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Thurs., Apr. 9, 2015
2:00-3:30pm

EECS 16A: SPRING 2015—MIDTERM 2

Important notes: Please read every question carefully and completely – the setup may or may not be the same as what you have seen before. Also, be sure to show your work since that is the only way we can potentially give you partial credit.

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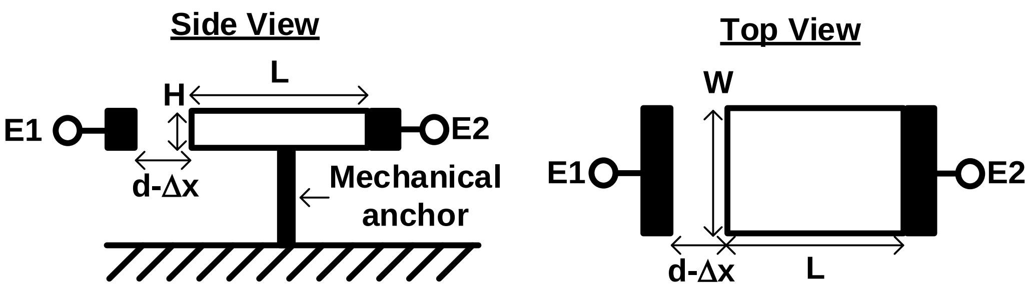
Total: _____/ 91

PROBLEM 1. MEMS Sensors (13 pts)

We saw in lecture that our smart phones use capacitance measurement based techniques to enable the touchscreen, but it turns out that many of the other capabilities that make a smart-phone "smart" rely on capacitance measurement techniques as well. In particular, to first order, all of the motion sensors in the phone (accelerometer, gyroscope, etc.) use capacitance measurement to electrically sense the position of some (small) movable object (this is mechanical parts of "MEMS", which stands for Micro-Electro-Mechanical System), where the position (and/or overall trajectory) of that object is directly influenced by the type of motion we are trying to detect.

In this problem we'll examine the highly simplified sensor shown below. When the whole device is accelerated to the left or to the right at a certain rate, the "proof" mass will shift to the left or the right by a distance Δx proportional to the acceleration.



a) **(3 pts)** Assuming that no acceleration is applied (i.e., $\Delta x = 0$), that H=10μm, L=100μm, W = 20μm, d = 0.5μm, and that the proof mass is made out of nickel (i.e., a conductive metal), what is the capacitance between E1 and E2?

$$C = \varepsilon_0 \cdot \frac{A}{d} \implies C_{E1-E2} = \varepsilon_0 \cdot \frac{W \cdot H}{d - \Delta x}$$

$$C_{E1-E2} = 8.85e-12 \cdot \frac{20e\text{-}6\,m \cdot 10e\text{-}6\,m}{0.5e\text{-}6\,m}$$

$$\boxed{C_{E1-E2} = 3.54\ fF}$$

b) **(4 pts)** Keeping the other parameters the same as in part (a), then if Δx = 10e-9 s^2 * a (where a is the acceleration with units of m/s^2), write an equation relating the capacitance between E1 and E2 with the acceleration a.

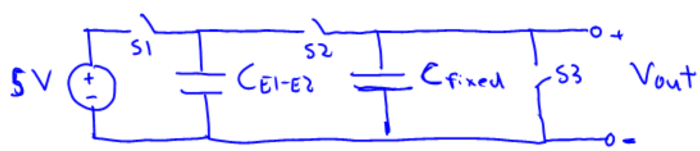
$$C_{E1\text{-}E2}(\Delta x) = \varepsilon_0 \cdot \frac{W \cdot H}{d - \Delta x} = \varepsilon_0 \cdot \frac{W \cdot H}{d} \cdot \frac{d}{d - \Delta x} = C_{E1\text{-}E2}(0) \cdot \frac{1}{1 - \Delta x / d}$$

So: $$C_{E1\text{-}E2}(a) = 3.54\text{ fF} \cdot \frac{1}{1 - 10e\text{-}9\ s^2 \cdot a / 0.5e\text{-}6\,m}$$

$$= \boxed{3.54\text{ fF} \cdot \frac{1}{1 - a/(50\ m/s^2)}}$$

c) **(6 pts)** Using any combination of ideal circuit elements you'd like to, design a circuit that produces a voltage that increases along with the capacitance between E1 and E2. (I.e., if $C_{E1\text{-}E2}$ increases your output voltage V_{out} also increases, but you don't specifically need a purely linear relationship between $C_{E1\text{-}E2}$ and V_{out}.)

Use the same circuit we built in the touch screen lab!



For this circuit:

$$V_{out} = \frac{5V \cdot C_{E1\text{-}E2}}{C_{E1\text{-}E2} + C_{fixed}} \rightarrow \text{increases with } C_{E1\text{-}E2} \checkmark$$

PROBLEM 2. Communication with Noise and Interference (28 points)

You and your friend Alice have set up your own wireless communication system where Alice transmits to you messages that consist of a sequence of length 4 whose values are ±1. Calling this signal *x[n]*, you should assume that *x[n]* is 0 for n≤0 and for n>4. Due to noise and interference from other wireless signals, your antenna and the circuit you use to amplify/record the signal produced by the antenna can't exactly reproduce the values of *x[n]*, but you do have a circuit that computes the cross-correlation of *x[n]* with sequence *y[n]*, where *y[1]* = 1, *y[2]* = -1, and *y[3]* = 1.

a) **(6 pts)** Alice sends you a message and the cross-correlation signal that you get *c[n]* is *c[-2]* = -1, *c[-1]* = 2, *c[0]* = -1, *c[1]* = 1, *c[2]* = 0, *c[3]* = 1, with *c[n]* = 0 for all other values of *n*. Set up a system of equations relating the measured *c[n]* with the (still unknown) message *x[n]*.

Cross-correlation looks like this:

	0	0	x[1]	x[2]	x[3]	x[4]
c[-2]	1	-1	1			
c[-1]		1	-1	1		
c[0]			1	-1	1	

...

i.e., $c[n] = \sum_{i=-\infty}^{\infty} x[i] \cdot c[i+n]$

So:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \underset{6\times 4}{A_{xcorr}} \cdot \underset{4\times 1}{x} = \underset{6\times 1}{c}$$

b) **(4 pts)** Show how to use the system of equations from part a) to estimate Alice's transmitted signal *x[n]*.

Have an over-determined system, so use least squares:

$$\hat{x} = \left(A_{xcorr}^T A_{xcorr}\right)^{-1} \cdot A^T c$$

In general $\hat{x}$ could contain continuous values, but we know the message should be only ±1. So, our actual estimate should be:

$$\boxed{x = \text{sign}(\hat{x})}$$

Alternatively, it turns out that without any noise you can simply solve the equations and directly compute x[n]. Note that the least squares approach will also give you this precise result.

c) **(6 pts)** You end up communicating with Alice quite often, and you notice that occasionally the output from your cross-correlation circuit is noisier than the first few times you tested it. For example, let's say you recorded a cross-correlation signal *c[n]* where *c[-2]* = -0.7, *c[-1]* = 1.8, *c[0]* = -2.8, *c[1]* = 3.3, *c[2]* = -2.1, *c[3]* = 1.3, with *c[n]* = 0 for all other *n*. In this case, would you change the strategy you use to estimate Alice's transmitted signal? If so, how would you change it? If not, why not?

* If you used least squares in part (b), then no change in method is needed.
* We still have six equations with four unknowns, and hence we still have an over-determined problem. Least squares would thus still give us the values of x[n] that result in the closest (in the squared error sense) match to the measured correlation values.

d) **(6 pts)** On one particular occasion you are trying to communicate with Alice, but you happen to walk by a cell phone tower and notice that some of the values coming out of your cross-correlation circuit have been horribly corrupted (probably by the large interference created by the signals transmitted by the cell phone tower). In fact, you get *c[-2]* = 100, *c[-1]* = 100, *c[0]* = -2.5, *c[1]* = 3.3, *c[2]* = -2.1, *c[3]* = 100, and *c[n]* = 0 for all other values of *n*. Knowing that Alice is relatively far away and that her transmitter can't put out too large of a signal (after all, she doesn't want her battery to die), you decide to throw out some of the unreasonable values. Set up a new system of equations relating the measured *c[n]* with *x[n]*, but with the unreasonable values thrown out. Please be sure to explicitly highlight which of the *c[n]* measurements you intend to throw out.

Values that are 100 seem pretty unreasonable given the earlier data, so throw out c[-2], c[-1], and c[3].

So:

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} = \begin{bmatrix} -2.5 \\ 3.3 \\ -2.1 \end{bmatrix}$$

or

$$\underset{3\times 4}{A_{x\text{-corr}}} \cdot \underset{4\times 1}{x} = \underset{3\times 1}{c}$$

e) **(6 pts)** Having thrown out the unreasonable values of *c[n]* from part (d), and knowing that Alice would want to send a signal with as little energy in it as possible (again, she doesn't want her battery to die), now how you would you estimate Alice's transmitted signal *x[n]*?

* Now we have only three equations but four unknowns – hence the system is under-determined.

* Out of the infinite possible solutions, we were told we want the one with minimum energy (norm), so we should find the least-norm solution:

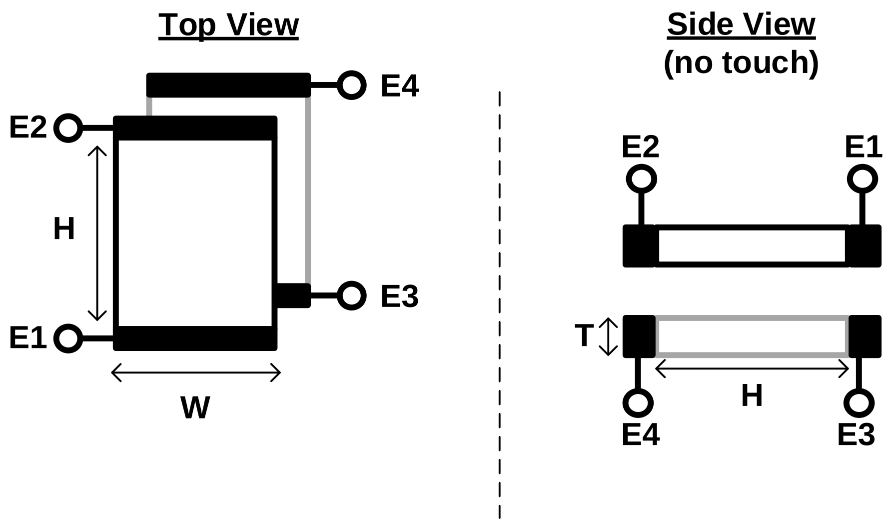
$$\hat{x} = A_{x\text{-}corr}^T \left(A_{x\text{-}corr} \cdot A_{x\text{-}corr}^T \right)^{-1} \cdot c$$

We can still get continuous values out of this procedure, so we should once again take the sign to get the final message – i.e.,

$$x = \text{sign}(\hat{x})$$

PROBLEM 3. Multitouch Resistive Touchscreen (22 points + BONUS 16 pts)

In this problem we will look at a simplified version of the multitouch resistive touchscreen. In particular, rather than measuring the position of two potential touch points in both dimensions (i.e., a pair of coordinates (x_1, y_1) and (x_2, y_2) corresponding to two touch positions), let's think about a version where we are interested in measuring only the vertical position of the two touch points (i.e., y_1 and y_2). Therefore, unlike the touchscreens we looked at in class and as shown below, both of the resistive plates (i.e., both the top and the bottom plate) would have conductive strips placed along their top and bottom edges.



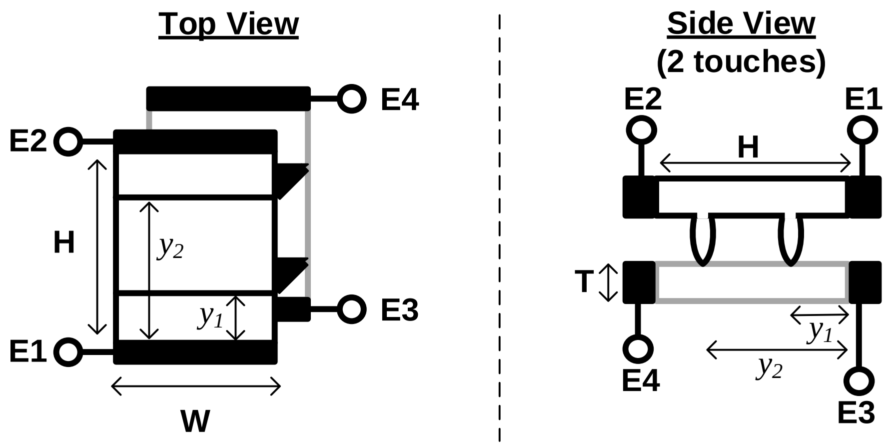
a) **(4 pts)** Assuming that both of the plates are made out of a material with ρ = 1 Ω*m and that the dimensions of the plates are W = 3cm, H = 12cm, and T = 0.5mm, with no touches at all, what is the resistance between terminals E1 and E2 (which would be the same as the resistance between terminals E3 and E4)?

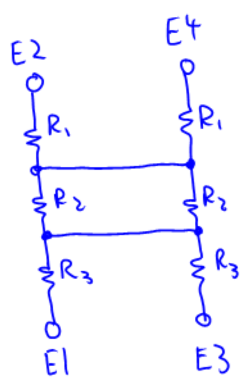
$$R = \rho \cdot \frac{L}{A} \Rightarrow R_{E1-E2} = \rho \cdot \frac{H}{W \cdot T}$$

$$= 1\Omega \cdot m \cdot \frac{12e\text{-}2\, m}{3e\text{-}2\, m \cdot 0.5e\text{-}3\, m}$$

$$\boxed{R_{E1-E2} = 8k\Omega}$$

b) **(6 pts)** Now let's look at what happens when we have two touch points. Let's assume that at wherever height the touch occurs, a perfect contact is made between the top plate and the bottom plate along the entire width of the plates (i.e., you don't have to worry about any lateral resistors), but that otherwise none of the electrical characteristics of the plates change. Defining the bottom of the plate as being y = 0cm (i.e., a touch right at E1 would be at y = 0cm), let's assume that the two touches happen at y_1 = 3cm and y_2 = 7cm, and that your answer to part (a) was 5kΩ (which may or may not be the right answer), draw a model with 6 resistors that captures the electrical connections between E1, E2, E3, and E4. Note that for clarity, the system has been redrawn below to depict this scenario.



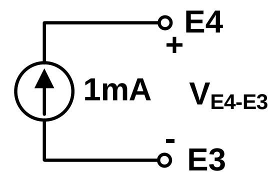


$R_3 = \frac{3cm}{12cm} \cdot R_{E2-E1} = 1.25k\Omega$

$R_2 = \frac{7cm-3cm}{12cm} \cdot R_{E2-E1} = 1.667k\Omega$

$R_1 = \frac{12cm-7cm}{12cm} \cdot R_{E2-E1} = 2.0833k\Omega$

c) **(6 pts)** Using the same assumptions as part b), if you drove terminals E3 and E4 with a 1mA current source (as shown below) but left terminals E1 and E2 open-circuited, what is the voltage you would measure across E4-E3 (i.e., V_{E4-E3})?



Equivalent resistance between E4-E3 is:

$$R_{E4-E3} = R_1 + R_2 || R_2 + R_3 = R_1 + \frac{R_2}{2} + R_3$$

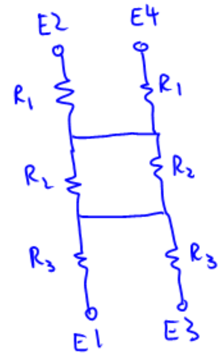
$$= 1.2\bar{5}\,k\Omega + \frac{1.667\,k\Omega}{2} + 2.0833\,k\Omega$$

$$\approx 4.167\ k\Omega$$

$$V_{E4-E3} = \underset{1mA}{I} \cdot R_{E4-E3} \implies \boxed{V_{E4-E3} = 4.167\ V}$$

d) **(6 pts)** Now let's try to generalize the situation by assuming that the two touches can happen at any two arbitrary points y_1 and y_2, but with y_1 defined to always be less than y_2 (i.e., y_1 is always the bottom touch point). Leaving the setup the same as in part c) except for the arbitrary y_1 and y_2, by measuring only the voltage between E4 and E3, what information can you extract about the two touch positions? Please be sure to provide an equation relating $V_{E4\text{-}E3}$ to y_1 and y_2 as a part of your answer, and note that you may want to redraw the model from part (b) to help you with this.

Model:



For general y_1 & y_2:

$$R_3 = \frac{y_1}{12cm} \cdot 5k\Omega$$

$$R_2 = \frac{y_2 - y_1}{12cm} \cdot 5k\Omega$$

$$R_1 = \frac{12cm - y_2}{12cm} \cdot 5k\Omega$$

$$R_{E4-E3} = R_1 + \frac{R_2}{2} + R_3 = \left(12cm - y_2 + \frac{y_2 - y_1}{2} + y_1\right) \cdot \frac{5k\Omega}{12cm}$$

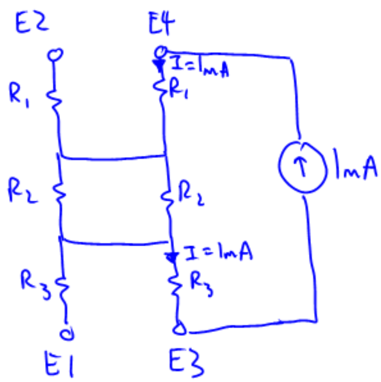
$$= \left(12cm + \frac{y_1}{2} - \frac{y_2}{2}\right) \cdot \frac{5k\Omega}{12cm}$$

So: $\boxed{V_{E4-E3} = \frac{12cm - (y_2 - y_1)/2}{12cm} \cdot 5V}$

This means that by measuring V_{E4-E3}, we can only measure the <u>distance</u> between the two touch points $(y_2 - y_1)$

e) **(BONUS: 8 pts)** One of your colleagues claims that by measuring the appropriate voltages, not only can they extract what both y_1 and y_2 are in this system, they can even do so in a way that would have a set of three independent voltage equations related to y_1 and y_2 and hence gain some robustness to noise in the voltage measurements.

In order to facilitate this, write equations relating $V_{E4\text{-}E2}$ and $V_{E1\text{-}E3}$ to y1 and y2. (The third voltage we'll use is $V_{E4\text{-}E3}$, which you should have already derived an equation for in the previous part of the problem.)



$$\boxed{V_{E4-E2} = I \cdot R_1 = \frac{12\text{cm} - y_2}{12\text{cm}} \cdot 5\text{V}}$$

$$\boxed{V_{E1-E3} = I \cdot R_3 = \frac{y_1}{12\text{cm}} \cdot 5\text{V}}$$

f) **(BONUS: 8 pts)** Using the equations you derived in part e), show the procedure you would use to estimate y_1 and y_2. Note that your final result should be in the form of a matrix-vector equation that predicts y_1 and y_2 as a function of the measured voltages. (Note that you can plug in numbers for all of the dimensions/material constants such as H, L, etc.)

We have three equations and two unknowns, so we'll use least squares. First we need to massage the equations in to the right format however.

$$V_{E4-E3} = \frac{12cm - (y_2 - y_1)/2}{12cm} \cdot 5V \longrightarrow \frac{1}{24cm} \cdot y_1 - \frac{1}{24cm} \cdot y_2 = \frac{V_{E4-E3}}{5V} - 1$$

$$V_{E4-E2} = \frac{12cm - y_2}{12cm} \cdot 5V \longrightarrow -\frac{1}{12cm} \cdot y_2 = \frac{V_{E4-E2}}{5V} - 1$$

$$V_{E1-E3} = \frac{y_1}{12cm} \cdot 5V \longrightarrow \frac{1}{12cm} \cdot y_1 = \frac{V_{E1-E3}}{5V}$$

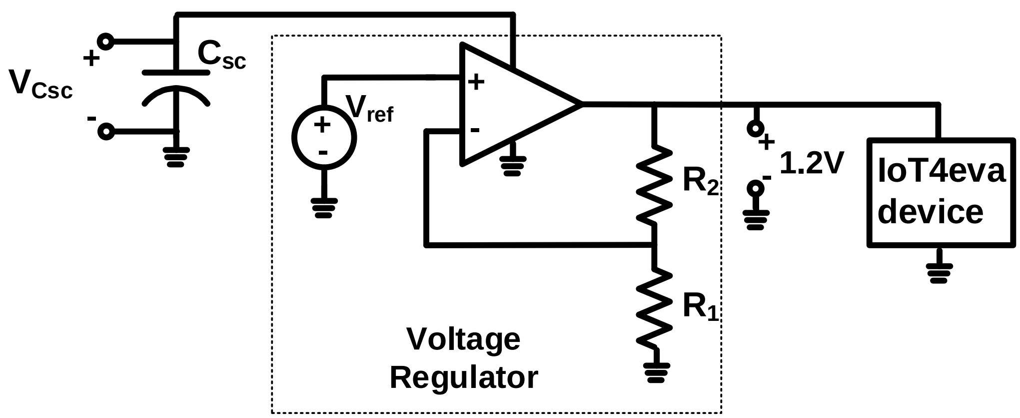
Rewriting in matrix format:

$$\underbrace{\begin{bmatrix} \frac{1}{24cm} & -\frac{1}{24cm} \\ 0 & -\frac{1}{12cm} \\ \frac{1}{12cm} & 0 \end{bmatrix}}_{A\ (3\times2)} \underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\vec{y}\ (2\times1)} = \underbrace{\begin{bmatrix} \frac{V_{E4-E3}}{5V} - 1 \\ \frac{V_{E4-E2}}{5V} - 1 \\ \frac{V_{E1-E3}}{5V} \end{bmatrix}}_{\vec{v}\ (3\times1)}$$

So: $\boxed{\vec{y}_{ls} = (A^T A)^{-1} A^T \vec{v}}$

PROBLEM 4. IoT4eva, Again (28 pts)

In this problem we'll revisit yet again (we called the company "4eva" for a reason) some aspects related to powering the device being made by the start-up IoT4eva. Just like in HW6 and as shown below, the device will be powered by a super-capacitor (which is basically just a big capacitor) charged to some initial voltage $V_{Csc}(0) = V_{init}$, and we will use the voltage regulator shown below to provide a fixed 1.2V output voltage to the rest of the circuits in the device.



This time around we're going to be looking at how to build some circuits that will help us to keep track of how long the device will continue to operate off of the charge stored in the super-capacitor, or if it is about to die.

a) **(6 pts)** Assuming that the super-capacitor has a capacitance C_{sc} = 10kF and an initial voltage V_{init} = 1.5V, how much charge can we pull out of the super-capacitor before its voltage gets too low to ensure that 1.2V comes out of the voltage regulator? In other words, we want to calculate the "**usable charge**" stored in the super-capacitor.

* The voltage regulator can't produce 1.2V at its output if $V_{SC} < 1.2V$, so the "usable charge" is just $(V_{init} - 1.2V) \cdot C_{SC}$

$$Q_{usable} = (1.5V - 1.2V) \cdot 10\,kF$$

$$\boxed{Q_{usable} = 3k\,C}$$

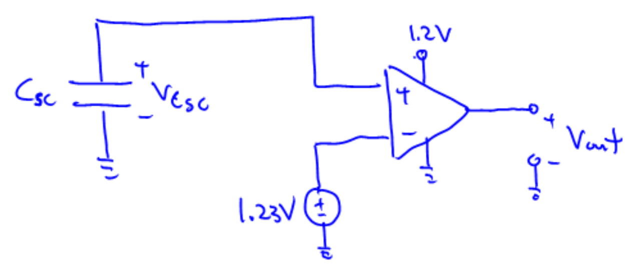
(Coulombs is the unit of charge)

b) **(10 pts)** We'd like to know when the device is getting close to dying so that we can give a warning to the user to charge it. Using the same V_{init} as part a), design a circuit that outputs 0V once the super-capacitor has 10% or less of the charge you computed in part a) (i.e., the **usable charge**), and that outputs 1.2V otherwise. You are allowed to use ideal op-amps and resistors to build this circuit, the 1.2V coming from the regulator, the voltage on the super-capacitor itself (V_{Csc}), and one ideal voltage source whose voltage you can select. Please remember that as shown in the figure, the 1.2V voltage produced by the regulator is relative to the ground connected to the super-capacitor.

Hint: What is the voltage on the super-capacitor V_{Csc} when only 10% of its usable charge is left?

Note that in this part as well as in part c), partial credit will be provided for correct descriptions and/or block diagrams indicating the strategy you intend to use to meet the goals of the design.

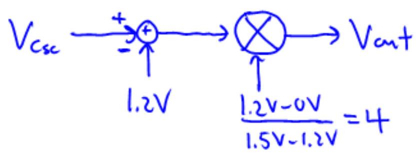
* Since $Q = CV$, $V = Q/C$ → i.e., the voltage on the super-capacitor directly indicates how much charge is left. We have to remember however that only charge associated with voltage <u>above</u> 1.2V is "usable".
* 10% of the value from part (a) is 300C, and the change in voltage associated with this is $300C / 10kF = 30mV$ (i.e., 10% of the 300mV of "usable voltage").
* So, really all we need to is check for when $V_{Csc} < 1.2V + 30mV$:



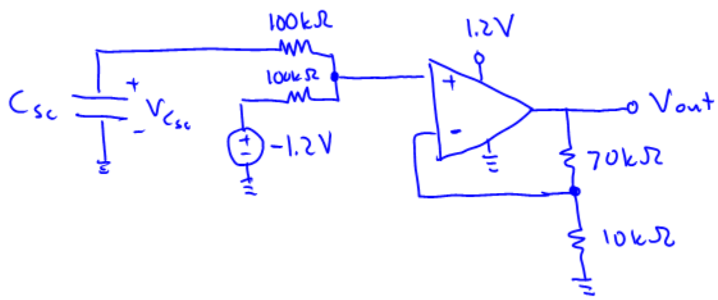
c) **(12 pts)** Instead of only a binary indication of the super-capacitor being almost out of usable charge, we might instead want a voltage that is linearly proportional to the remaining usable charge. Under the same constraints as part b), design a circuit that outputs 1.2V when 100% of the usable charge is left, 0V when 0% of the usable charge is left, and that drops linearly from 1.2V for any percentage in between (e.g. 0.6V for 50%, 0.3V for 25%, etc.).

* The key is again to realize that we can look at V_{Csc} and simply subtract off 1.2V to tell us how much usable charge is left. We then just have to scale that remaining voltage appropriately to meet the 0V–1.2V requirement given in the problem.

* In block diagram form, we want to implement:



* Finally, one circuit that realizes this:



Note that actual block diagram implemented by this circuit is shown below, but the overall functionality is identical.