Remote Discussion Section (Starting @ 7:10PM, Berkeley Time)

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- What is discussion about?
- Discussion attendance for remote

Icebreaker!

- Share with your breakout room peers your name and what you're looking forward to with in person classes and on campus
- Consider the following jobs and topics in EECS16B. What kind of connections do you think there are that show what concepts would be encountered in certain careers?

**Jobs**

- Analog circuit design
- Digital circuit design
- Chip verification
- Control Engineer
- ML engineer
- Graphics/Animation
- CPU design / Architecture design
- Data scientist
- Scientists in Physics, chem, biology
- Software engineer

**Topics**

- Compression
- filtering
- Control systems
- Feedback loops
- Classification
- Sampling (sensor measurements)
- Modeling systems that evolve with time.
- Linear transformations / coordinate changes
- Modeling nonlinear systems with linear systems: $\vec{y} = A\vec{x}$

## 1. Linear Algebra Review

For the following matrices, find the following properties:

i. What is the column space of the matrix?

ii. What is the null space of the matrix?

$A\vec{x} = \vec{0}$ ← can use gaussian elimination to answer

iii. What are the eigenvalues and corresponding eigenspaces for the matrix?

(a) $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$

$\mathbb{R}^2$ = every vector with 2 entries: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for any value of $x_1$ and $x_2$

(b) $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$

Hint: $\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -2 \\ -4 \end{bmatrix}$

(a) ⓘ $A\vec{x}$

$\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 4 \\ 3 \end{bmatrix} x_2$

$\text{Colspace}\left(\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\right) = \mathbb{R}^2$ (columns are linearly independent)

$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \neq \alpha \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

Columns are linearly independent. They're not multiples of each other. The only solution is $\vec{x} = \vec{0}$

$\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{0}$

ⓘⓘ $\text{Nulspace}\left(\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\right)$ = what does this mean?

the set of solutions of all $\vec{x}$ to → $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\vec{x} = \vec{0}$ → The set of solutions is $\{\vec{0}\}$

$\text{Nulspace}\left(\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}\right) = \{\vec{0}\}$

(a) (iii) $A\vec{x} = \lambda\vec{x}$

polynomial in $\lambda$

$\det(A - \lambda I) = 0$

roots of this polynomial are the eigenvalues

$$\det\left(\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 2-\lambda & 4 \\ 0 & 3-\lambda \end{bmatrix}\right) = (2-\lambda)(3-\lambda) - 4\cdot 0 = 0$$

$$(2-\lambda)(3-\lambda) = 0$$

$$\lambda_1 = 2,\ \lambda_2 = 3$$

Calculating eigenvectors

$\lambda_1 = 2$

$(A - 2I)\vec{x} = \vec{0}$

$A\vec{x} = 2\vec{x}$

$\vec{x} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$ is an eigenvector for $\lambda = 2$

$\text{Span}\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ → eigenspace for $\lambda_1 = 2$

## 2. KVL/KCL Review

Use Kirchhoff's Laws on the circuit below to find $V_x$ in terms of $V_{in}$, $R_1$, $R_2$, $R_3$.

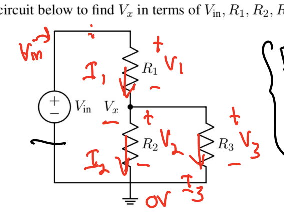



Figure 1: Example Circuit

Review: Read notes 11-20 from 16A

(a) Recall Node Voltage Analysis (NVA). Determine $V_x$ by labeling the circuit and writing equations to solve a system of equations in node voltages.

(b) In EECS16A, you learned you can simplify analysis by replacing series or parallel resistors with equivalents and memorizing common circuit design blocks. Determine $V_x$ by leveraging resistor equivalence and recognition of a design block.

(c) As a check, as $R_3 \to \infty$, what is $V_x$ for what you found in (a) and (b)? The $V_x$'s of each part should approach the same value. What is the name we used for this type of circuit?

NVA

1. Label GND ✓
2. Label node (voltages)
3. Label V's, I's (passive sign convention)
4. Write KCL equations at nodes (with unknown voltage)
5. Substitute Ohm's law expressions
6. Solve the system

KCL @ $V_x$: $I_1 = I_2 + I_3$

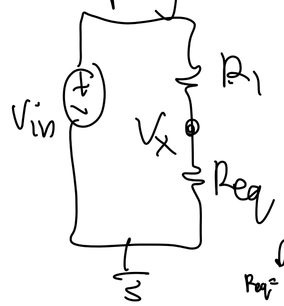
$\left(\frac{V_1}{R} = \frac{V_{in} - V_x}{R}\right)$

$$\frac{V_{in} - V_x}{R_1} = \frac{V_x - 0V}{R_2} + \frac{V_x - 0V}{R_3} \rightarrow \frac{V_{in}}{R_1} = \frac{V_x}{R_1} + \frac{V_x}{R_2} + \frac{V_x}{R_3}$$

$$\frac{V_{in}}{R_1} = V_x\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V_x = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} V_{in}$$

(b) Simplify to:

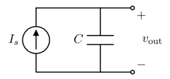



$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

3. **Current Sources And Capacitors** (The following problem has been adapted from EECS16A Fall 20 Disc 9A.)

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.
It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

Given the circuit below, find an expression for $v_{\text{out}}(t)$ in terms of $I_s$, $C$, $V_0$, and $t$, where $V_0$ is the initial voltage across the capacitor at $t = 0$.




Then plot the function $v_{\text{out}}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

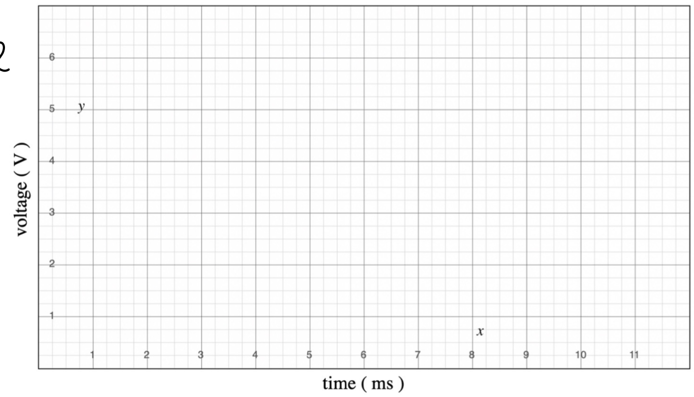
(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.
(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.
(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.

charge

$Q = CV$ voltage

$I = C\frac{dV}{dt}$




4. **(Take-Home) Op-Amp Summer**

Review Golden Rules

Consider the following circuit (assume the op-amp is ideal):

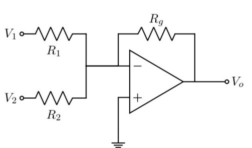



Figure 2: Op-amp Summer

What is the output $V_o$ in terms of $V_1$ and $V_2$? You may assume that $R_1$, $R_2$, and $R_g$ are known.