EECS 16B Designing Information Devices and Systems II
Fall 2021 Discussion Worksheet Discussion 7B

The following notes are useful for this discussion: Note 10, Note 11

1. Eigenvalue Placement in Discrete Time

Consider the following linear discrete time system

$$\vec{x}[i+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[i] + \vec{w}[i] \tag{1}$$

(a) **Is the system given in eq. (1) stable?**

(b) **Derive a state space representation of the resulting closed loop system**. Use state feedback of the form:

$$u[i] = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \vec{x}[i] \tag{2}$$

Hint: If you're having trouble parsing the expression for $u[i]$, note that $\begin{bmatrix} f_1 & f_2 \end{bmatrix}$ is a row vector, while $\vec{x}[i]$ is column vector. What happens when we multiply a row vector with a column vector like this?)

(c) **Find the appropriate state feedback constants, f_1, f_2, that place the eigenvalues of the state space representation matrix at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$.**

(d) **Is the system now stable in closed-loop, using the control feedback coefficients f_1, f_2 that we derived above?**

(e) Suppose that instead of $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u[i]$ in eq. (1), we had $\begin{bmatrix} 1 \\ 1 \end{bmatrix} u[i]$ as the way that the discrete-time control acted on the system. In other words, the system is as given in eq. (3). As before, we use $u[i] = \begin{bmatrix} f_1 & f_2 \end{bmatrix} \vec{x}[i]$ to try and control the system.

$$\vec{x}[i+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[i] + \vec{w}[i] \tag{3}$$

What would the desired eigenvalues now be? Can you move all the eigenvalues to where you want? In particular, can you make this system stable given the form of the input?

(f) **[Practice]** Can you place the eigenvalues at complex conjugates, such that $\lambda_1 = a + \mathrm{j}b, \lambda_2 = a - \mathrm{j}b$ using only real feedback gains f_1, f_2? How about placing them at any arbitrary complex numbers, such that $\lambda_1 = a + \mathrm{j}b, \lambda_2 = c + \mathrm{j}d$?

2. Uncontrollability

Consider the following discrete-time system with the given initial state:

$$\vec{x}[i+1] = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u[i] \tag{4}$$

$$\vec{x}[0] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

(a) **Is the system controllable?**

(b) **Is it possible to reach $\vec{x}[\ell] = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$ for some ℓ? For what input sequence $u[i]$ up to $i = \ell - 1$?**

(c) **Is it possible to reach** $\vec{x}[\ell] = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$ **for some** ℓ**? For what input sequence** $u[i]$ **up to** $i = \ell - 1$**?**

Hint: look at the intermediate results of the previous subpart, where you wrote down what $x[0], x[1]$*, etc. were. Apply these new values to those expressions.*

(d) **Find the set of all** $\vec{x}[2]$**, given that you are free to choose the** $u[0]$ **and** $u[1]$ **of your choice.**

Contributors:

- Ioannis Konstantakopoulos.
- John Maidens.
- Anant Sahai.
- Regina Eckert.
- Druv Pai.
- Neelesh Ramachandran.
- Titan Yuan.
- Kareem Ahmad.