EECS 16B Designing Information Devices and Systems II
Fall 2021 UC Berkeley

Homework 06

This homework is due on Friday, October 8, 2021, at 11:59PM. Self-grades and HW Resubmission are due on Tuesday, October 12, 2021, at 11:59PM.

1. **Reading Lecture Notes**

   Staying up to date with lectures is an important part of the learning process in this course. Here are links to the notes that you need to read for this week: Note 7, Note 8, Note 9, and Note 10.

   You should also read the "Discipline Bonus" post on Piazza.

   (a) In the magnitude Bode plot for a first-order-low-pass filter drawn on a log-log graph (i.e., $|H(j\omega)|$ is plotted on a log scale), **what is the slope of the straight line approximation at frequencies higher than the cut-off frequency? What is the slope for a third-order-low-pass filter (three identical first-order-low-pass filters cascaded with unity gain buffers)?**

   (b) If you know a filter has an attenuation of $\left|H(j\omega_0)\right| = 0.95$ at ω_0, **what would the attenuation or gain be when you cascade** 10 **of these filters at** ω_0**?** If you know this same filter has a gain of $\left|H(j\omega_1)\right| = 1.1$ at some different angular frequency ω_1, **what would the gain be when you cascade** 10 **of these filters at** ω_1**?**

   (c) Suppose we had a continuous-time differential equation model

   $$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}_c(t) = A_c\vec{x}_c(t) + B_c\vec{u}_c(t) \tag{1}$$

   and we wanted to discretize it to get a model of the form

   $$\vec{x}_d[i+1] = A_d\vec{x}_d[i] + B_d\vec{u}_d[i] \tag{2}$$

   where $\vec{x}_d[i] = \vec{x}_c(i\Delta)$ and the input $\vec{u}_c(t)$ is generated to be piecewise constant so $\vec{u}_d[i]$ is the value of $\vec{u}_c(t)$ for the whole time interval $t \in [i\Delta, (i+1)\Delta)$.

   Describe the steps that you would take to get A_d **and** B_d **from** A_c **and** B_c**.** Feel free to assume that A_c is diagonalizable.

   (d) **What relevance does the following property of matrix multiplication have to system identification of vector systems by means of least-squares?**

   $$AB = A\begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_N \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ A\vec{b}_1 & \cdots & A\vec{b}_N \\ | & & | \end{bmatrix} = C = \begin{bmatrix} | & & | \\ \vec{c}_1 & \cdots & \vec{c}_N \\ | & & | \end{bmatrix}$$

   (e) **What is a condition for a discrete time scalar system to be BIBO stable? What is a condition on the eigenvalues for a discrete time vector system to be BIBO stable? What is a condition on the eigenvalues for a continuous time system to be BIBO stable?**

   (f) **Do you intend to take advantage of the "Discipline Bonus" being offered as extra credit in the course?** If you want to, feel free to tell us why or why not.

2. Bandpass Filter: Lowpass and Highpass Cascade

In lecture, you heard about how you can go through the design of a bandpass filter by cascading lowpass and highpass filters via buffers (op-amps in unity-gain negative feedback to prevent loading effects). In this problem, you will do this for yourself.

Consider an input signal that is composed of the superposition of:

- $20\,\mathrm{mV}$ level pure tone at $60\,\mathrm{Hz}$ corresponding to power line noise.
- $1\,\mathrm{mV}$ level pure tone at $600\,\mathrm{Hz}$ corresponding to a voice signal.
- $10\,\mathrm{mV}$ level pure tone at $60\,\mathrm{kHz}$ corresponding to fluorescent light noise.

We would like to keep the $600\,\mathrm{Hz}$ tone, which could correspond to a voice signal.

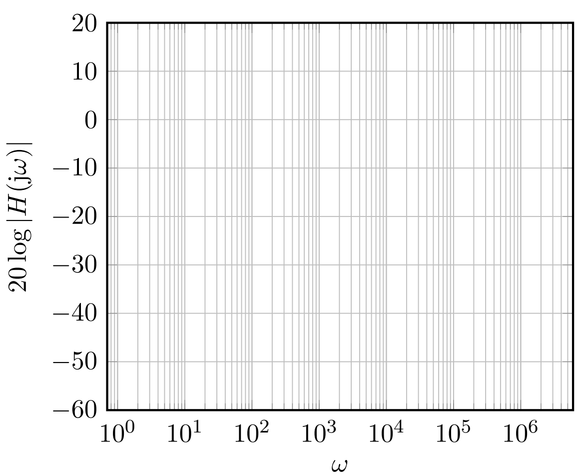
(a) Ignoring any phase offset for each signal (i.e., set the phases to zero), **write the $V_{\mathrm{in}}(t)$ that describes the above input in time domain, in the following format:**.

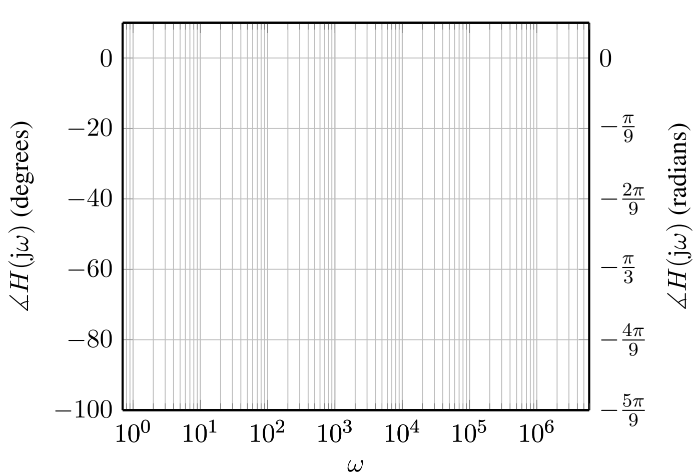
$$V_{\mathrm{in}}(t) = A_p \cos\bigl(2\pi f_p t\bigr) + A_v \cos(2\pi f_v t) + A_f \cos\bigl(2\pi f_f t\bigr) \tag{3}$$

(b) **What are the angular frequencies (i.e., $\omega_p, \omega_v, \omega_f$) involved and the phasors associated with each tone?** Remember that the frequencies of the tones are provided in Hz. To convert these frequencies to angular frequencies, we use $\omega = 2\pi f$.

(c) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the knee or cutoff-frequency for the lowpass filters**? *Hint: To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.*

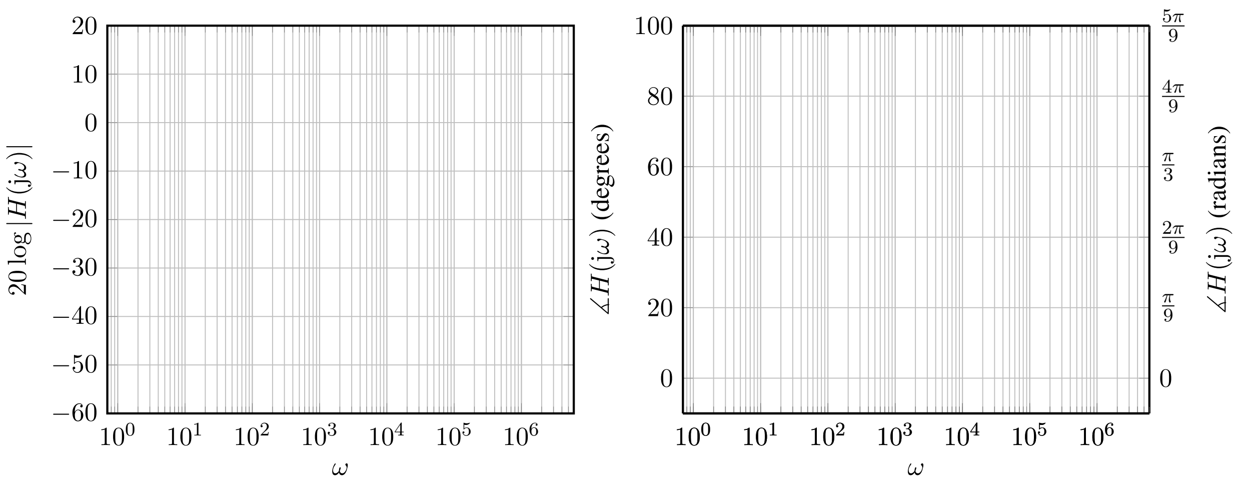
(d) **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude (using $20\log|H(\mathrm{j}\omega)|$) and phase of the lowpass filter.**





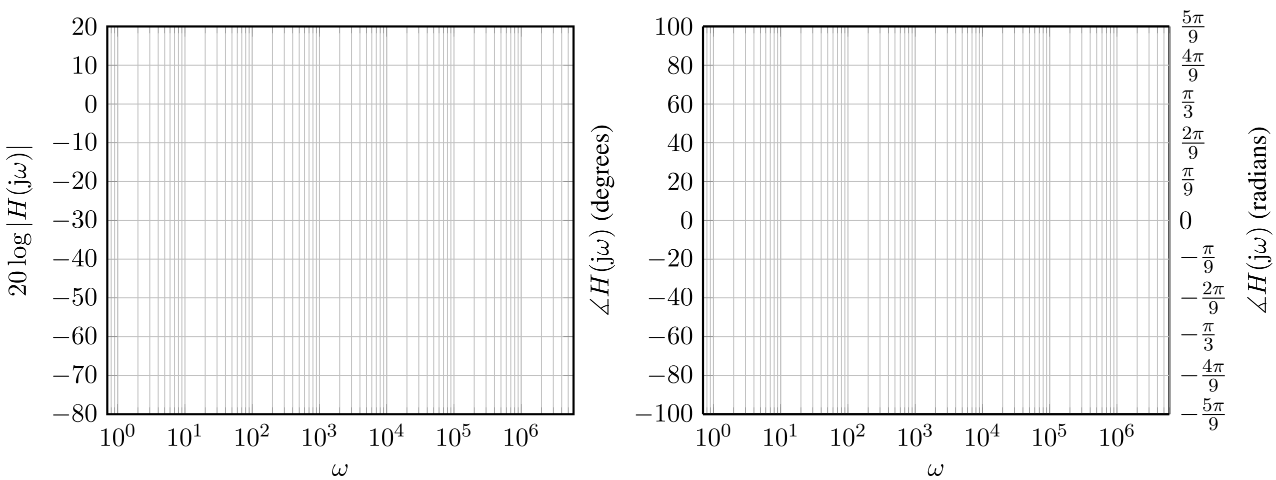
(e) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the knee or cutoff-frequency for the highpass filters**? *Hint: To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.*

(f) **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude (using $20\log|H(\mathrm{j}\omega)|$) and phase of the highpass filter.**



(g) For the following questions, assume your cut-off frequencies for lowpass and highpass are 6 kHz and 189 Hz respectively. Suppose that you only had 1 µF capacitors to use. **What resistance values would you choose for your highpass and lowpass filters so that they have the desired cutoff frequencies?**

(h) The overall bandpass filter that is created by cascading the lowpass and highpass with ideal buffers in between. **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude and phase of the overall bandpass transfer function.** *Hint: You should think about how the Bode plot of a cascade of two filters can be derived based on the Bode plots of the lower-level filters.*



(i) Suppose that the bandpass filter does not have enough suppression at 60 Hz and 60 kHz and you decide to use a cascade of three bandpass filters (with unity-gain buffers in between) to mitigate the issue (as shown in Figures 1 and 2). **What are the phasors for each of the frequency tones after all three bandpass filters?** *Hint: Remember how you determined the transfer function of the bandpass filter from the transfer functions of the lowpass and highpass filters.* Feel free to use a computer to help you evaluate both the magnitudes and the phases here.

(j) **Draw the Bode plots (straight-line approximations to the transfer function) for the magnitude and phase of the 3^{rd} order bandpass filter.** To highlight the difference between the 3^{rd} and 1^{st} order filters, please draw both Bode plots on a single figure.

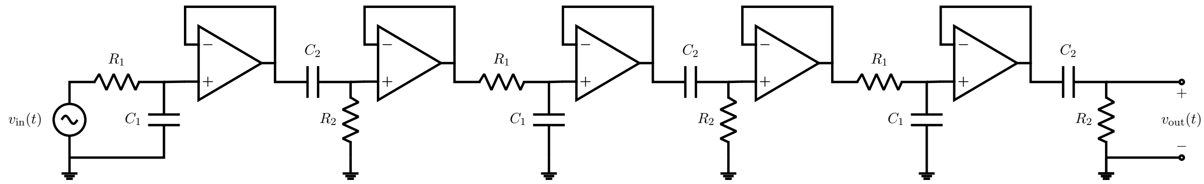


Figure 1: "Time-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.

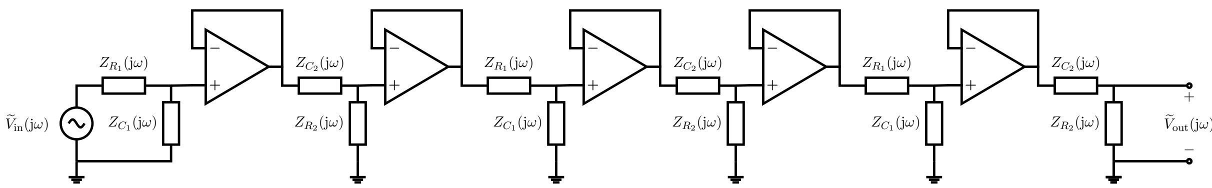
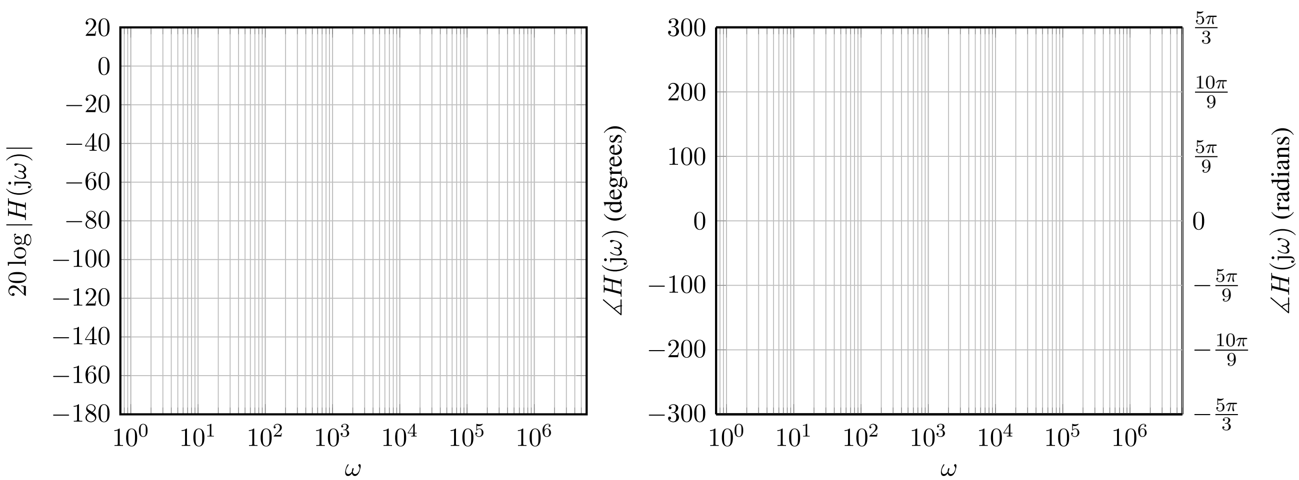


Figure 2: "Phasor-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.



(k) **Write the final time domain voltage waveform that would be present after the filter.**

(l) The included jupyter notebook `filter_cascade.ipynb` sets up the same problem described above. In the notebook, you can use the slider bars to play around with:

- highpass cutoff frequency f_{highpass} (i.e., the knee frequency of the highpass filters)
- lowpass cutoff frequency f_{lowpass} (i.e., the knee frequency of the lowpass filters)
- Filter order N. Filter order means the number of lowpass filters and highpass filters that are used in a row. Here, N means that there are N lowpass filters and N highpass filters, so the overall order of the entire filter is actually $2N$.

The notebook will plot the magnitude and phase, the input voltage waveform, and the output waveform at the end of the filter.

Play around with the values for the highpass and lowpass cutoff frequencies, and N.

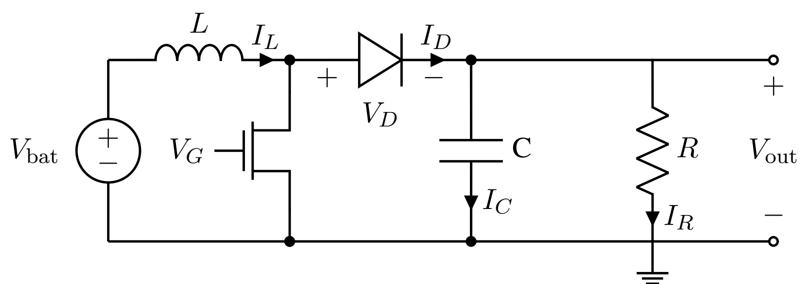
Observe the waveforms at the output of the filter. **Comment on the limits of f_{lowpass}, f_{highpass}, and N that you can use to successfully isolate the desired $600\,\text{Hz}$ tone. What happens if you keep f_{lowpass} and f_{highpass} constant, and just increase N?**

3. Time-Domain: Boost Converter

Below is a circuit called a Boost Converter. The boost converter is a simple version of a circuit that is very common in battery-operated devices. While a single alkaline battery has a voltage that varies from $1.6\,\text{V}$ when it is fresh to 0.8V at end of life, most electronic devices require a constant supply voltage that stays within $\pm 10\%$ of a certain nominal value. That nominal value is often greater than $1.6\,\text{V}$. So how do we power electronics with batteries? And in particular, how do we power electronics that wants $3\,\text{V}$ or $5\,\text{V}$ to function when our battery is providing significantly less than that?

The boost converter is a commonly used circuit to provide that stable supply voltage, even from a battery whose voltage is slowly decreasing as it discharges. With proper choice of components and proper control of the gate voltage V_G of the transistor, power from the battery can be converted to higher voltages with almost perfect efficiency.

Consider the following circuit:



with

$$V_{\text{bat}} = 1\,\text{V} \tag{4}$$
$$L = 1\,\mu\text{H} \tag{5}$$
$$C = 1\,\text{mF} \tag{6}$$
$$R = 1\,\text{k}\Omega. \tag{7}$$

We model the electronic component that is consuming power as a resistor R with voltage V_{out} across it. Assume that the gate voltage at the NMOS is low (and thus the NMOS is an open circuit) for $t < 0$.

This circuit introduces a new nonlinear circuit element called a diode — this is also called a rectifier and its behavior is related to something that is very popular these days in the design of neural networks. The fact that the simple nonlinearity of the diode is powerful enough to do interesting computations is what allows neural networks based on ReLU units (basically diodes) to be useful.

An ideal diode allows current to flow when $V_D > 0$ and blocks current flows through an ideal diode when $V_D \leq 0$. A physical diode is a circuit element that allows current to flow when the voltage V_D is above a certain threshold voltage. (We approximate this threshold voltage as being zero for an ideal diode, but in reality, this threshold like behavior is actually the consequence of an underlying exponential behavior. Take EE130 to understand the very interesting physics that underlies this behavior.) That means, when the voltage is below the threshold, this circuit element behaves like an open circuit. This is very similar to how a transistor can behave as a switch, but the diode is directly switching current through it without having a separate gate controlling it.

In the robot car project, you will use the diode in a manner similar to this problem.

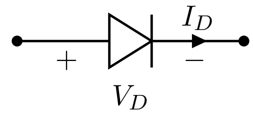


Figure 3: Diagram of a diode in isolation. The arrow denotes the direction that current is allowed to flow.

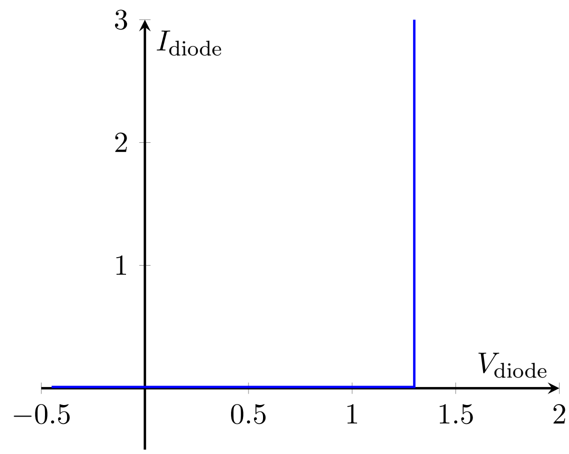


Figure 4: Basic current voltage response of a diode, which shows an impedance of 0 when on, and ∞ when off. (*In this example, the threshold voltage is about* $1.3V$ — most physical diodes are treated as having a threshold that is closer to 0.7V.)

(a) Assume that there is no current flowing through the diode — treat it like an open circuit. **Calculate the time constant of the RC circuit. How long will it take for the output voltage to decay by** 10% **from any non-zero value?** If the voltage on the capacitor is $3\,\mathrm{V}$ at $t = 0$, **what is the current in the resistor at time** $t = 0$**? What is the voltage in the capacitor for** $t > 0$**?**

(b) Let's assume that at $t = 0$, the voltage on the capacitor is $3\,\mathrm{V}$. At $t = 0$, the gate voltage on the NMOS V_G goes high, turning the transistor on (effectively shorting that circuit path). **What is the rate of change of current in the inductor? How long does it take for the current in the inductor to increase to** $100\,\mathrm{mA}$**?**

Don't be scared by the fact that the voltage across an inductor is permitted to change discontinuously.

(c) Based on your two previous answers, **how much has the capacitor voltage decayed during this time?** Your answer should be such a small number that in the following parts, we will consider the output voltage constant.

(d) When the current in the inductor hits $100\,\mathrm{mA}$, the transistor gate goes low, turning the transistor off. The energy stored in the magnetic field in the inductor quickly causes the voltage on the positive side of the diode to increase to $3\,\mathrm{V}$, turning the ideal diode on (to understand why, think about what a rapid decrease in current will do to the inductor voltage when the transistor turns off). **What is the rate of change of current in the inductor when the diode is on? How long does it take the current in the inductor to go to zero?**

(e) **Sketch the following currents vs time:** i_L**,** i_{NMOS}**,** i_{diode} **for a single inductor charge/discharge cycle described above.**

If you like circuits like this, take EE113 Power Electronics.

(f) To maintain V_{out} at $3\,\mathrm{V}$, we need the average current through the diode to bring enough charge to the capacitor. **Calculate the charge that flows through the diode in a single cycle. At what frequency**

does the transistor need to cycle through the on/off phases you analyzed above in order to supply the current consumed by the resistor, computed in part (a)?

(g) The included jupyter notebook `boost_converter.ipynb` sets up the same problem described above. In the notebook, you can use the slider bars to play around with:

- The cycle frequency for the transistor
- The component values R, L, and C
- The battery voltage V_{bat}

How does the inductor current curve in part (e) change if the battery voltage changes? What is the effect of a larger or smaller capacitor? A larger or smaller load?

4. Circuit Design

In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

Assume that the op-amp is *ideal*. *A special note on op amps in frequency domain analysis: The op-amps you learned about in 16A can be used in exactly the same way for setting up differential equations and even Phasor analysis in 16B. Treat them as ideal op-amps and invoke the Golden Rules.*

You have at your disposal *only one of each* of the following components (not including R_1):

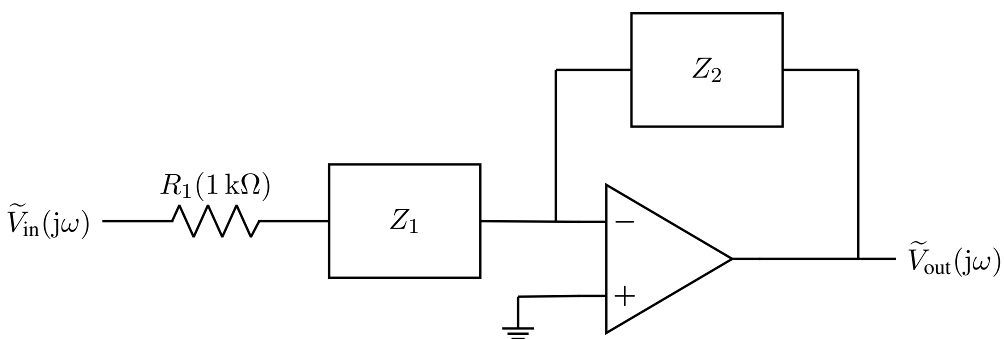
(a) an open circuit

(b) a short circuit

(c) a resistor (you choose from the values $R = 1\,\mathrm{k\Omega}, 10\,\mathrm{k\Omega}, 20\,\mathrm{k\Omega}$)

(d) a capacitor (you choose from the values $C = 0.5\,\mathrm{\mu F}, 1\,\mathrm{\mu F}, 2\,\mathrm{\mu F}$)

Consider the circuit below. The voltage source $v_{\text{in}}(t)$ has the form $v_{\text{in}}(t) = v_0 \cos(\omega t + \phi)$. The labeled voltages $\widetilde{V}_{\text{in}}(\mathrm{j}\omega)$ and $\widetilde{V}_{\text{out}}(\mathrm{j}\omega)$ are the phasor representations of $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$. The transfer function $H(\mathrm{j}\omega)$ is defined as $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}(\mathrm{j}\omega)}{\widetilde{V}_{\text{in}}(\mathrm{j}\omega)}$.



(a) Let the left box have impedance $Z_1(\mathrm{j}\omega)$ and the right box have impedance $Z_2(\mathrm{j}\omega)$. **Write the expression of the transfer function $H(\mathrm{j}\omega)$.**

(b) Let R_1 be $1\,\mathrm{k\Omega}$. We have to find Z_1 and Z_2, such that the circuit's transfer function $H(\mathrm{j}\omega)$ has the following properties:

- It is a high-pass filter.
- $|H(\mathrm{j}\infty)| = 10$.
- $|H(\mathrm{j}10^3)| = \sqrt{50}$.

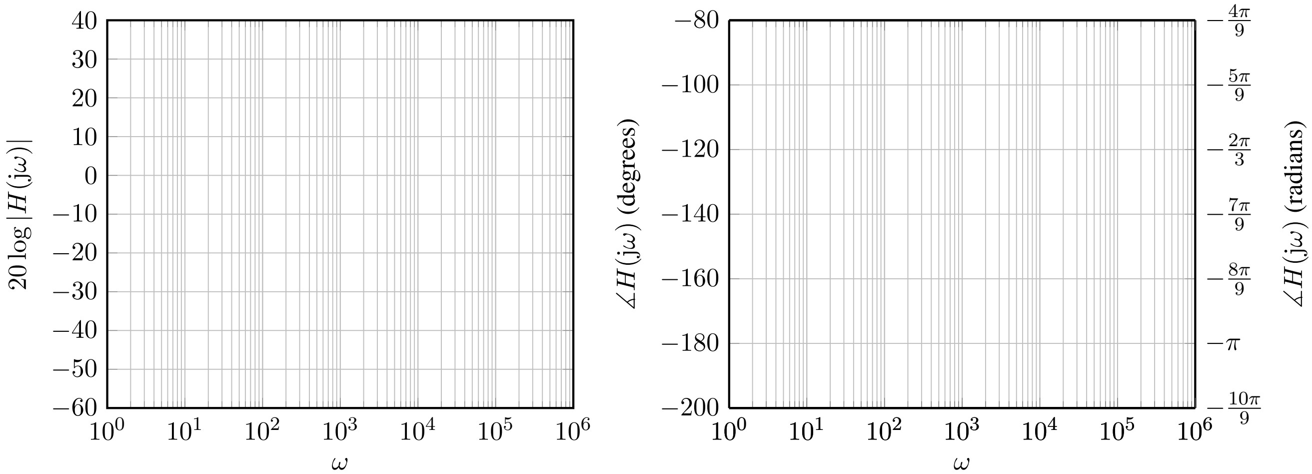
Using the fact that the circuit is a high pass filter, infer the components (we will find values later) of Z_1 and Z_2. Write the transfer function $H(\mathrm{j}\omega)$ using these components.

Hint: Try method of elimination: figure out what Z_2 cannot be. Once you find what Z_2 is, what does Z_1 have to be for the circuit to be a filter?

(c) **Now use the facts that $|H(\mathrm{j}\infty)| = 10$ and $R_1 = 1\,\mathrm{k\Omega}$ to find the component value of Z_2.**

(d) **Finally use the fact that $|H(\mathrm{j}10^3)| = \sqrt{50}$ and the values of R_1 and Z_2 to find the component value of Z_1.**

(e) **Draw the magnitude and phase Bode plots (straight-line approximations to the transfer function) of this transfer function.** Blank plots are provided here for you to use.



5. System Identification

You are given a discrete-time system as a black box. You don't know the specifics of the system but you know that it takes one scalar input and has two states that you can observe. You assume that the system is linear and of the form

$$\vec{x}[t+1] = A\vec{x}[t] + Bu[t] + \vec{w}[t], \tag{8}$$

where $\vec{w}[t]$ is an external unseen disturbance that you hope is small, $u[t]$ is a scalar input, and

$$A = \begin{bmatrix} a_0 & a_1 \\ a_2 & a_3 \end{bmatrix}, \quad B = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}, \quad x[t] = \begin{bmatrix} x_0[t] \\ x_1[t] \end{bmatrix}. \tag{9}$$

You want to identify the system parameters from measured data. You need to find the unknowns: a_0, a_1, a_2, a_3, b_0 and b_1. However, you can only interact with the system via a black box model, i.e., you can see the states $\vec{x}[t]$ and set the inputs $u[t]$ that allow the system to move to the next state.

(a) You observe that the system has state $\vec{x}[t] = \begin{bmatrix} x_0[t] & x_1[t] \end{bmatrix}^\top$ at time t. You pass input $u[t]$ into the black box and observe the next state of the system: $\vec{x}[t+1] = \begin{bmatrix} x_0[t+1] & x_1[t+1] \end{bmatrix}^\top$.

Write scalar equations for the new states, $x_0[t+1]$ **and** $x_1[t+1]$. Write these equations in terms of the a_i, b_i, the states $x_0[t]$, $x_1[t]$ and the input $u[t]$. Here, assume that $\vec{w}[t] = \vec{0}$ (i.e., the model is perfect).

(b) Now we want to identify the system parameters. We observe the system at the start state $\vec{x}[0] = \begin{bmatrix} x_0[0] \\ x_1[0] \end{bmatrix}$. We can then input $u[0]$ and observe the next state $\vec{x}[1] = \begin{bmatrix} x_0[1] \\ x_1[1] \end{bmatrix}$. We can continue this for a sequence of m inputs.

Let us define an m-length trajectory to be

$$\begin{bmatrix} x_0[0] & x_1[0] & u[0] & x_0[1] & x_1[1] & u[1] & \cdots & x_0[m-1] & x_1[m-1] & u[m-1] & x_0[m] & x_1[m] \end{bmatrix}. \tag{10}$$

Assuming there is no noise ($\vec{w}[t] = \vec{0}$)**, what is the minimum value of** m **you need to identify the system parameters?**

(c) Now assume that there is a nonzero noise/disturbance $\vec{w}[t]$. **Would using more than equations than in part (b) help you in this case? If so, explain why.**

(d) Say we feed in a total of 4 inputs $\begin{bmatrix} u[0] & u[1] & u[2] & u[3] \end{bmatrix}^\top$ into our black box. This allows us to observe the following states $\begin{bmatrix} x_0[0] & x_0[1] & x_0[2] & x_0[3] & x_0[4] \end{bmatrix}^\top$ and $\begin{bmatrix} x_1[0] & x_1[1] & x_1[2] & x_1[3] & x_1[4] \end{bmatrix}^\top$, which we can use to identify the system.

To identify the system we need to set up an approximate (because of potential disturbances) matrix equation

$$D\vec{p} \approx \vec{s} \tag{11}$$

using the observed values above and the unknown parameters we want to find. We know our parameter vector should be $\vec{p} = \begin{bmatrix} a_0 & a_1 & b_0 & a_2 & a_3 & b_1 \end{bmatrix}^\top$. **Find the corresponding** D **and** $\vec{s}$ **to do system identification. Write out both explicitly.**

Note: This is a different representation of the parameters than was done in lecture. This is intentional to make sure that you actually understand and can adapt.

(e) Now that we have set up $D\vec{p} \approx \vec{s}$, **explain how you would use this approximate equation to estimate the unknown values** a_0**,** a_1**,** a_2**,** a_3**,** b_0, b_1**.** In particular, give an expression for your estimate $\vec{\widehat{p}}$ in terms of the D and $\vec{s}$. Assume that the columns of D are linearly independent.

(Hint: Don't forget that D is not a square matrix. It is taller than it is wide.)

6. Identifying systems from their responses to known inputs

In many problems, we have an unknown system, and would like to characterize it. One of the ways of doing so is to observe the system response with different initial conditions (or inputs). This problem is also called system identification. It is a prototypical example of a problem that today is called machine learning — inferring an underlying pattern from data, and doing so well enough to be able to exploit that pattern in some practical setting. Go through the attached Jupyter notebook `demo_system_id.ipynb` and answer the following questions.

(a) In Example 2, we assume that instead of measuring the state $\vec{x}$, we are instead measuring a transformation of the state $\vec{y} = T\vec{x}$ where T is a full rank matrix. Assume that we perform system ID on our observations $\vec{y}[t]$ to recover A_y, B_y such that $\vec{y}[t+1] = A_y\vec{y}[t] + B_y u[t]$. **How do the identified A_y and B_y matrices relate to the original A and B matrices in the dynamics of $\vec{x}$?** Remember that our original state dynamics are $\vec{x}[t+1] = A\vec{x} + B\vec{u}$.

Hint: The answer is given in the Jupyter notebook but remember to show your work.

(b) **Please share your observations on Example 2.**

(c) **Prove that for any full rank transformation matrix T, the eigenvalues of A_y and A from part (a) are the same.**

(d) **Please share your observations on Example 3.**

(e) **Please share your observations on Example 4.**

(f) **Please share your observations on Example 5.**

7. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning (from the bottom up) is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.

You need to write your own question and provide a thorough solution to it. The scope of your question should roughly overlap with the scope of this entire problem set. This is because we want you to exercise your understanding of this material, and not earlier material in the course. However, feel free to combine material here with earlier material, and clearly, you don't have to engage with everything all at once. A problem that just hits one aspect is also fine.

Note: One of the easiest ways to make your own problem is to modify an existing one. Ordinarily, we do not ask you to cite official course materials themselves as you solve problems. This is an exception. Because the problem making process involves creative inputs, you should be citing those here. It is a part of professionalism to give appropriate attribution.

Just FYI: Another easy way to make your own question is to create a Jupyter part for a problem that had no Jupyter part given, or to add additional Jupyter parts to an existing problem with Jupyter parts. This often helps you learn, especially in case you have a programming bent.

8. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

(a) **What sources (if any) did you use as you worked through the homework?**

(b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

(c) **Roughly how many total hours did you work on this homework? Write it down here where you'll need to remember it for the self-grade form.**

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