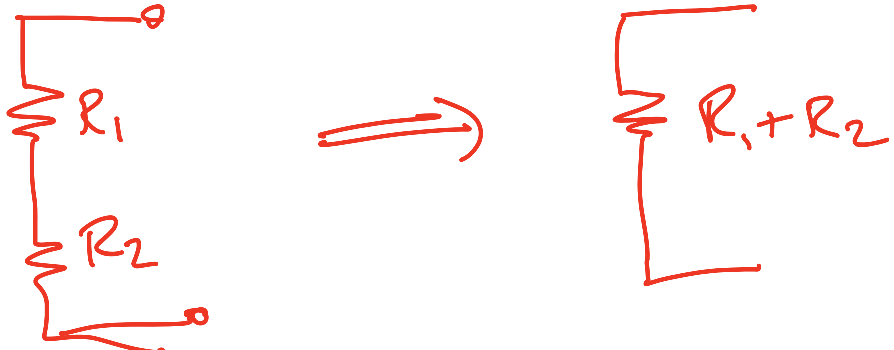
$I = C\frac{dV}{dt}$ $\underline{Q = CV}$

Equivalent Circuits with Capacitors
*Capacitor – only circuits

$$\boxed{I_C = C\frac{dV_C}{dt}}$$

Capacitive Circuit



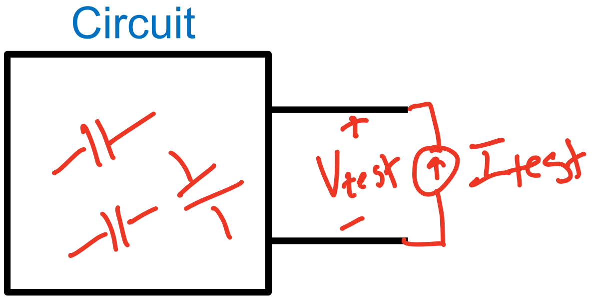


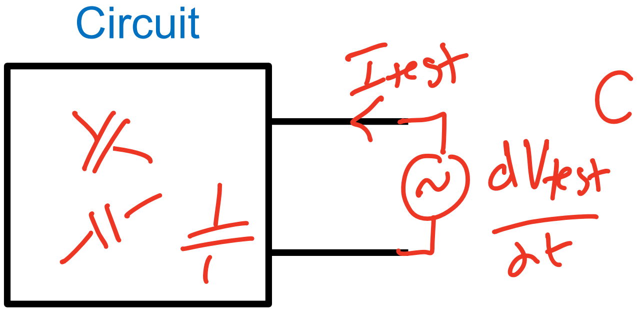
Resistor

$V = IR \quad R = \frac{V}{I}$

Two Methods

$$\boxed{I_C = C\frac{dV_C}{dt}}$$





$$C = \frac{I_C}{\partial V_C / \partial t}$$

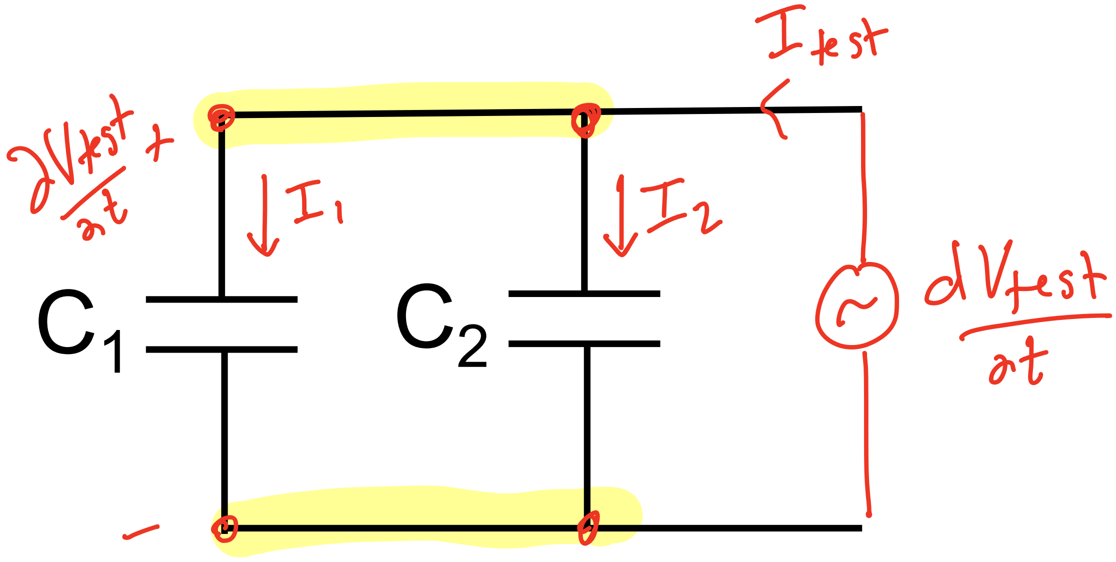
① Method 1:
Apply I_{test}
Measure dV_{test}/dt

Method 2:
Apply dV_{test}/dt
Measure I_{test}

find: $\dfrac{I_{test}}{\partial V_{test}/dt} = C_{eq}$

Equivalence Example 1

$$\boxed{I_C = C\frac{dV_C}{dt}}$$



KCL: $I_{test} = \underline{I_1} + \underline{I_2}$

I-V:

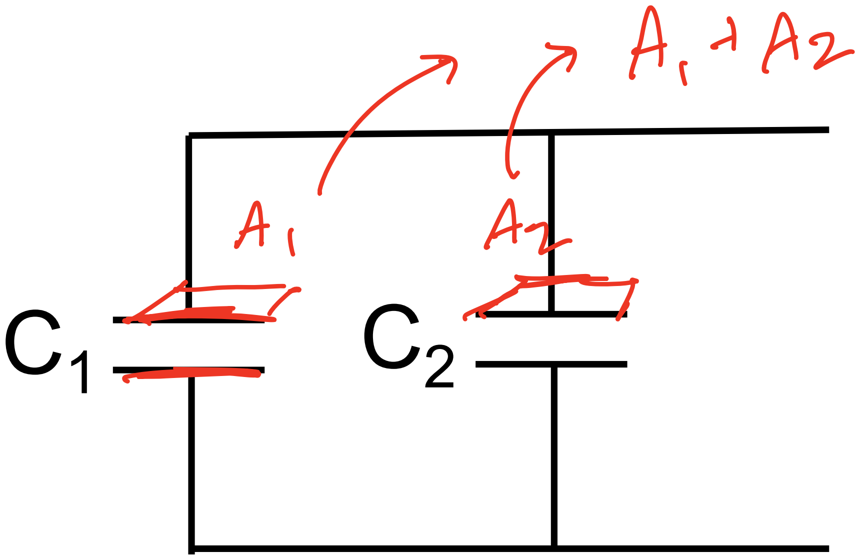
$I_1 = \frac{C_1 \, dV_{test}}{\partial t}$

$I_2 = C_2 \frac{\partial V_{test}}{\partial t}$

(2) Method 2:
Apply dV_{test}/dt
Measure I_{test}

Equivalence Example 1

$$I_C = C\frac{dV_C}{dt}$$



$$I_{test} = I_1 + I_2$$

$$= C_1 \frac{\partial V_{test}}{\partial t} + C_2 \frac{\partial V_{test}}{\partial t}$$

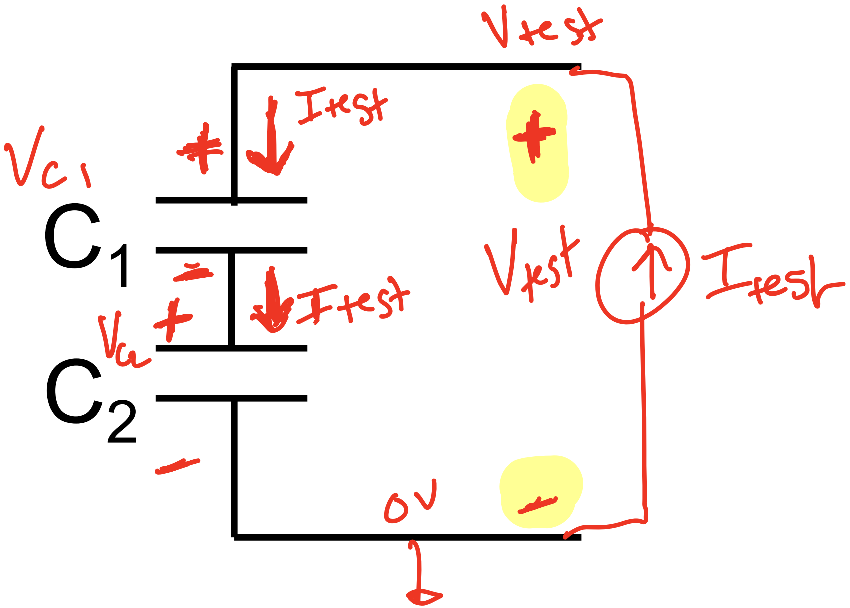
$$I_{test} = (C_1 + C_2)\frac{\partial V_{test}}{\partial t}$$

$$\frac{I_{test}}{\partial V_{test}/\partial t} = C_{eq} = \boxed{C_1 + C_2}$$

Method 2:
Apply dV_{test}/dt
Measure I_{test}

Equivalence Example 2

$$\boxed{I_C = C\frac{dV_C}{dt}}$$



KVL: $V_{test} = V_{c1} + V_{c2}$

$V_{test} - V_{c1} - V_{c2} = 0$

$\frac{\partial}{\partial t}$

① $\frac{\partial V_{test}}{\partial t} = \frac{\partial V_{c1}}{\partial t} + \frac{\partial V_{c2}}{\partial t}$

I-V

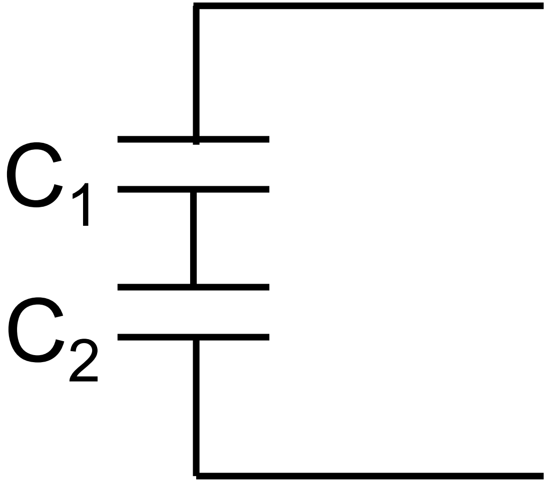
② $I_{test} = C_1 \frac{\partial V_{c1}}{\partial t} = C_2 \frac{\partial V_{c2}}{\partial t}$

$\frac{\partial V_{c1}}{\partial t} = \frac{I_{test}}{C_1}$ $\frac{\partial V_{c2}}{\partial t} = \frac{I_{test}}{C_2}$

① Method 1:
Apply I_{test}
Measure dV_{test}/dt

Equivalence Example 2

$$I_C = C\frac{dV_C}{dt}$$



$$\frac{\partial V_{test}}{\partial t} = \frac{\partial V_{C1}}{\partial t} + \frac{\partial V_{C2}}{\partial t}$$

$$\frac{\partial V_{test}}{\partial t} = \frac{I_{test}}{C_1} + \frac{I_{test}}{C_2}$$

$$* \; C_{eq} = \frac{I_{test}}{\partial V_{test}/\partial t} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1}$$

$$\frac{\partial V_{test}}{\partial t} = I_{test}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Method 1:

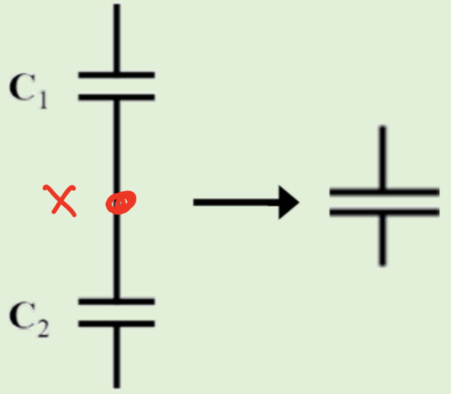
Apply I_{test}

Measure dV_{test}/dt

Equivalent Capacitors Summary

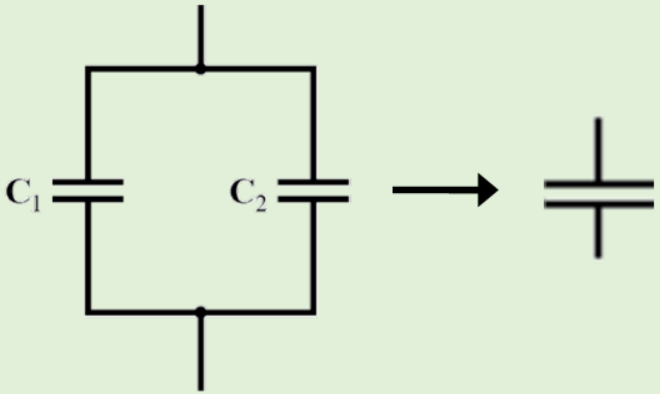
Capacitors in <u>Series</u>

$C_1 // C_2$

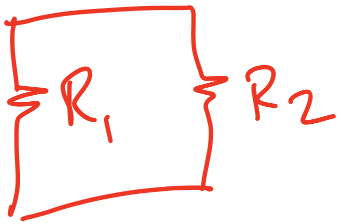


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in <u>Parallel</u>

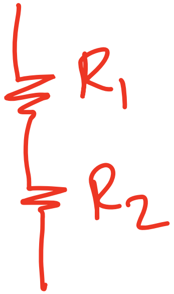


$$C_{eq} = C_1 + C_2$$

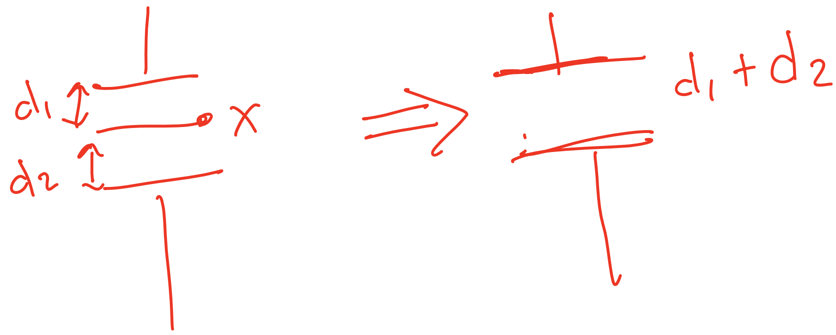


$$\frac{R_1 R_2}{R_1 + R_2}$$

$R_1 // R_2$

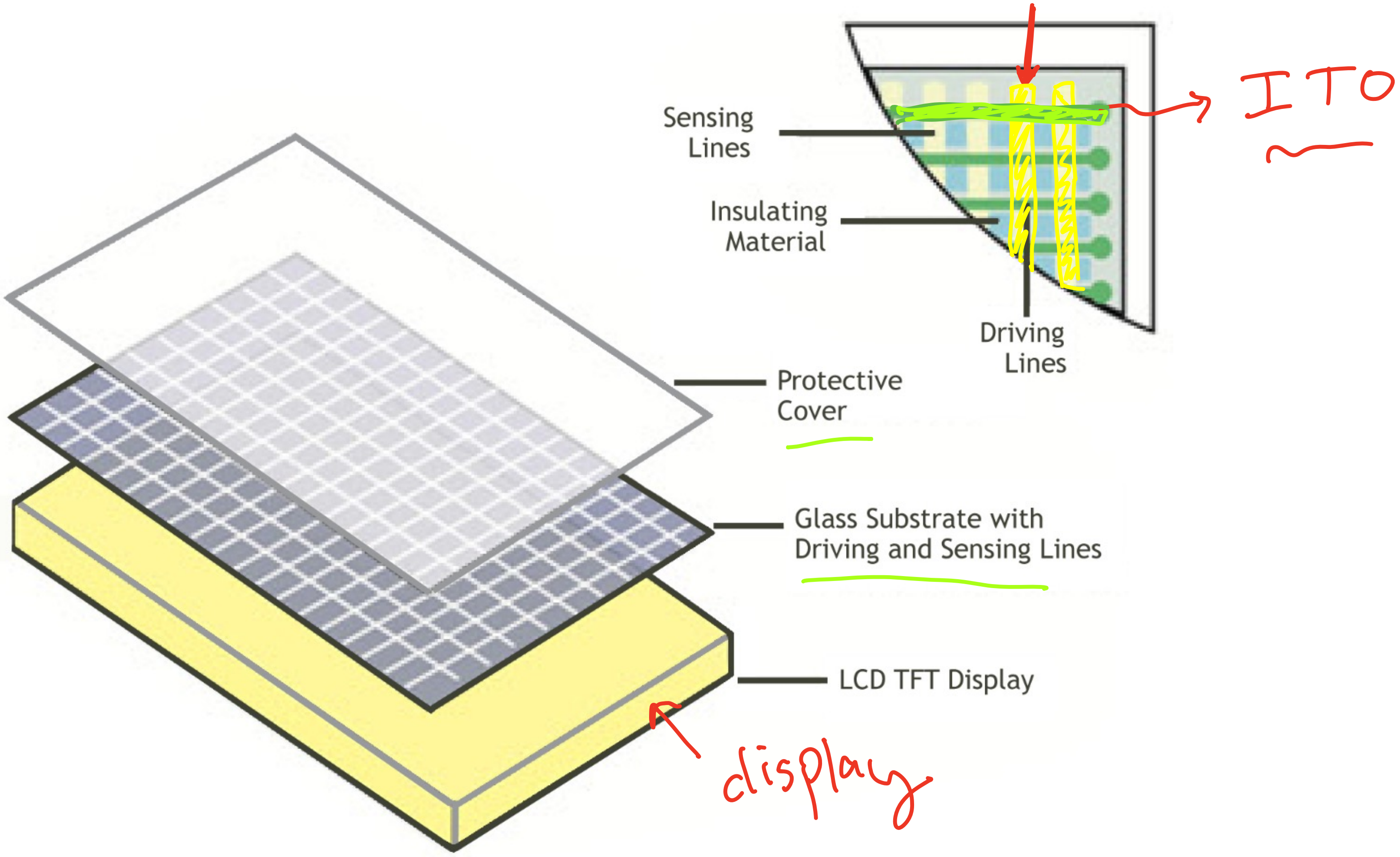


$R_{eq} = R_1 + R_2$



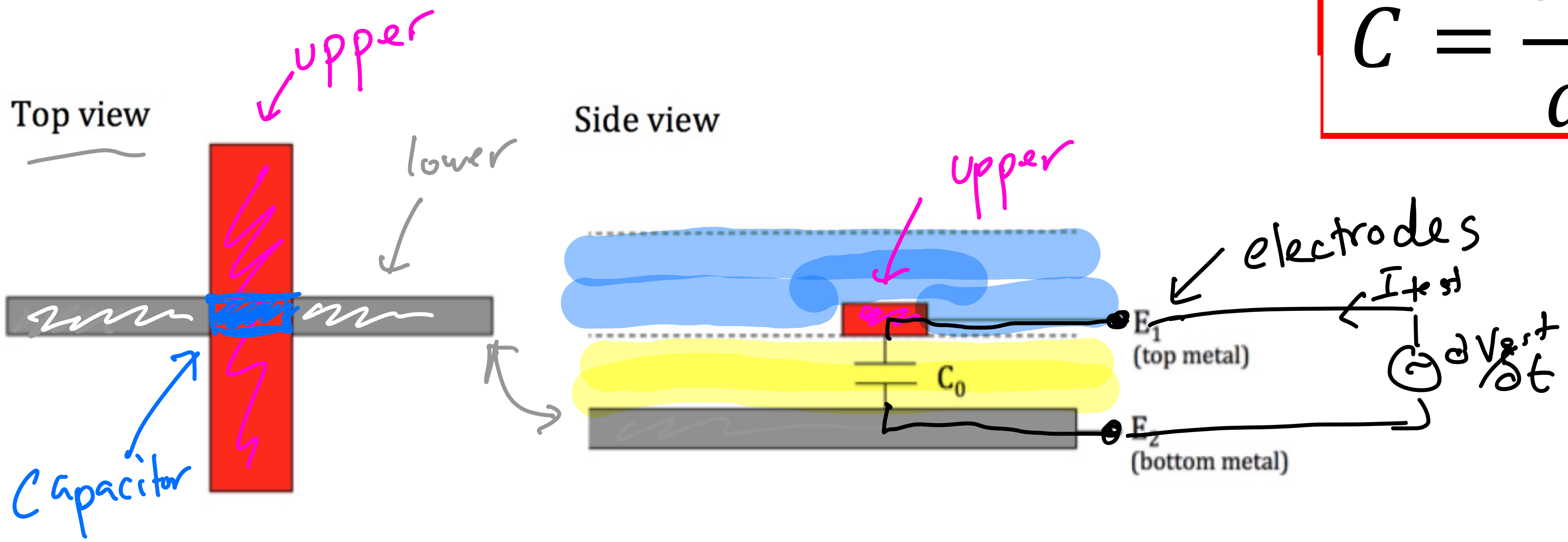
$$C = \frac{\epsilon A}{d}$$

Capacitive Touchscreens



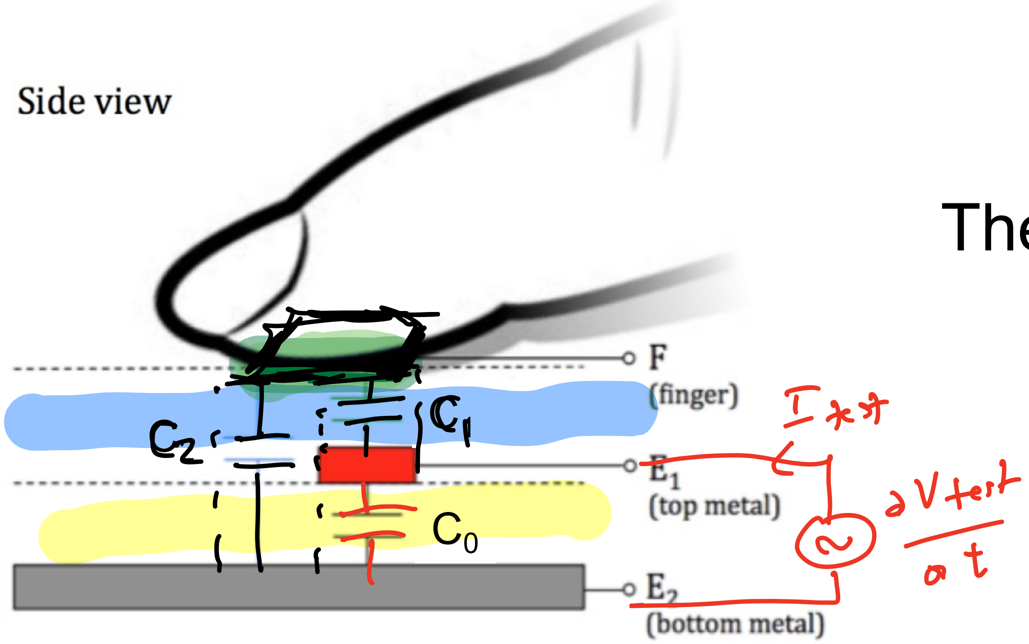
Capacitive Touchscreen – Model without Touch

$$\boxed{C = \frac{\epsilon A}{d}}$$



Capacitive Touchscreen – Model with Touch

*
$$\boxed{C = \frac{\epsilon A}{d}}$$

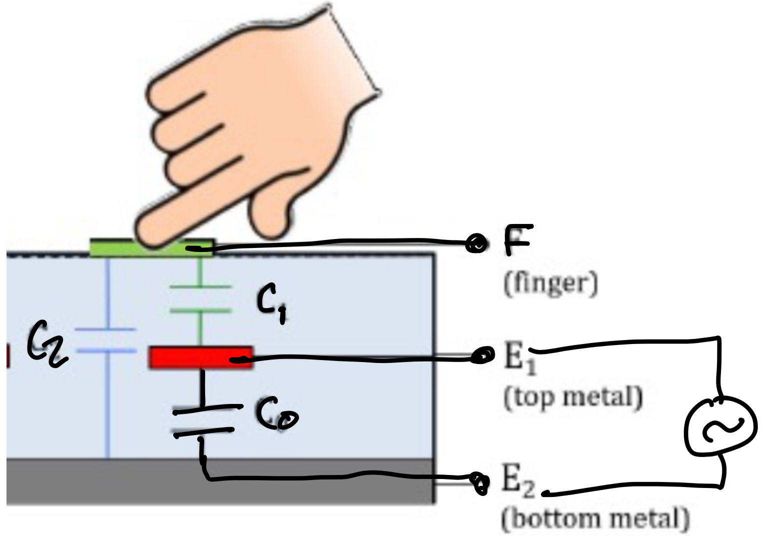


The finger forms a capacitor!

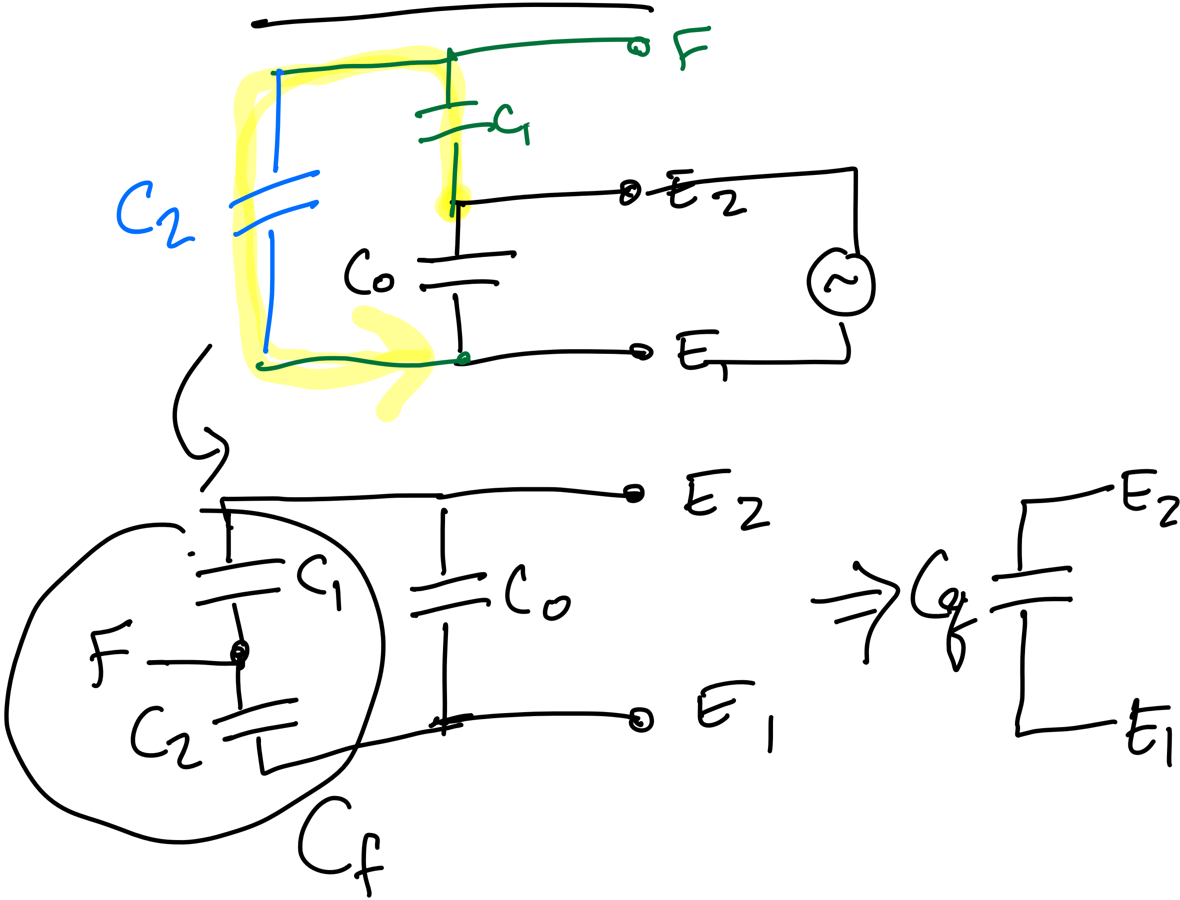
← the measurement will change with a finger!

*Problem: One of the terminals is a finger?!?

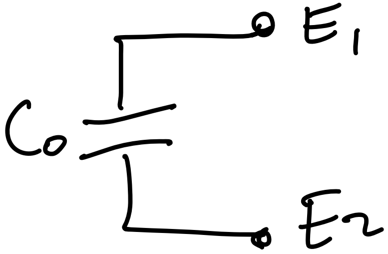
Capacitive Touchscreen – Model with Touch



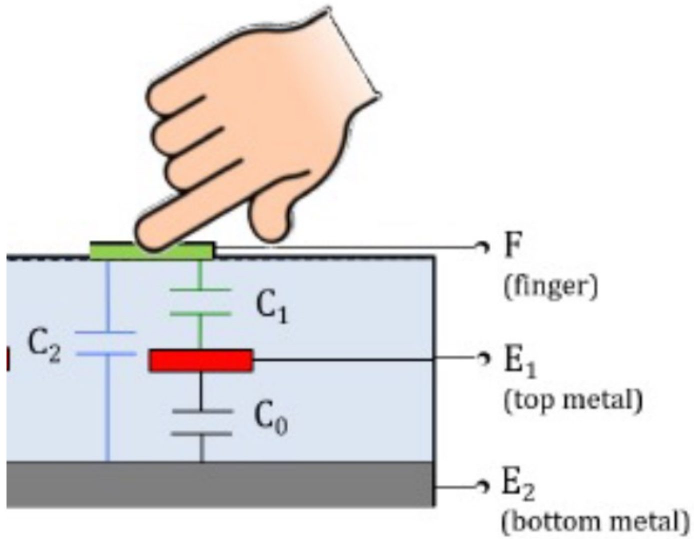
with touch

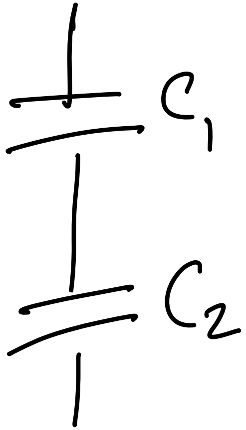


No touch



Capacitive Touchscreen – Model with Touch





$$\Longrightarrow \quad \frac{C_1 C_2}{C_1 + C_2} = C_f$$

w/finger
extra cap