EECS 16B Designing Information Devices and Systems II
Fall 2021 Discussion Worksheet **Discussion 14B**

The following notes are useful for this discussion: Note 2j.

1. Gram Schmidt on Complex Vectors

(a) Consider the three complex vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ \mathrm{j} \\ 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{1}$$

Compute an orthonormal basis from this list of vectors with Gram Schmidt.

(b) **Derive the least-squares solution for the case of a complex tall matrix of data and a tall matrix of values.** We want to find the best (complex) linear combination of the columns for predicting the observed values in a least-squares sense — we want to minimize the norm of the residual.

This can be formulated as having a feature matrix of data $D \in \mathbb{C}^{m \times n}$ where $m > n$ and measurements $\vec{y} \in \mathbb{C}^m$. In this case, feel free to assume that the columns of D are linearly independent even when we allow complex linear combinations. **First assume that the columns of D are orthonormal.** *Hint: You may find it useful to define a matrix $U = \begin{bmatrix} D & D_+ \end{bmatrix}$, obtained via Gram-Schmidt.* Recall that the procedure here resembles that used in the SVD derivation (one portion of U contains the data/info we "care about", and the remainder is there via extension, to span the space.)

(c) **Repeat the previous part without the assumption of orthonormality for the columns of D.** You can keep the assumption of linear independence.

2. Q&A time! [$\approx$ 20 minutes]

This time is here for you all to ask any questions from discussion 14A and 14B to the TAs to review the material on complex vectors. If there are no further questions, then feel free to discuss anything else related to the course content, as this is the last non-review discussion.

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