EECS 16B Designing Information Devices and Systems II
Fall 2021 Discussion Worksheet

Discussion 7A

The following notes are useful for this discussion: Note 10 and Note 11.

1. Changing behavior through feedback

In this question, we discuss how feedback control can be used to change the effective behavior of a system.

(a) Consider the scalar system:

$$x[i+1] = 0.9x[i] + u[i] + w[i] \tag{1}$$

where $u[i]$ is the control input we get to apply based on the current state and $w[i]$ is the external disturbance, each at time i.

Is the system stable? If $|w[i]| \leq \epsilon$, what can you say about $|x[i]|$ at all times i if you further assume that $u[i] = 0$ and the initial condition $x[0] = 0$? How big can $|x[i]|$ get?

(b) Suppose that we decide to choose a control law $u[i] = fx[i]$ to apply in feedback. **For what values of λ can you get the system to behave like:**

$$x[i+1] = \lambda x[i] + w[i]? \tag{2}$$

How would you pick f?

(*Note*: In this case, $w[i]$ can be thought of like another input to the system, except we can't control it.)

(c) **For the previous part, which f would you choose to minimize how big $|x[i]|$ can get?**

(d) **What if instead of a 0.9, we had a 3 in the original eq. (1). What, if anything, would change?**

(e) Now suppose that we have a vector-valued system with a vector-valued control:

$$\vec{x}[i+1] = A\vec{x}[i] + B\vec{u}[i] + \vec{w}[i] \tag{3}$$

where we further assume that B is an invertible square matrix. Futher, suppose we decide to apply linear feedback control using a square matrix F so we choose $\vec{u}[i] = F\vec{x}[i]$.
For what values of matrix G can you get the system to behave like:

$$\vec{x}[i+1] = G\vec{x}[i] + \vec{w}[i]? \tag{4}$$

How would you pick F given knowledge of A, B and the desired goal dynamics G?

2. Controlling states by designing sequences of inputs

This is something that you saw in 16A in the Segway problem. In that problem, you were given a semi-realistic model for a segway. Here, we are just going to consider the following matrix chosen for ease of understanding what is going on:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Let's assume we have a *discrete-time* system defined as follows:

$$\vec{x}[i+1] = A\vec{x}[i] + \vec{b}u[i]. \tag{6}$$

(a) We are given the initial condition $\vec{x}[0] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Let's say we want to achieve $\vec{x}[\ell] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ for some specific $\ell \geq 0$. We don't need to stay there, we just want to be in this state at that time. **What is the smallest ℓ such that this is possible? What is our choice of sequence of inputs $u[i]$?**

(b) **What if we started from $\vec{x}[0] = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$? What is the smallest ℓ and what is our choice of $u[i]$?**

(c) **If we start from** $\vec{x}[0] = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$**, what is smallest** ℓ **such that** $\vec{x}[\ell] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$**, what is corresponding** $u[i]$**?**

(d) **If you would like to make sure that at time** ℓ **we are at** $\vec{x}[\ell] = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ **for the state, what controls could you use to get there? How big does** ℓ **have to be for this strategy to work?**

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