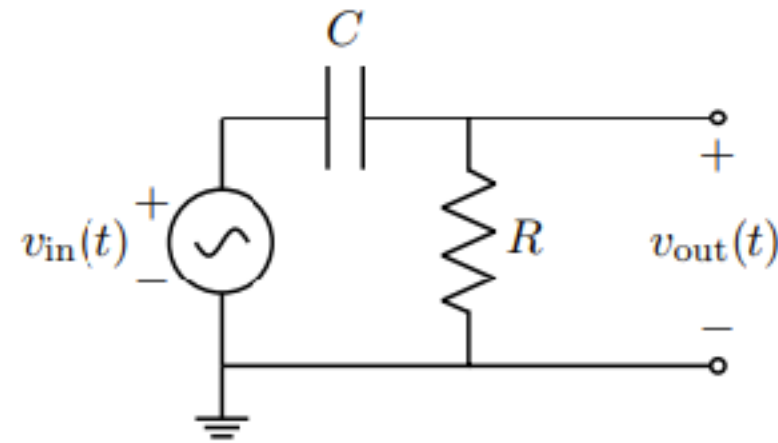


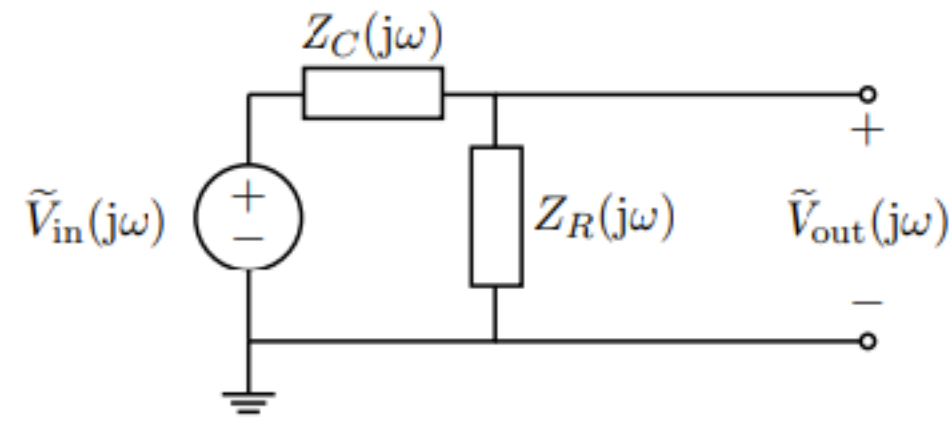
1. Transfer function practice

In this problem, you'll be deriving some transfer functions on your own. **For each circuit, determine the transfer function $H(j\omega) = \frac{\tilde{V}_{out}(j\omega)}{\tilde{V}_{in}(j\omega)}$. How does each circuit respond as $\omega \rightarrow 0$ (low frequencies), as $\omega \rightarrow \infty$ (high frequencies)? Is the circuit high-pass filter, low-pass filter, or band-pass filter?**

(a) RC circuit



(a) Circuit in "time domain"



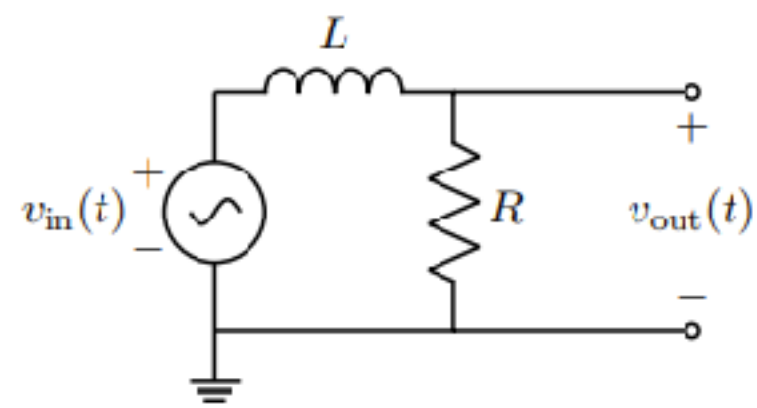
(b) Circuit in "phasor domain"

$$H(j\omega) = \frac{Z_R}{Z_R + Z_C} = \frac{R}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega R}{j\omega R} = \frac{j\omega R}{j\omega R + \frac{1}{C}}$$

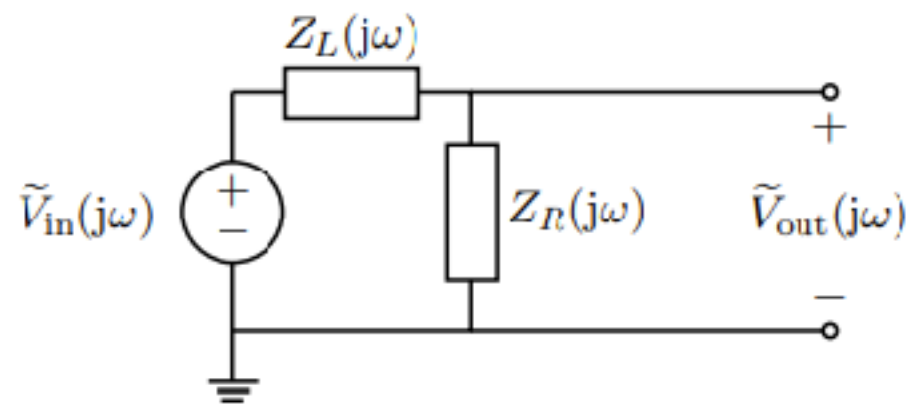
$$\lim_{\omega \rightarrow 0} H(j\omega) = \frac{0}{0 + \frac{1}{RC}} = 0 \quad \text{high pass}$$

$$\lim_{\omega \rightarrow \infty} H(j\omega) = \frac{R}{R + \frac{1}{\infty}} = \frac{R}{R} = 1$$

(b) LR circuit



(a) Circuit in "time domain"



(b) Circuit in "phasor domain"

$$\begin{aligned} \tilde{V}_{out} &= \frac{Z_R}{Z_R + Z_L} \tilde{V}_{in} \\ &= H(j\omega) \cdot \tilde{V}_{in} \end{aligned}$$

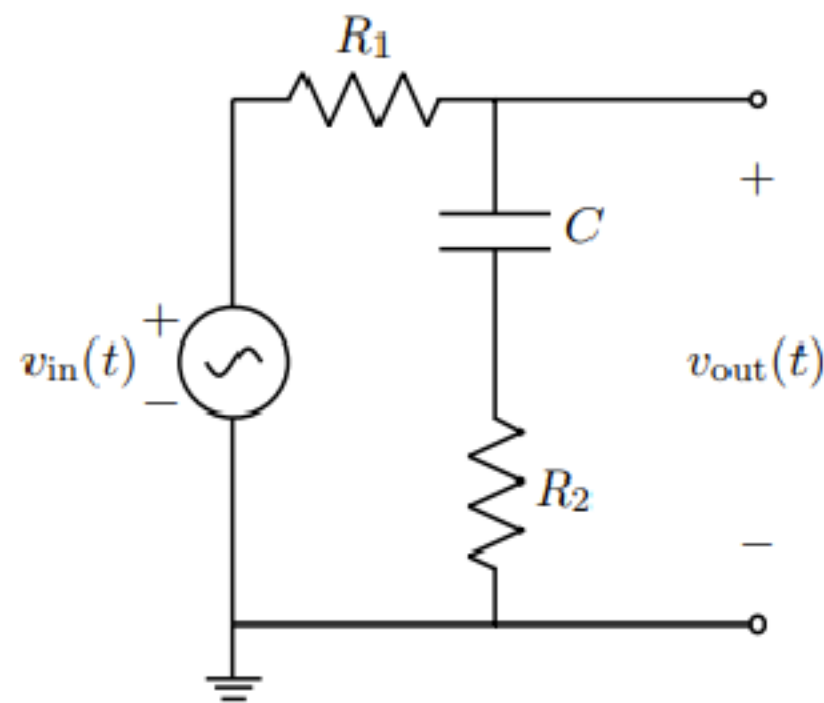
$$H(j\omega) = \frac{R}{R + j\omega L} = \frac{R/L}{\frac{R}{L} + j\omega}$$

$$\lim_{\omega \rightarrow 0} H(j\omega) = \frac{R/L}{R/L + 0} = 1$$

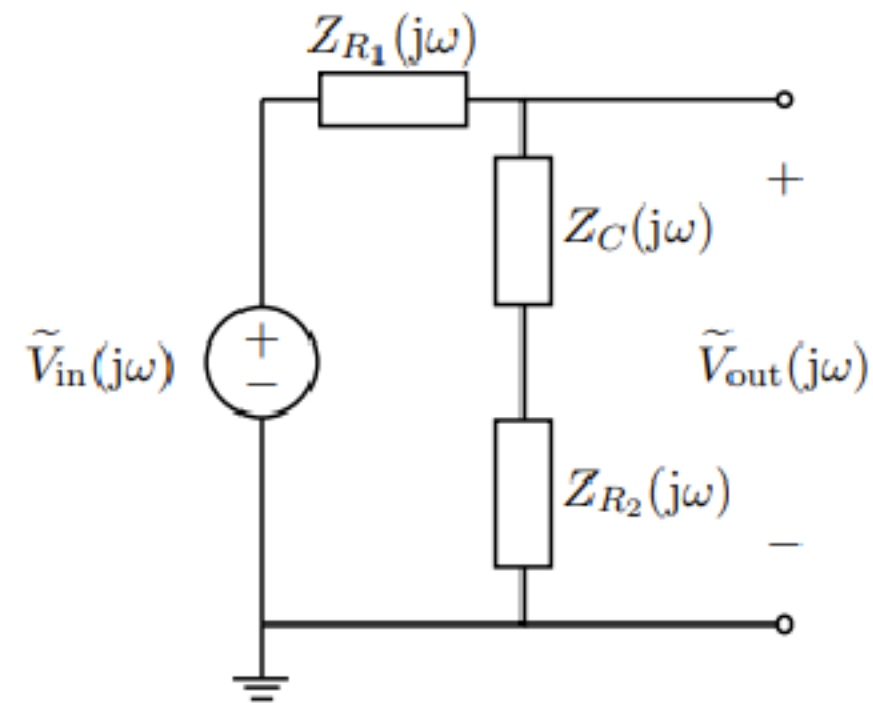
$$\lim_{\omega \rightarrow \infty} H(j\omega) = \frac{R/L}{R/L + \infty} = 0$$

low pass

(c) RCR circuit



(a) Circuit in "time domain"



(b) Circuit in "phasor domain"

$$H(j\omega) = \frac{Z_C + Z_{R_2}}{Z_{R_1} + Z_{R_2} + Z_C} = \frac{\frac{1}{j\omega C} + R_2}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{1 + j\omega C R_2}{1 + j\omega (R_1 + R_2) C}$$

$$\lim_{\omega \rightarrow 0} H(j\omega) = \frac{1}{1} = 1$$

$$\lim_{\omega \rightarrow \infty} H(j\omega) = \frac{R_2}{R_1 + R_2}$$

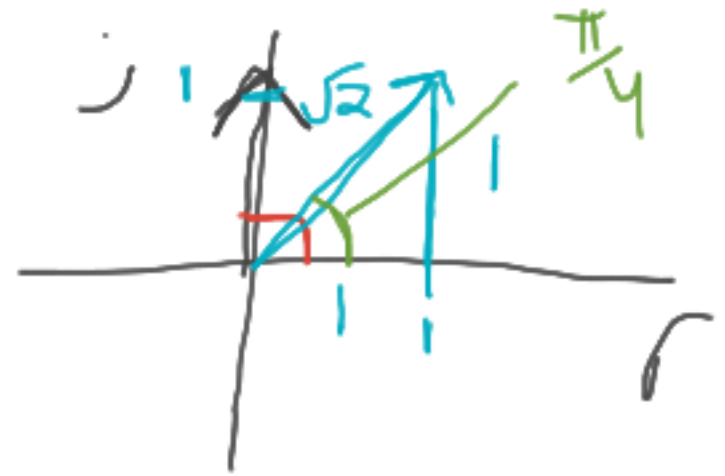
(d) Assuming $v_{in}(t) = 12 \sin(\omega_{in}t)$ compute the $v_{out}(t)$ using transfer functions computed in part

(a). For this part, we assume that $R = 1 \text{ k}\Omega$, $L = 25 \mu\text{H}$, $C = 10 \mu\text{F}$, $\omega_{in} = 100 \text{ Hz}$

$$V_{in}(t) = 12 \sin(\omega t) = 12 \cos(\omega t - \frac{\pi}{2}) \quad |a + bj| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\tilde{V}_{in} = \frac{A}{2} e^{j\phi} = 6 e^{-j\frac{\pi}{2}}$$

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}} = \frac{100j}{100j + \frac{1}{10^3 \cdot 10 \cdot 10^{-6}}} = \frac{100j}{100j + 100} = \frac{j}{j+1}$$



$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1^2 + 1^2}}$$
$$\angle H(j\omega) = \angle j - \angle 1 + j = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

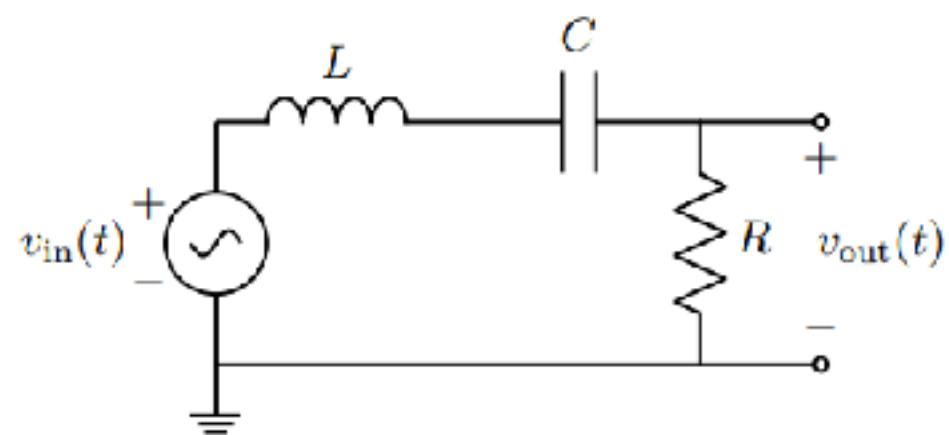
$$H(j\omega) = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}$$

$$\tilde{V}_{out} = \tilde{V}_{in} \cdot H(j\omega)$$
$$= 6 e^{-j\frac{\pi}{2}} \cdot \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} = \frac{6}{\sqrt{2}} e^{-j\frac{\pi}{4}}$$

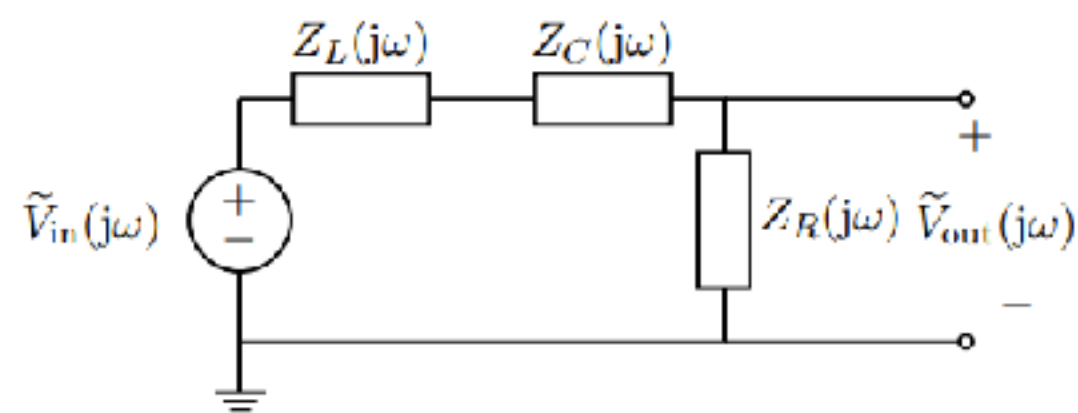
$$v_{out}(t) = \frac{12}{\sqrt{2}} \cos(\omega t - \frac{\pi}{4})$$

2. Band-pass filter

It is quite common to need to design a filter which selects only a narrow range of frequencies. One example is in WiFi radios, it is desirable to select only the 2.4GHz frequency containing your data, and reject information from other nearby cellular or bluetooth frequencies. This type of filter is called a band-pass filter; we will explore the design of this type of filter in this problem.



(a) Circuit in "time domain"



(b) Circuit in "phasor domain"

- (a) Write down the impedance of the series RLC combination in the form $Z_{RLC}(j\omega) = A(\omega) + jX(\omega)$, where $A(\omega)$ and $X(\omega)$ are real valued functions of ω .

$$R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - \frac{j}{\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\frac{1}{j} = -j$$

(b) Write down the transfer function $H(j\omega) = \frac{\tilde{V}_{\text{out}}(j\omega)}{\tilde{V}_{\text{in}}(j\omega)}$ for this circuit.

$$\frac{Z_R}{Z_{LR}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

(c) **At what frequency ω_n does $X(\omega_n) = 0$?** (i.e. at what frequency is the impedance of the series combination of RLC purely real — meaning that the imaginary terms coming from the capacitor and inductor completely cancel each other. This is called the *natural frequency*.)

$$j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

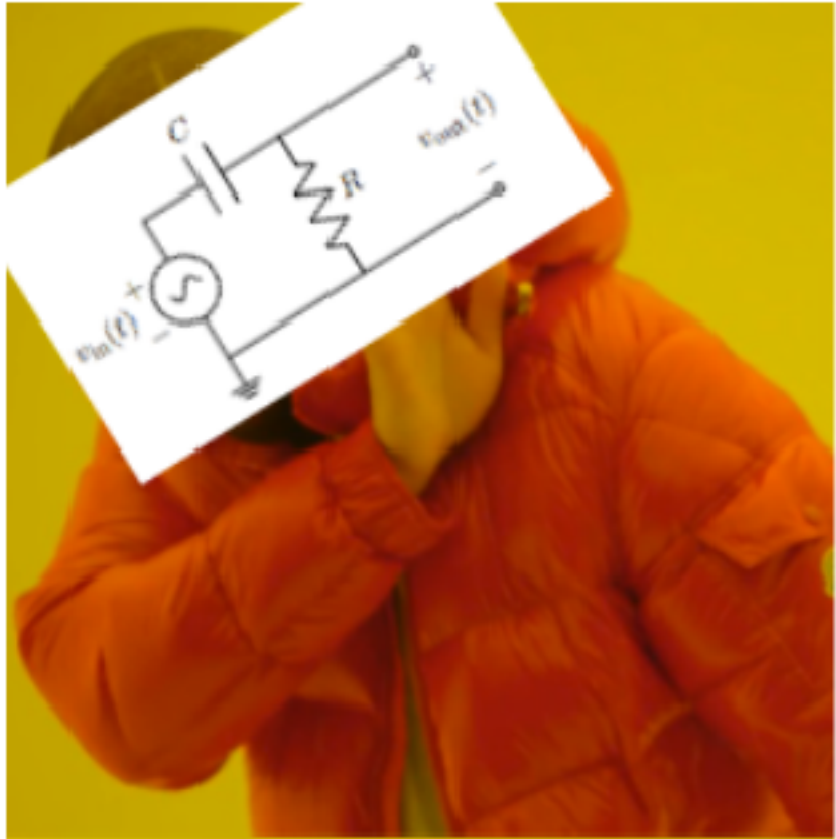
$$\Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

(d) What happens to the relative magnitude of the impedances of the capacitor and inductor as ω moves above and below ω_n ? What is the value of the transfer function at this frequency ω_n ?

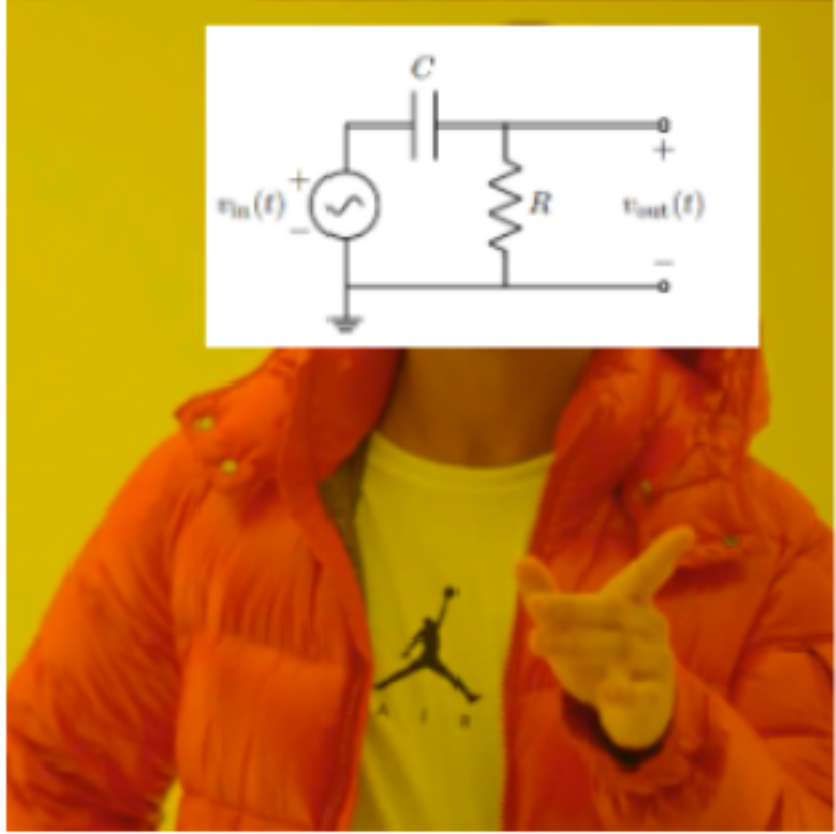
$$\omega \rightarrow 0 \quad H \rightarrow 0$$

$$\omega \rightarrow \infty \quad H \rightarrow 0$$

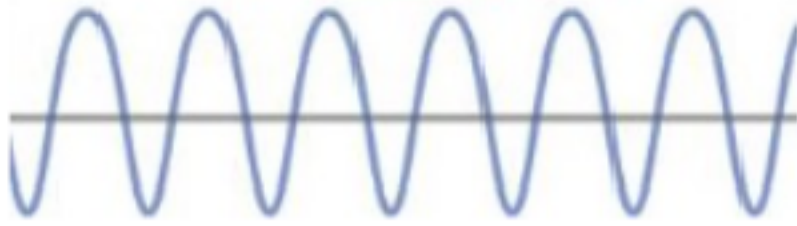
NOTEBOOK TIME!



Low Frequency Wave



High Frequency Wave



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