EECS 16A Designing Information Devices and Systems I
Fall 2022 Midterm 2

Midterm 2 Solution

1. HONOR CODE

Please read the following statements of the honor code, and sign your name (you don't need to copy it).

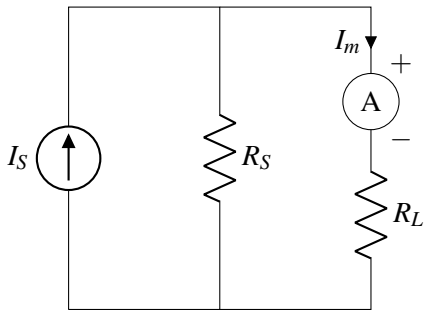
I will respect my classmates and the integrity of this exam by following this honor code. I affirm:

- *I have read the instructions for this exam. I understand them and will follow them.*
- *All of the work submitted here is my original work.*
- *I did not reference any sources other than my one reference cheat sheet.*
- *I did not collaborate with any other human being on this exam.*

2. Tell us a joke or pun about circuits (1 point) *All answers will be awarded full credit.*

3. Why is the ammeter so hot? (14 points)

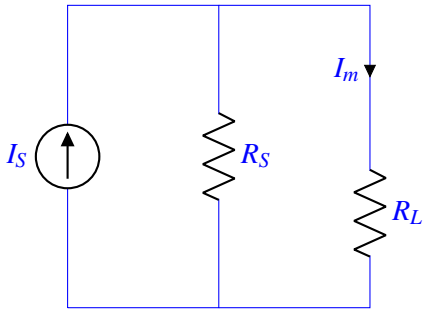
In the following circuit, we would like to measure the current I_m through resistor R_L using an ammeter.



(a) (4 points) If $I_S = 4\,\text{A}$, $R_S = 3\,\text{m}\Omega$, and $R_L = 1\,\text{m}\Omega$, then what is the measured current I_m? Assume an ideal ammeter.

Solution:

The ideal ammeter is an equivalent short circuit.



Using the current divider equation, we have

$$I_m = \frac{R_S}{R_S + R_L} \cdot I_s = \frac{(3\,\text{m}\Omega)}{(3\,\text{m}\Omega) + (1\,\text{m}\Omega)}(4\,\text{A})$$
$$I_m = 3\,\text{A}.$$

(b) (4 points) Now suppose we have different values of I_s and R_s such that $I_m = 2\,\text{A}$, then **what is the power dissipated by $R_L = 1\,\text{m}\Omega$? How much energy does R_L dissipate in 20 seconds?**

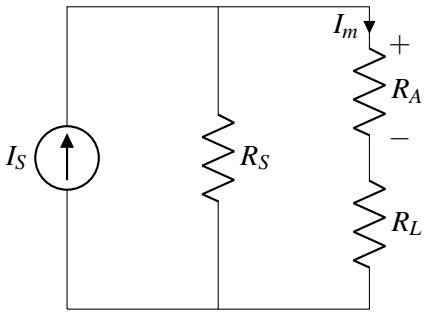
Solution: To find the energy dissipated by R_L we can consider the current I_L through it and then use one of the power equations for a resistor $P = I^2L = R$. Doing this yields

$$I_L = I_m = 2\,\text{A}$$
$$P_L = I_L^2 R_L = (2\,\text{A})^2(1\,\text{m}\Omega) = 4\,\text{mW}$$

The amount of energy dissipated by R_L in 20 seconds is just the product of power and time (since power is constant) which gives

$$E_L = P_L \Delta t = (4\,\text{mW})(20\,\text{s})$$
$$= 80\,\text{mJ}$$

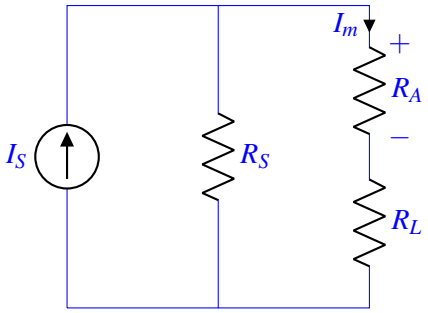
(c) (6 points) Now assume the ammeter is not ideal and can instead be modeled with equivalent resistance R_A. That is, we can replace the ammeter with a resistor R_A:



If $I_S = 4\,\text{A}$, $R_S = 3\,\text{m}\Omega$, $R_L = 1\,\text{m}\Omega$, and the current measured by the ammeter (i.e., the current through R_A) is $I_m = 2\,\text{A}$, then **what must be the internal resistance R_A of the ammeter? How much power is dissipated by the ammeter?**

Solution:

The equivalent circuit replaces the non-ideal ammeter with an equivalent resistance R_A.



Using a current divider and combining R_A and R_L we get,

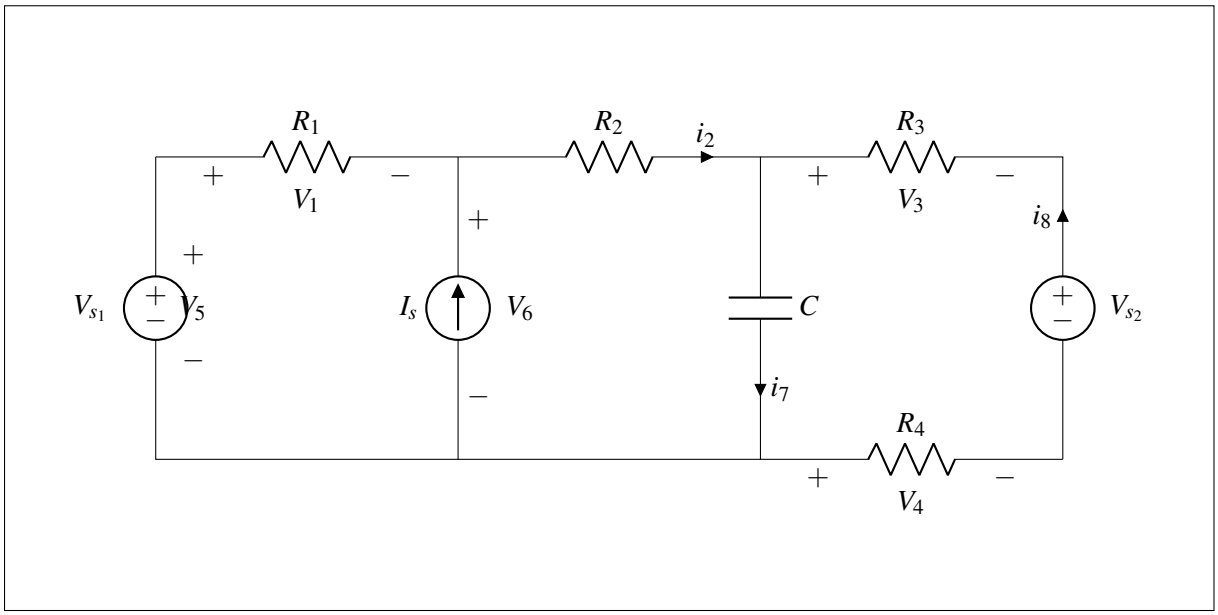
$$\begin{aligned} I_m &= \frac{R_S}{R_S + (R_A + R_L)} \cdot I_s \\ I_m R_A &= I_S R_s - I_m \cdot (R_S + R_L) \\ R_A &= \frac{I_S}{I_m} R_S - (R_S + R_L) \\ &= \frac{(4\,\text{A})}{(2\,\text{A})}(3\,\text{m}\Omega) - (3\,\text{m}\Omega + 1\,\text{m}\Omega) \\ R_A &= 2\,\text{m}\Omega \end{aligned}$$

The power dissipated by the ammeter is then

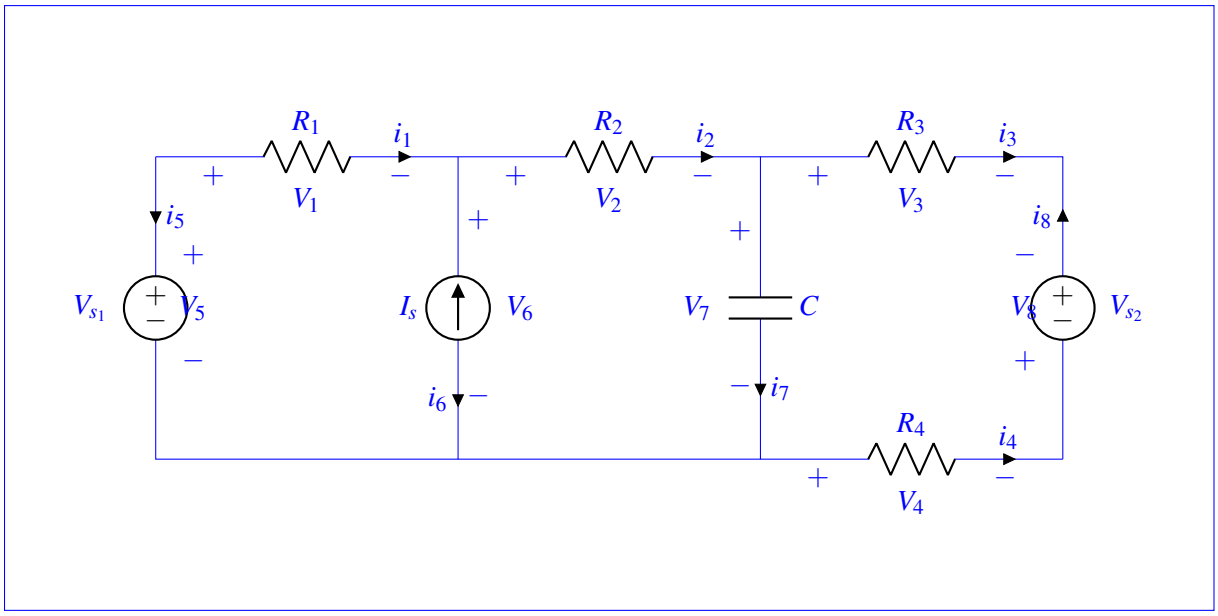
$$\begin{aligned} P_A &= I_m^2 R_A = (2\,\text{A})^2 (2\,\text{m}\Omega) \\ &= 8\,\text{mW} \end{aligned}$$

4. Just Solvin' a Circuit (18 points)

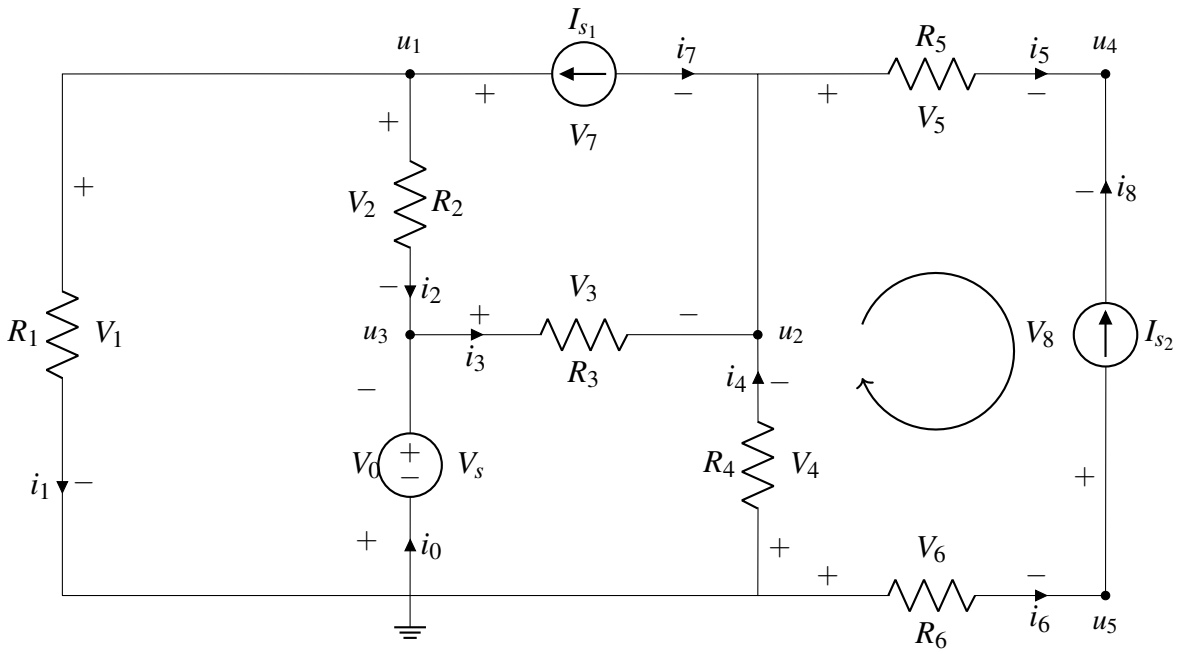
(a) (2 points) For **this part only**, consider the following circuit. For *each* element in the circuit **(including the sources!)**, label the missing current or voltage **using passive sign convention**. Please label directly on the given circuit below. Voltage labels should have +/- terminals. For example, the voltage source V_{s_1} has its voltage already labeled (V_5) but is missing its current label.



Solution: Every element should have a voltage, current, and $+/-$ terminals labeled. The current should flow from the positive terminal to the negative terminal following passive sign convention.


R_1
i_1
R_2
i_2
R_3
i_3
V_1
V_2
V_3
i_5
i_8
V_{s_1}
V_5
I_s
V_6
V_7
C
V_8
V_{s_2}
i_6
i_7
R_4
i_4
V_4

Now, **for the remaining parts**, consider an entirely different circuit which is already labeled.



(b) (2 points) Write the KVL equation for the labeled loop in terms of element voltages (V_1, V_2, ... ,V_8) only.

Solution: Depending on which convention is used,

$$-V_4 - V_5 + V_8 + V_6 = 0$$

or

$$V_4 + V_5 - V_8 - V_6 = 0$$

or any equivalent expression.

(c) (4 points) Write the KCL equations for nodes u_1 and u_2 **ONLY** in terms of I_{s1}, I_{s2}, node voltages (**not element voltages!**), and resistances.

Solution:

For u_1,

$$i_1 + i_2 + i_7 = 0$$
$$i_1 + i_2 - I_{s1} = 0$$

$$\frac{u_1}{R_1} + \frac{u_1 - u_3}{R_2} - I_{s1} = 0$$

or equivalent equations

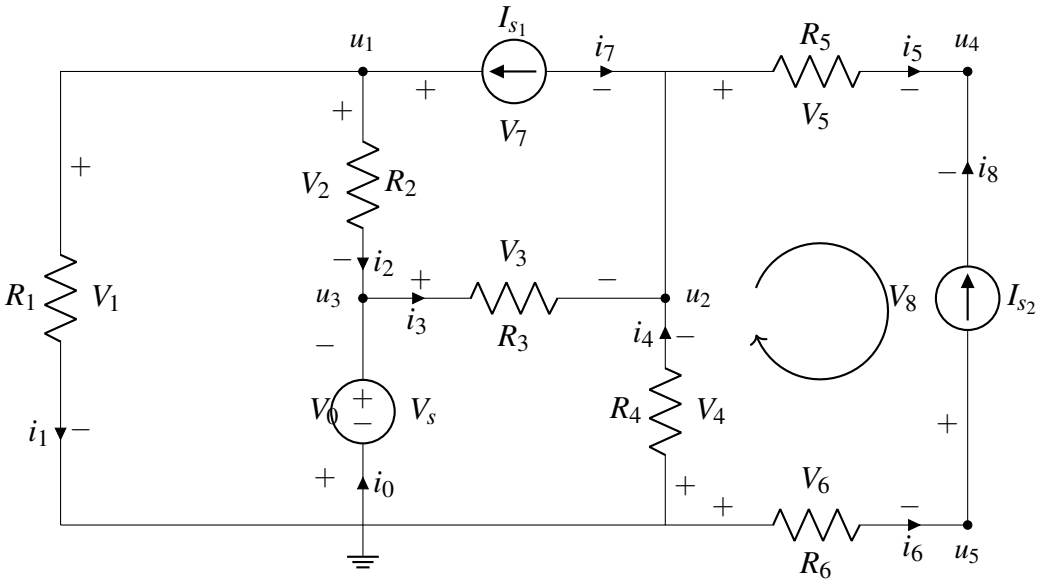
For u_2,

$$i_3 + i_4 + i_7 = i_5$$

$$\frac{u_3 - u_2}{R_3} - \frac{u_2}{R_4} - I_{s1} + I_{s2} = 0$$

or equivalent equations. Note that I_{s2} could also be replaced with $\frac{u_2-u_4}{R_5}$ or $-\frac{u_5}{R_6}$

Here the circuit is redrawn for your convenience:

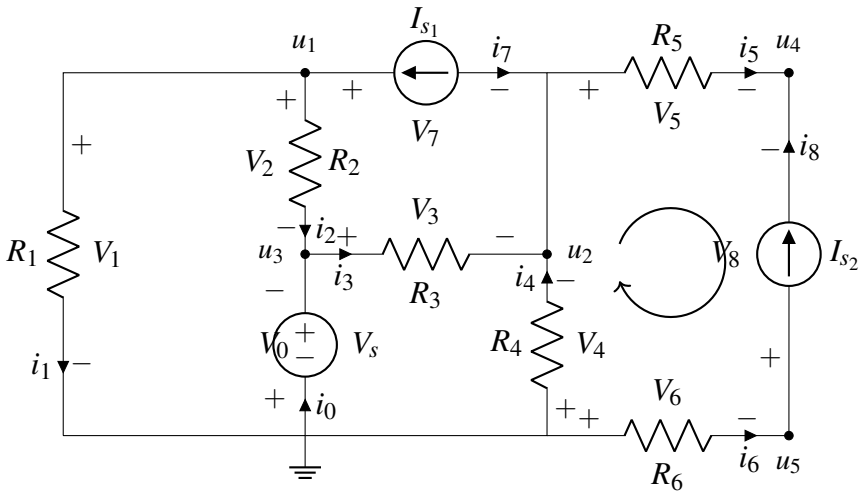


(d) (10 points) Solve for the voltage V_1 across and current I_1 through R_1 given the following values:

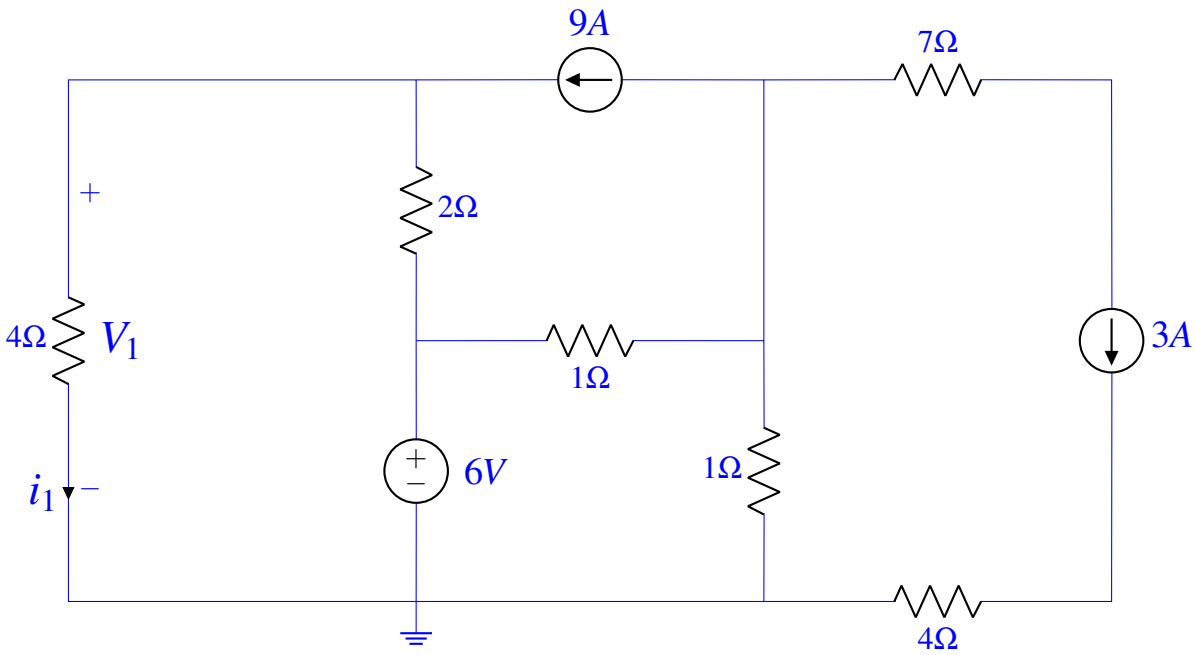
$$R_1 = 4\Omega,\ R_2 = 2\Omega,\ R_3 = 1\Omega,\ R_4 = 1\Omega,\ R_5 = 7\Omega,\ R_6 = 4\Omega$$
$$V_s = 6V,\ I_{s1} = 9A,\ I_{s2} = 3A$$

You do **NOT** need to solve for all quantities in the circuit nor use all of the above given values in your answer.

Here the circuit is redrawn for your convenience:

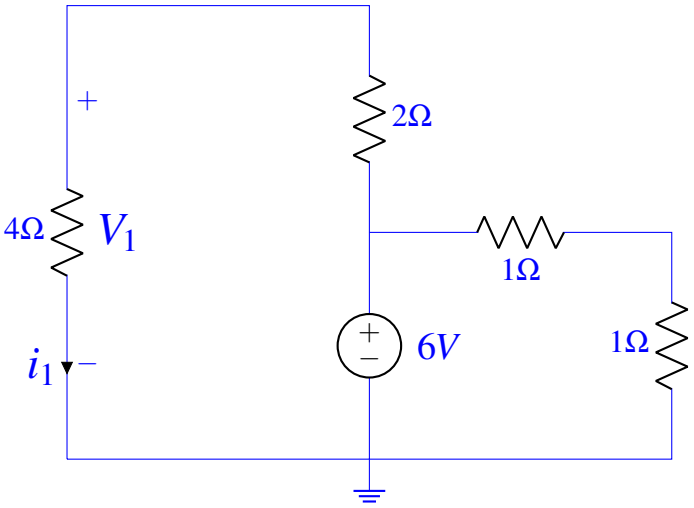


Solution: With these given values, we have the following circuit:



To find i_1 and V_1, we perform superposition on the circuit.

First, turn V_s on and zero out all other independent sources. This means that I_{s1} and I_{s2} behave like open circuits.



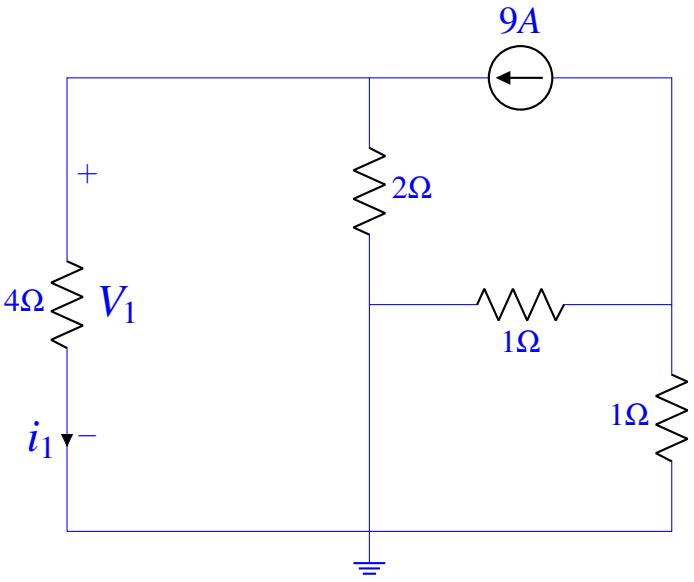
By inspection, the left half of the circuit is a voltage divider. By the voltage divider formula,

$$V_1 = \frac{4\Omega}{2\Omega + 4\Omega} \cdot 6V = \frac{2}{3} \cdot 6V = 4V$$

Using Ohm's Law, we can find i_1 from V_1:

$$i_1 = \frac{V_1}{R_1} = \frac{4V}{4\Omega} = 1A$$

Next, turn on I_{s1} and zero out all other independent sources. This means that V_s behaves like a wire and I_{s2} behaves like an open circuit.



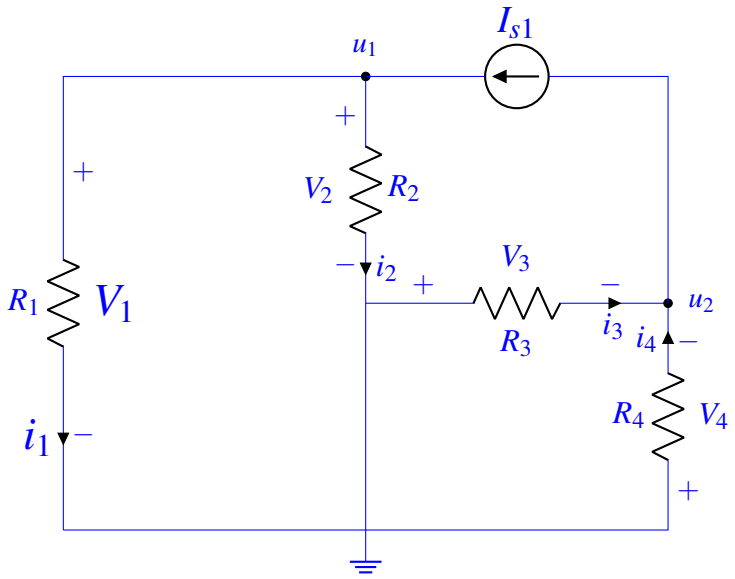
By inspection, we have a current divider, where the $9A$ current splits into the 4Ω and 2Ω branches. We can then apply the current divider formula as follows:

$$i_1 = \frac{2\Omega}{2\Omega + 4\Omega} \cdot 9A = \frac{1}{3} \cdot 9A = 3A$$

Using Ohm's Law, we can find V_1 from i_1:

$$V_1 = i_1 R_1 = 3A \cdot 4\Omega = 12V$$

Alternatively, we can solve this via NVA:



We write KCL for node u_1 and solve:

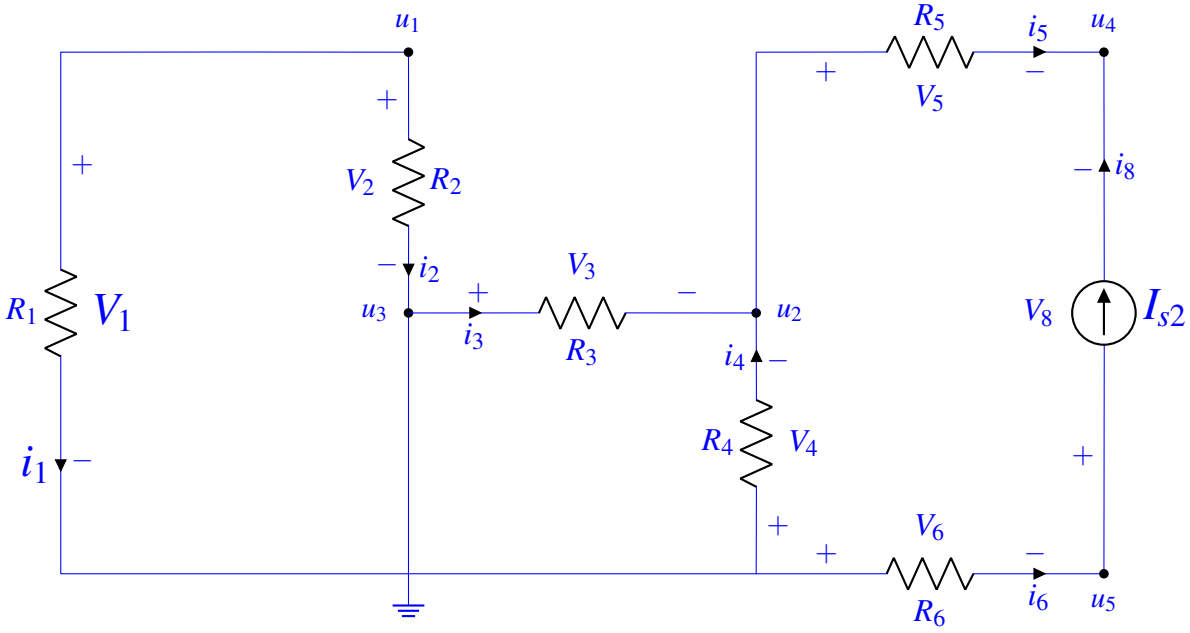
$$\frac{u_1}{R_1} + \frac{u_1}{R_2} = I_{s1}$$

$$\frac{R_2 u_1 + R_1 u_1}{R_1 R_2} = \frac{(R_1 + R_2) u_1}{R_1 R_2} = I_{s1}$$

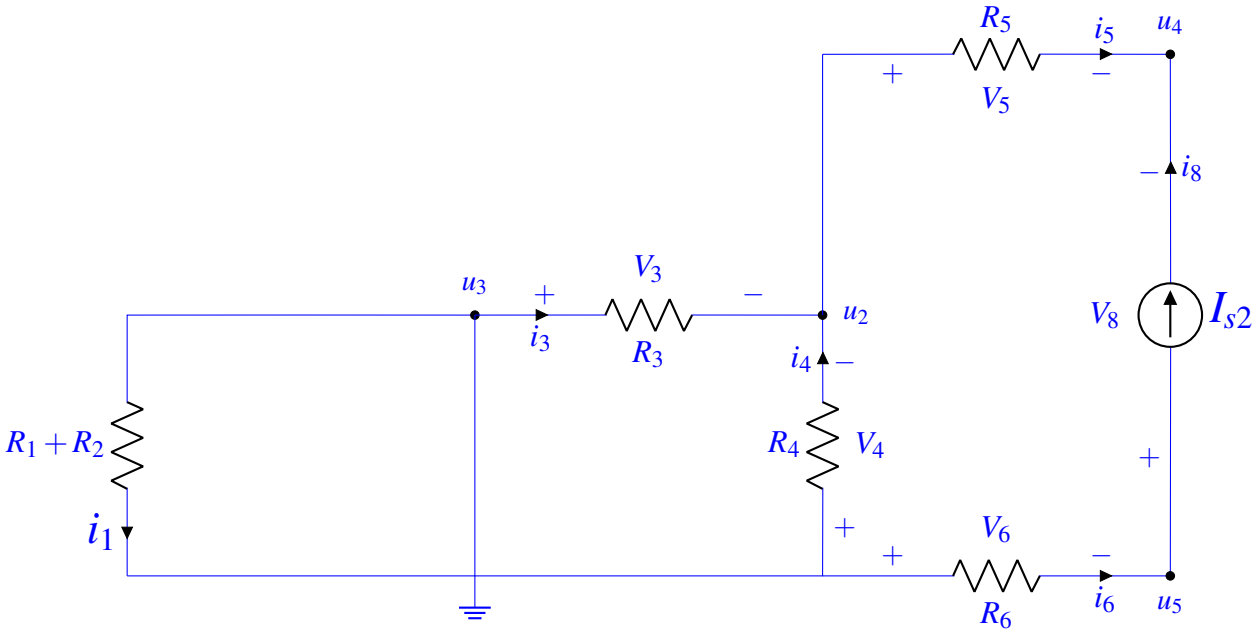
$$V_1 = u_1 = \frac{R_1 R_2}{R_1 + R_2} I_{s1} = \frac{8\Omega^2}{6\Omega} \cdot 9A = 12V$$

$$i_1 = \frac{V_1}{R_1} = \frac{R_2}{R_1 + R_2} I_{s1} = \frac{2\Omega}{4\Omega + 2\Omega} \cdot 9A = 3A$$

Lastly, turn on I_{s2} and zero out all other independent sources. This means that V_s behaves like a wire and I_{s1} behaves like an open circuit.



We can simplify the circuit by combining resistors R_1 and R_2 which are in series:



By inspection, the R_1+R_2 resistor is shorted because it is connected in parallel to a wire. This means there is no current flowing through the R_1+R_2 resistor. To see this, we can use Ohm's Law:

$$u_3 = \text{ground} = 0$$
$$i_1 = \frac{u_3 - 0}{R_1+R_2} = \frac{0}{R_1+R_2} = 0A$$
$$V_1 = i_1 R_1 = 0V$$

Alternatively, we can solve this via NVA by using KCL at node u_1:

$$i_1 = i_2$$
$$\frac{u_1}{R_1} = \frac{u_1}{R_2}$$

Since $R_1 \neq R_2$, the above equation is only true if $u_1 = 0$. Then,

$$i_1 = \frac{u_1}{R_1} = 0A$$
$$V_1 = i_1 R_1 = 0V$$

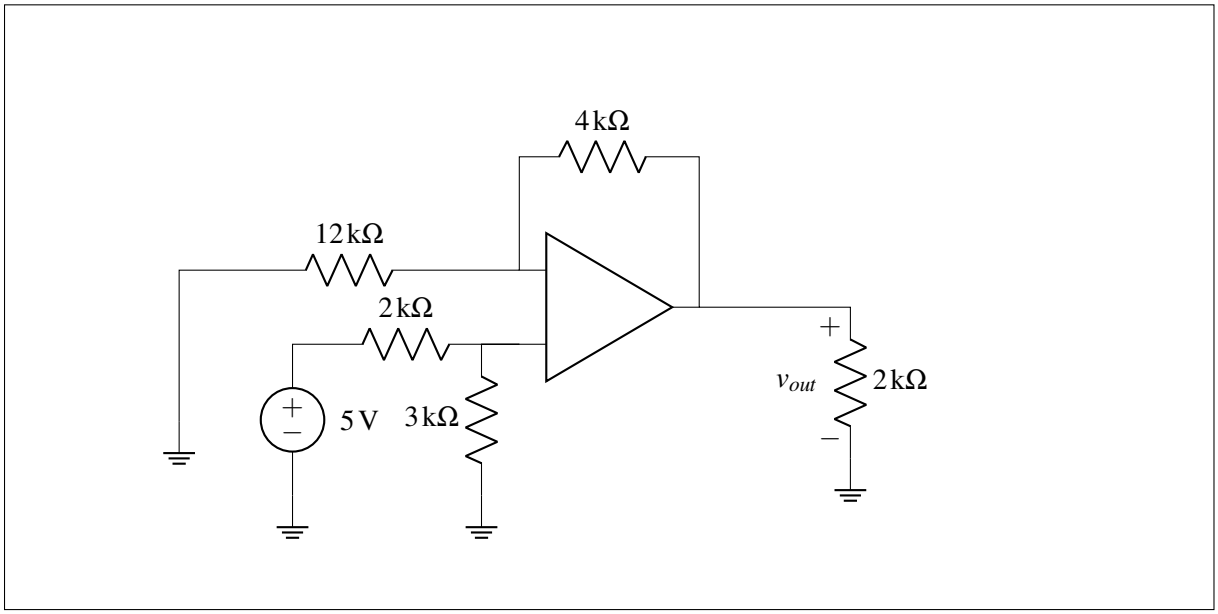
Finally, using superposition we can sum up the contributions from all three sources to find i_1 and V_1 of the complete circuit:

$$i_1 = i_1\Big|_{V_{s,on}} + i_1\Big|_{I_{s1,on}} + i_1\Big|_{I_{s2,on}} = 1A + 3A + 0A = 4A$$

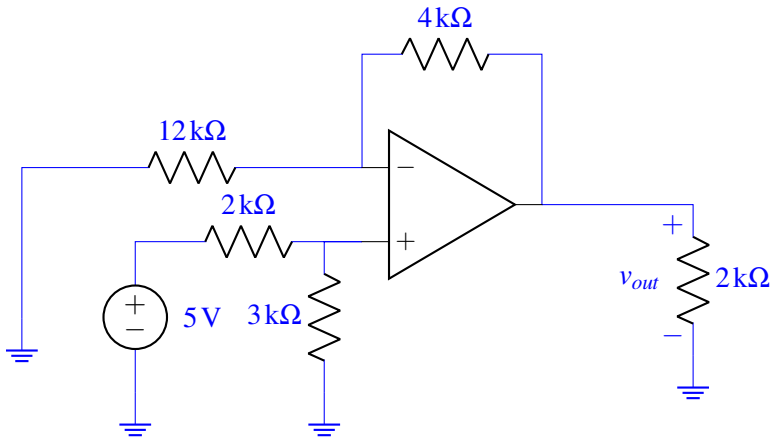
$$V_1 = V_1\Big|_{V_{s,on}} + V_1\Big|_{I_{s1,on}} + V_1\Big|_{I_{s2,on}} = 4V + 12V + 0V = 16V$$

5. Op Amps and Comparators (17 points)

(a) (2 point) Consider the following op-amp **for this part only**. Notice the + and - signs are missing on the op-amp input. Please label them such that the op-amp is in negative feedback.

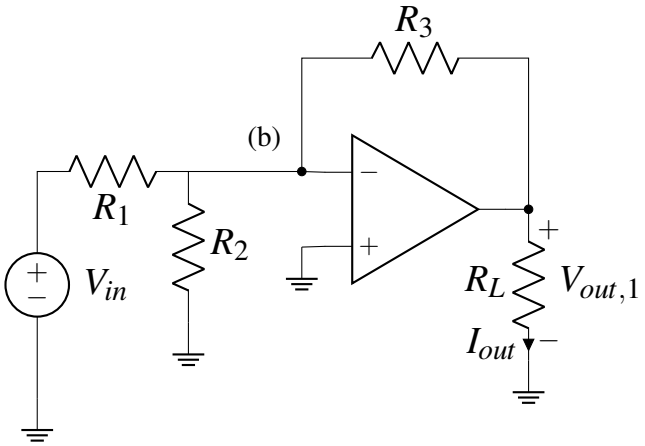


Solution:



This circuit is in negative feedback because "dinking" the output voltage of the Op-Amp causes U^- (the negative input terminal of the Op-Amp) to increase as well which then causes $V_{out} = A(U^+ - U^-)$ to decrease. Since U^- is in some way connected back to V_{out} and the dinked output led to a subsequent change in the opposite direction, the circuit is indeed in negative feedback.

For the next two parts consider an entirely new op-amp circuit. Determine the following (you can assume that the op-amp is ideal and in negative feedback):



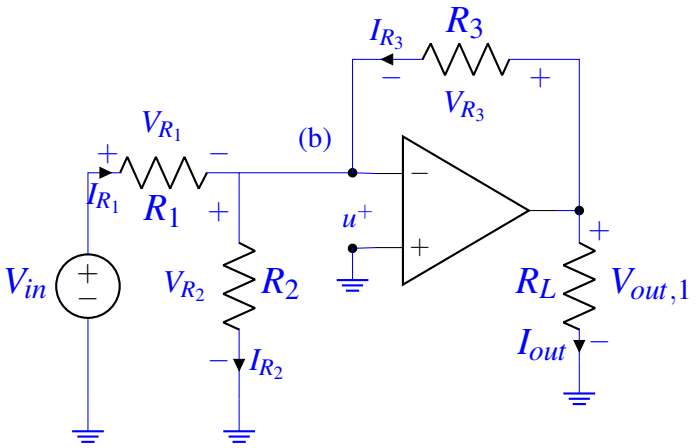
(b) (3 point) What is the voltage held at the negative terminal of the op-amp (the node labeled (b))? Express your answer in terms of the variables shown in the circuit diagram.

Solution:

Since it is given that we are working with an ideal Op-Amp and our Op-Amp is in negative feedback, we can apply the second golden rule to this circuit ($U^+ = U^-$). Since U^+ is connected to ground node and we know ($U^+ = U^-$), U^- must be 0V.

(c) (6 points) What is I_{out}, the current flowing through R_L? **Express your answer ONLY in terms of** $V_{in}, R_1, R_2, R_3, R_L$. You **cannot** use $V_{out,1}$ in your final expression.

Solution:



The above circuit uses ideal Op-Amps and is in negative feedback. Thus, we can apply the two golden rules:

1. $I^+ = I^- = 0A$
2. $U^+ = U^- = 0V$ (from part b)

We can solve for $V_{out,1}$ first and then use Ohm's law to get the desired I_{out}.

To begin, we perform KCL on U^-:

$I_{R_1} + I_{R_3} = I_{R_2}$

$\frac{V_{in}}{R_1} + \frac{V_{R_3}}{R_3} = \frac{U^-}{R_2}$

$\frac{V_{out,1}}{R_3} = -\frac{V_{in}}{R_1}$

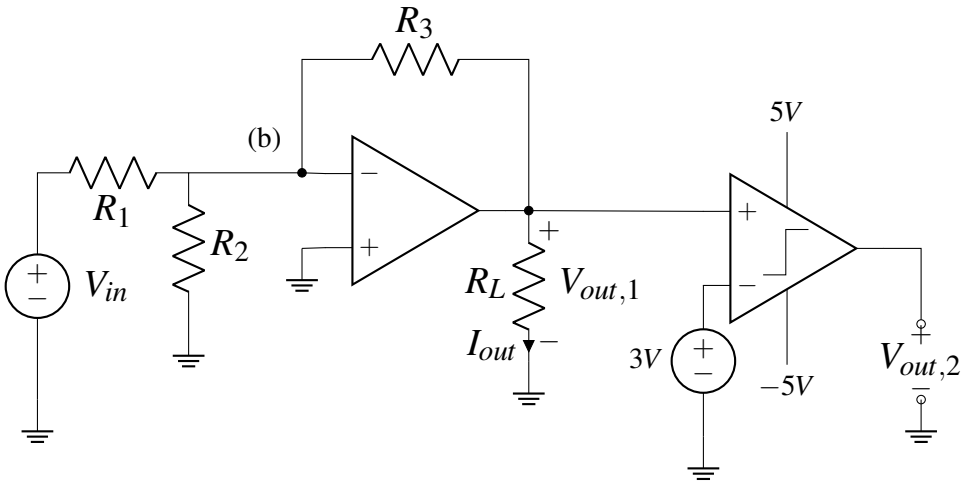
Solving for $V_{out,1}$, we get:

$V_{out,1} = -\frac{R_3}{R_1} V_{in}$

Finally, applying Ohm's law gives $I_{out} = \frac{V_{out,1}}{R_L} = -\frac{R_3}{R_1 R_L} V_{in}$

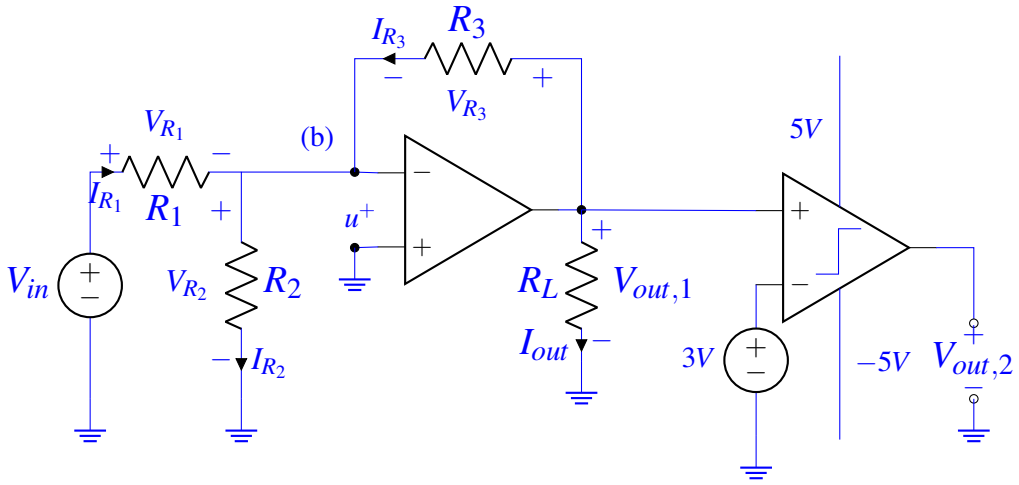
(d) (3 points)

Now consider attaching an ideal comparator to the output of the op-amp circuit from the previous part.



Suppose the output voltage of the op-amp is $V_{out,1} = -2V$ What is $V_{out,2}$, the voltage outputted by the comparator?

Solution:



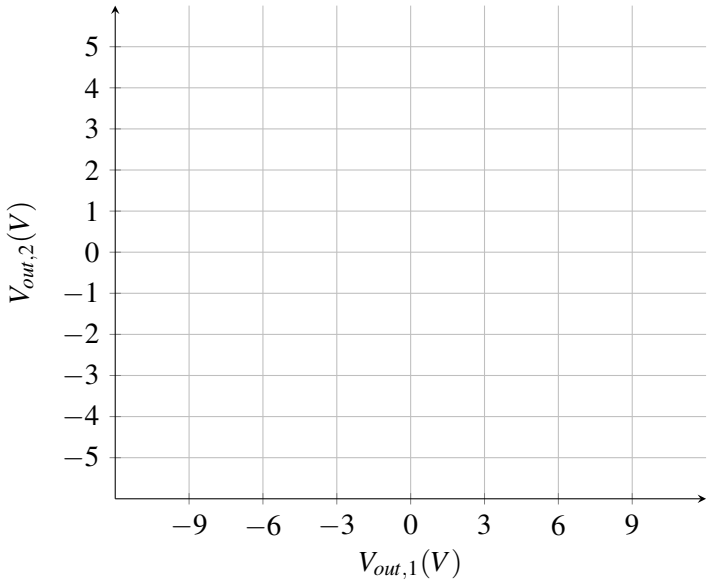
We know that when the input voltage at the negative terminal of the comparator exceeds that at the positive terminal, then we output V_{SS}. However, when the input voltage at the positive terminal of the comparator exceeds that at the negative terminal, then we output V_{DD}. When they are equivalent to each other (i.e. $U_c^+ = U_c^-$), then we see the comparator instantenously change from outputting V_{SS} to outputting V_{DD}. Therefore, we can compute the difference of the voltages at the two input terminals to determine the output of the comparator:

$U_c^+ = V_{out,1} = -2V$

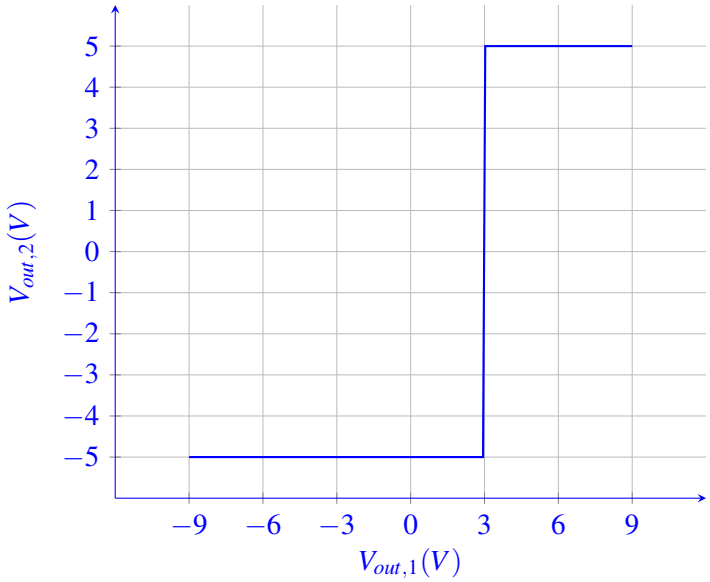
$U_c^- = 3V$

$U_c^+ - U_c^- = -2V - 3V = -5V < 0$ Therefore, $V_{out,2} = V_{SS} = -5V$

(e) (3 points) Sketch the output behavior of the comparator on the plot provided for different values of $V_{out,1}$.



Solution:



Observe that we always output the value of V_{SS} when $U_c^+ < U_c^-$, switch from outputting V_{SS} to outputting V_{DD} when $U_c^+ = U_c^-$, and remain outputting V_{DD} for the entire time that $U_c^+ > U_c^-$. Also observe that the shape of the graph takes on the shape of the internal labeling of the comparator!

6. Oski is Soldering a Resistive Touchscreen (12 points)

(a) (4 points) Oski attended Touchscreen 2 lab and was able to make a resistive touchscreen model that supports 9 touches. The schematic of the touchscreen bottom mesh is shown on Fig. 1.

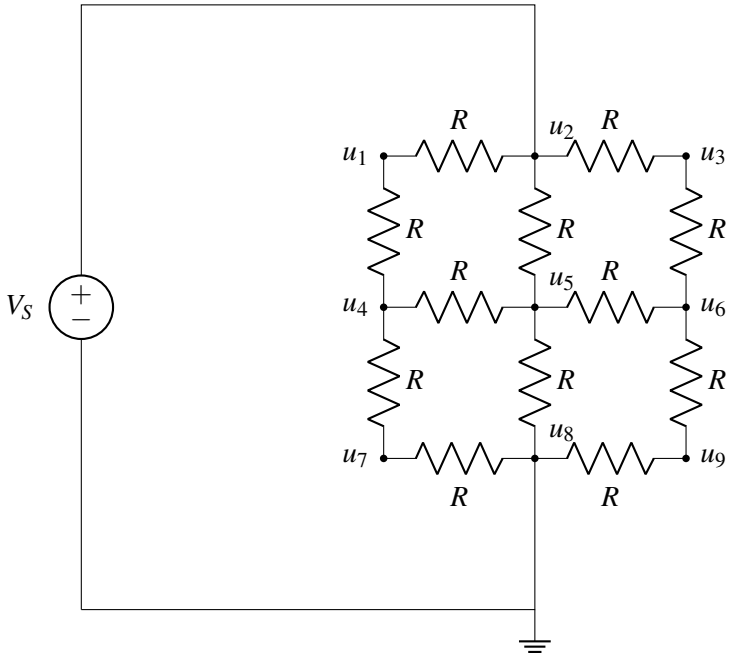


Fig. 1: The bottom mesh of a 2D touchscreen schematic.

Knowing that the lab model suggests to take all 12 resistors to have the same value, and $V_s = 8V$, what are the voltage values on each node? Fill out the table:

node name	voltage (V)
u1	
u2	
u3	
u4	
u5	
u6	
u7	
u8	
u9	

Solution:

node name	voltage (V)
u1	6
u2	8
u3	6
u4	4
u5	4
u6	4
u7	2
u8	0
u9	2

$$u_8 = 0V \text{ (ground node)}$$

$$u_2 = 8V \text{ (from the voltage source)}$$

$$u_5 = \frac{1}{2}u_2 = 4V$$

$$u_4 = u_6 = u_5 = 4V$$

$$u_1 = u_3 = 4V + \frac{1}{2}(8-4)V = 6V$$

$$u_7 = u_9 = \frac{1}{2}(4-0)V = 2V$$

(b) (2 points) The lab Oski is in only has a two different types of resistor, R_1 and R_2. They are both 4-band resistors but have different colored bands: R_1 has yellow-violet-red-gold and R_2 has red-violet-yellow-gold. What are the values of resistances **R_1 (yellow-violet-red-gold)** and **R_2 (red-violet-yellow-gold)**. Include the tolerance value in your answer (you may leave it as a percentage). You may put your answer in scientific notation. Refer to Fig. 2 for the color code convention.

Color	Color	1st Band	2nd Band	3rd Band Multiplier	4th Band Tolerance
Black		0	0	x1Ω	
Brown		1	1	x10Ω	±1%
Red		2	2	x100Ω	±2%
Orange		3	3	x1kΩ	
Yellow		4	4	x10kΩ	
Green		5	5	x100kΩ	±0.5%
Blue		6	6	x1MΩ	±0.25%
Violet		7	7	x10MΩ	±0.10%
Grey		8	8	x100MΩ	±0.05%
White		9	9	x1GΩ	
Gold				x0.1Ω	±5%
Silver				x0.01Ω	±10%

Fig. 2: 4-band resistor color code convention table.

Solution:

$$R_1(\text{yellow-violet-red-gold}) = 47 \times 10^2 \pm 5\% = 4.7k\Omega$$

$$R_2(\text{red-violet-yellow-gold}) = 27 \times 10^4 \pm 5\% = 270k\Omega$$

(c) (6 points) It turns out that Oski's bottom mesh does not work as it is proposed in the lab. Looking at the photo of Oski's circuit in Fig. 3, we see that Oski accidentally used R_2 instead of R_1 on the right side of the circuit. As a result, not all the resistor values are the same, which was a requirement we saw in part (a). Regardless of the values in the previous parts, assume that $R_1 = 1k\Omega$ and $R_2 = 2k\Omega$ and the voltage source $V_s = 8V$ remains the same as part a). What are the voltages at nodes u_3 and u_6?

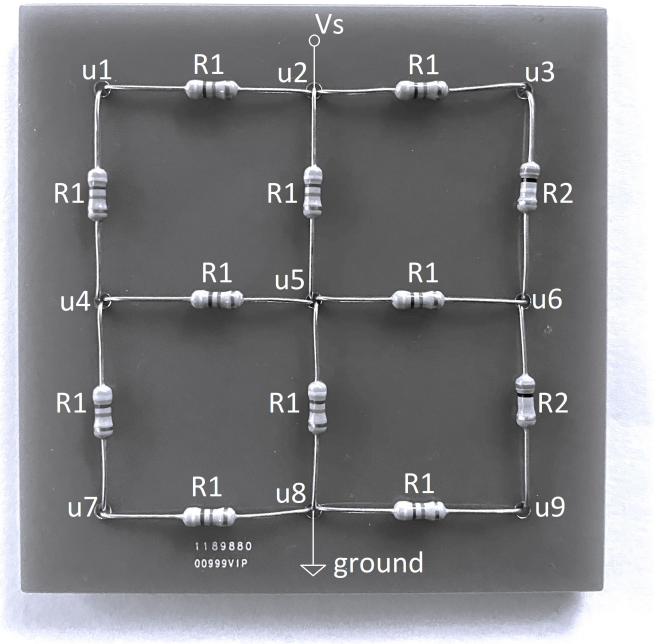


Fig. 3: A buggy bottom mesh of a 2D touchscreen.

Solution: They key thing to note is that the currents between u_4 and u_5 as well as between u_5 and u_6 is 0. Thus the desired node voltages can be found using voltage dividers

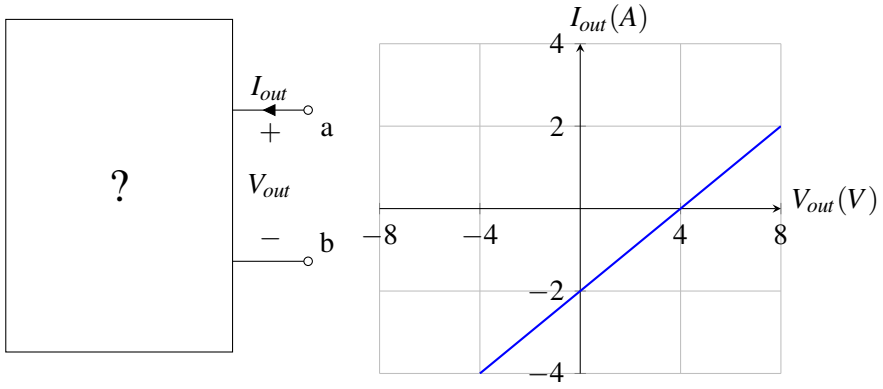
$$u_6 = \frac{R_1 + R_2}{2 \times (R_1 + R_2)} u_2 = \frac{1}{2} u_2 = 4V$$

$$u_3 = \frac{R_2 + R_2 + R_1}{2 \times (R_1 + R_2)} u_2 = \frac{20}{3} V$$

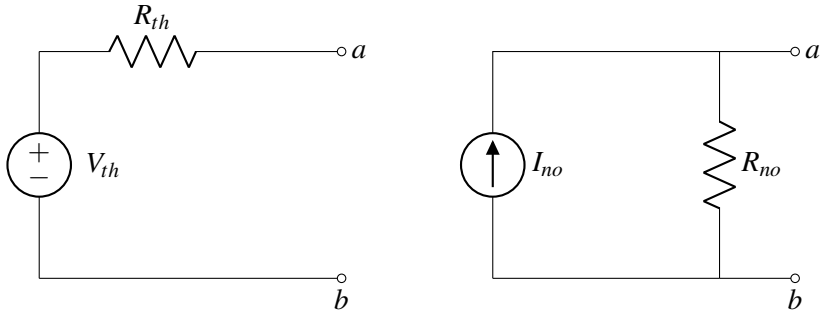
.

7. Making circuits equivalent (18 points)

(a) (8 points) Suppose you are given a mysterious circuit element shown below. You are also given the IV curve of the circuit at nodes a and b (also shown below).



Now you would like to model this mysterious circuit two different ways. One with each circuit below:

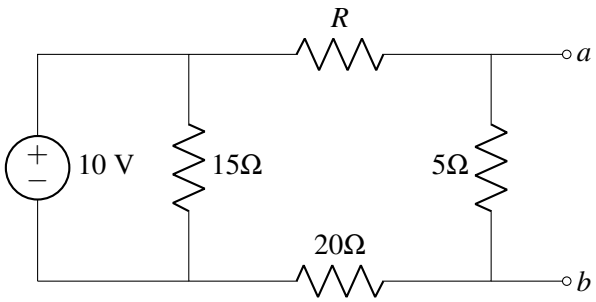


Find the values V_{th}, R_{th}, I_{no}, and R_{no} such that all 3 circuits (the mystery circuit and the two above) are equivalent. **Note the direction of I_{out} in the mysterious circuit!**

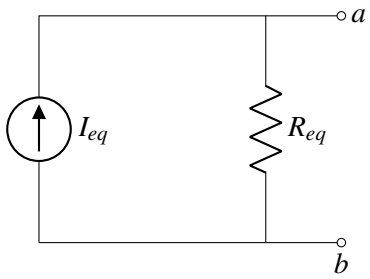
Solution: We recognize the below circuits as the Thevinin and Norton equivalents of the given mystery circuit. Recall that the Thevinin voltage V_{th} is the x-intercept of the IV curve, the Norton current I_{no} is the negative y-intercept of the IV curve, and $R_{th} = R_{no}$ is the inverse of the slope of the IV curve. Thus, just by reading the plot we can deduce that

$$V_{th} = 4V, \quad I_{no} = 2A, \quad R_{th} = R_{no} = \frac{4}{2} = 2\,\Omega.$$

(b) (10 points) Suppose now we are given a new circuit (shown below), but this time we know all the components except one resistor R:

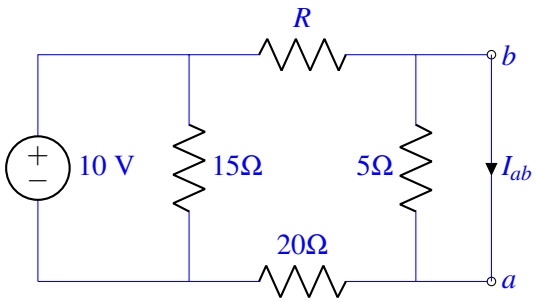


We also know that we can model the circuit at nodes a and b using the following equivalent circuit:

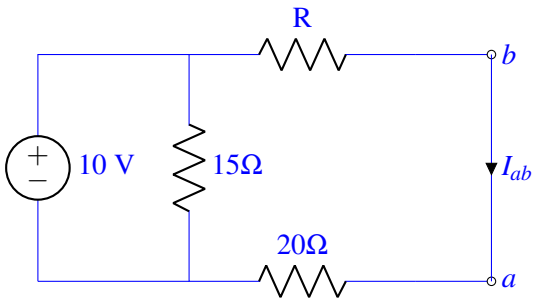


Find the resistor value R in the new circuit such that $I_{eq} = 0.25A$.

Solution: Connecting terminals a and b in the original circuit, we want to find the current I_{ab} going through it, which equals our I_{eq}.



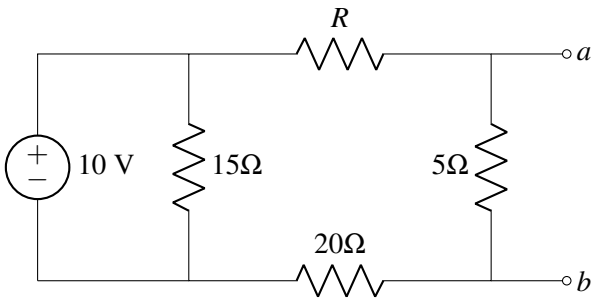
The wire ends up shorting the 5Ω resistor so our equivalent circuit looks like this:



Now we can simply combine R with the 20Ω resistor (they share the same current I_{ab}) and use the fact that resistors in parallel share the same voltage

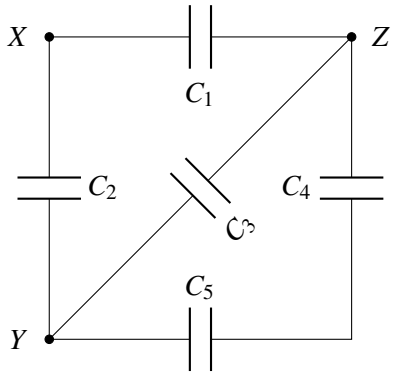
$$I_{ab} = \frac{10}{R+20} \quad R = \frac{10-20I_{ab}}{I_{ab}} = 20\Omega.$$

The circuit is redrawn here for your convenience:



8. Capacitors in the Wild (20 points)

(a) For the following circuit, find the equivalent capacitance between the specified nodes in terms of the given capacitors, C_i. You may use the *parallel operator* $\|$ for simplification (e.g. a valid answer format could be $C_{eq} = C_6 + (C_7 \parallel (C_8 + C_9))$).



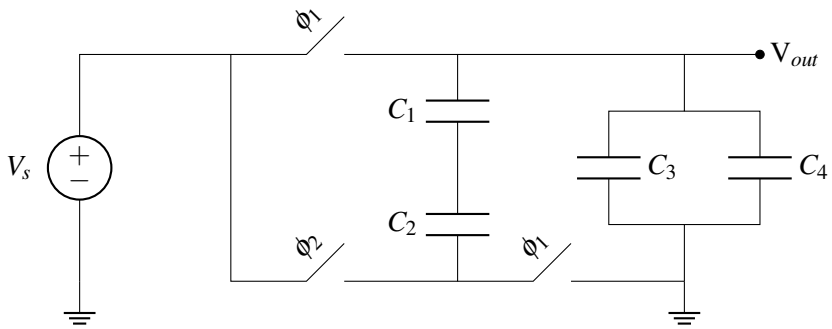
i. (4 points) Find the equivalent capacitance between nodes X and Y.
Solution:

$$C_{eq}^{XY} = C_2 + C_1 \parallel (C_3 + (C_4 \parallel C_5))$$

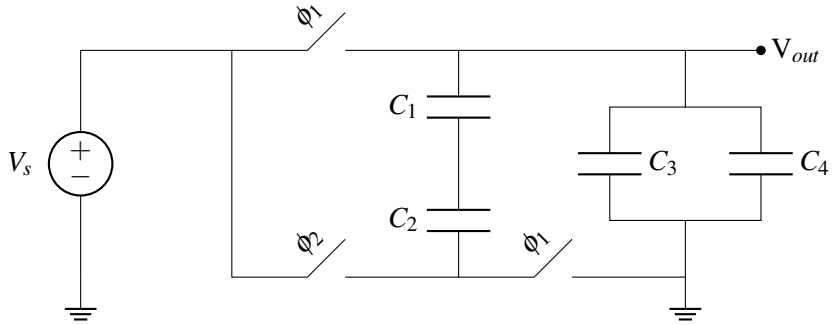
ii. (4 points) Find the equivalent capacitance between nodes Y and Z.
Solution:

$$C_{eq}^{YZ} = C_1 \parallel C_2 + C_3 + C_4 \parallel C_5$$

(b) (12 points) In the following circuit, switches labeled ϕ_1 are closed during phase 1 and open during phase 2; switches labeled ϕ_2 are closed during phase 2 and open during phase 1. Assume before phase 1 all capacitors are fully discharged. Each phase is long enough that the circuit reaches steady-state. **Find V_{out} in phase 2 in terms of V_s, C_1, C_2, C_3, and C_4.** You may use the *parallel operator* $\|$ in your final answer.

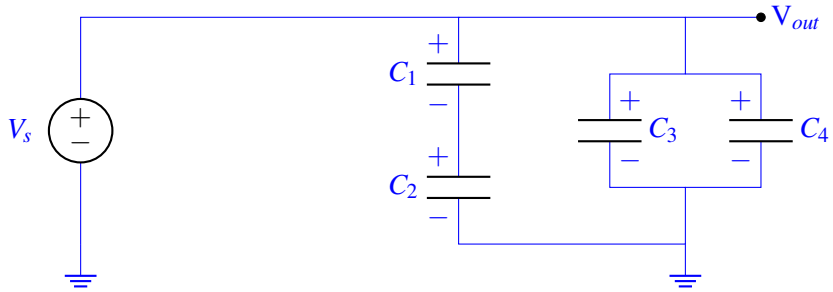


The circuit is redrawn here for your convenience.

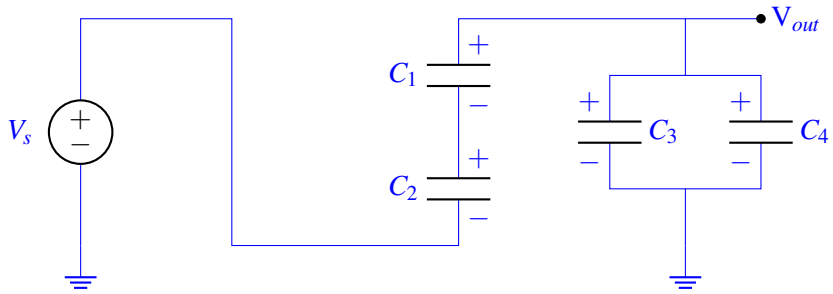


Solution:

First we draw the equivalent circuit diagrams in both phases. Phase 1:



Phase 2:

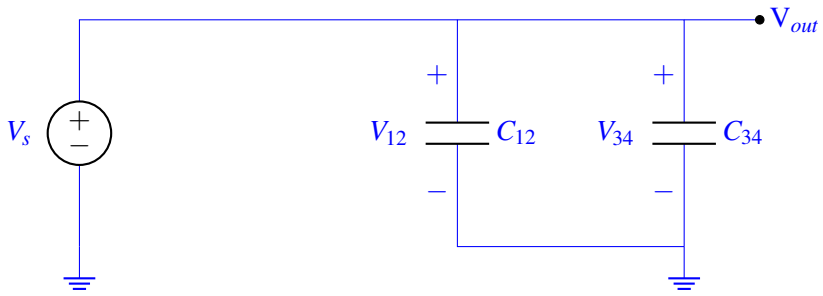


We note that C1 and C2 are in series for both phases, and C3 and C4 are in parallel for both phases as well. Therefore, as we are looking to compute the value of V_{out}, we can combine C1 and C2, and C3 and C4 and work with their equivalent capacitances to simplify calculations. It is possible to solve without this step, but it is more work. Let:

$$C_{12} = C_1 \parallel C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{34} = C_3 + C_4$$

We obtain the following equivalent circuit:

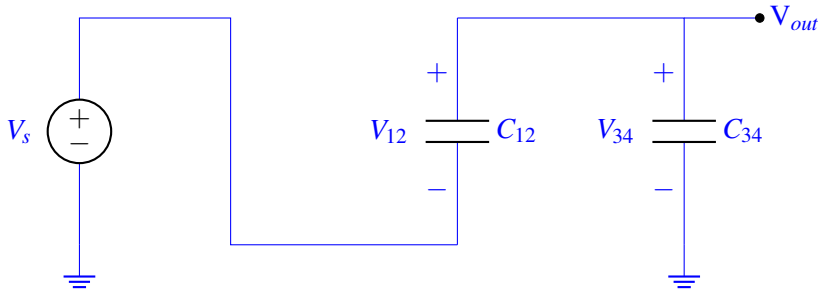


In this circuit, C_{12} and C_{34} are in parallel, and the voltage across both capacitors is V_s. Therefore, the magnitude of charges during phase 1 on both capacitors are

$$Q_{12}^{\phi_1} = C_{12}V_s \tag{1}$$

$$Q_{34}^{\phi_1} = C_{34}V_s \tag{2}$$

Now, during phase 2, the circuit looks like the following with our equivalent capacitors:



First, we have that in phase 2: $V_{out} = V_{34}^{\phi_2}$.
From KVL, we get:

$$V_s + V_{12}^{\phi_2} - V_{34}^{\phi_2} = 0 \tag{3}$$

Also, for each capacitor we have $Q = CV$:

$$Q_{12}^{\phi_2} = C_{12}V_{12}^{\phi_2} \tag{4}$$

$$Q_{34}^{\phi_2} = C_{34}V_{34}^{\phi_2} \tag{5}$$

Finally, we have one floating node, which is connected to both positive capacitor plates. Therefore, conserving the change between both phases we have:

$$+Q_{12}^{\phi_1} + Q_{34}^{\phi_1} = +Q_{12}^{\phi_2} + Q_{34}^{\phi_2} \tag{6}$$

Now, we solve!
First, plugging (1), (2), (4) and (5) into (6), we get:

$$V_s \cdot (C_{12} + C_{34}) = C_{12}V_{12}^{\phi_2} + C_{34}V_{34}^{\phi_2} \tag{7}$$

Dividing equation (7) by C_{12} and adding with equation (3), we get:

$$V_s + \cancel{V_{12}^{\phi_2}} - V_{34}^{\phi_2} + V_s\frac{C_{12}+C_{34}}{C_{12}} = \cancel{V_{12}^{\phi_2}} + \frac{C_{34}}{C_{12}}V_{34}^{\phi_2}$$

Where, we can solve for C_{34} (which is equal to V_{out}):

$$V_s\left(1+\frac{C_{12}+C_{34}}{C_{12}}\right)=V_{34}^{\phi_2}\left(1+\frac{C_{34}}{C_{12}}\right)$$

$$V_s\left(\frac{2C_{12}+C_{34}}{\cancel{C_{12}}}\right)=V_{34}^{\phi_2}\left(\frac{C_{12}+C_{34}}{\cancel{C_{12}}}\right)$$

Therefore,

$$V_{34}^{\phi_2}=V_s\left(\frac{2C_{12}+C_{34}}{C_{12}+C_{34}}\right)$$

Where,

$$C_{12}=C_1 \parallel C_2=\frac{C_1C_2}{C_1+C_2}$$

$$C_{34}=C_3+C_4$$

This answer is enough.

Answer without equivalent capacitances:

$$V_{34}^{\phi_2}=V_s\left(\frac{2(C_1 \parallel C_2)+C_3+C_4}{C_1 \parallel C_2+C_3+C_4}\right)$$

$$V_{34}^{\phi_2}=V_s\left(\frac{2\frac{C_1C_2}{C_1+C_2}+C_3+C_4}{\frac{C_1C_2}{C_1+C_2}+C_3+C_4}\right)$$

$$V_{34}^{\phi_2}=V_s\left(\frac{2C_1C_2+C_1C_3+C_1C_4+C_2C_3+C_2C_4}{C_1C_2+C_1C_3+C_1C_4+C_2C_3+C_2C_4}\right)$$