EECS 16B Designing Information Devices and Systems II
Fall 2021 Discussion Worksheet

Discussion 3A

The following notes are useful for this discussion: Note 3 (sections 1 and 2)

1. Changing Coordinates and Systems of Differential Equations, I

Suppose we have the pair of differential equations (valid for $t \geq 0$)

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -9x_1(t) \tag{1}$$

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = -2x_2(t) \tag{2}$$

with initial conditions $x_1(0) = -1$ and $x_2(0) = 3$.

(a) **Solve for $x_1(t)$ and $x_2(t)$ for $t \geq 0$.**

Now, suppose we are actually interested in a different set of variables with the following differential equations:

$$\frac{\mathrm{d}z_1(t)}{\mathrm{d}t} = -5z_1(t) + 2z_2(t) \tag{3}$$

$$\frac{\mathrm{d}z_2(t)}{\mathrm{d}t} = 6z_1(t) - 6z_2(t). \tag{4}$$

(b) **Write out the above system of differential equations in matrix form**. Can we solve this system in a similar way as we did above?

(c) Consider that in our frustration with the previous system of differential equations (which we cannot directly solve), we start hearing voices[1]. These voices whisper to us that that we should try writing out $\vec{z}(t)$ in terms of new variables, $y_1(t)$ and $y_2(t)$, as follows:

$$z_1(t) = -y_1(t) + 2y_2(t) \tag{5}$$

$$z_2(t) = 2y_1(t) + 3y_2(t). \tag{6}$$

Write out this transformation in matrix form ($\vec{z} = V\vec{y}$). What is V?

For each of the parts (d) - (f), solve the questions two ways: 1. using direct substitution, and 2. using matricies and vectors .

(d) Suppose that the following initial conditions are given: $\vec{z}(0) = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$. **How do these initial conditions for $z_i(t)$ translate into the initial conditions for $y_i(t)$?**

1. Solve this with direct substitution (For direct substitution, start with the governing equations in eqs. (5) to (6). Then, plug in known information and rearrange terms as needed to find the desired quantities):

2. Solve this with matrices and vectors (Recall that $\vec{y} = V^{-1}\vec{z}$):

[1]Friendly voices, so let's assume they're correct :)

(e) In eqs. (5) to (6), we are given $z_i(t)$ in terms of $y_i(t)$. Now, **write the corresponding equations for** $y_i(t)$ **in terms of** $z_i(t)$. Can we solve this system of differential equations?

1. Solve this with direct substitution:

2. Solve this with matrices and vectors:

(f) **What are the solutions for** $z_i(t)$**?**

1. Solve this with direct substitution:

2. Solve this with matrices and vectors:

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