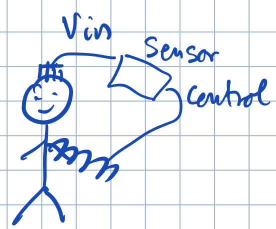
# Lecture 10

## * Design example – bandpass filter




Vin components:

| | freq | amplitude | |
|---|---|---|---|
| signal | 600 Hz | 1 mV | } desired |
| AC (alternate current) | 60 Hz | 10 mV | } ~~interference~~ |
| fluorescent light | 60 kHz | 20 mV | |

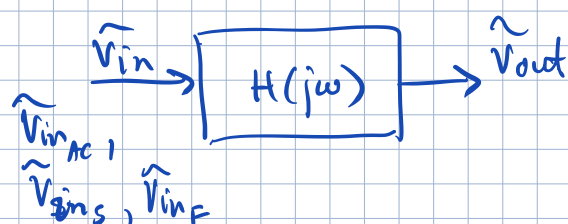
Design goal: Want to attenuate the interference AC and fluorescent components by 100X.

$$V_{in}(t) = V_{AC}\cos(\omega_{AC}t + \phi_{AC}) + V_S\cos(\omega_S t + \phi_S) +$$
$$+ V_F\cos(\omega_F t + \phi_F)$$

$\omega_{AC} = 2\pi \cdot 60\,Hz = 377\ \frac{rad}{s}$

$\omega_S = 2\pi \cdot 600\,Hz$

$\omega_F = 2\pi \cdot 60\,kHz$




Through superposition:

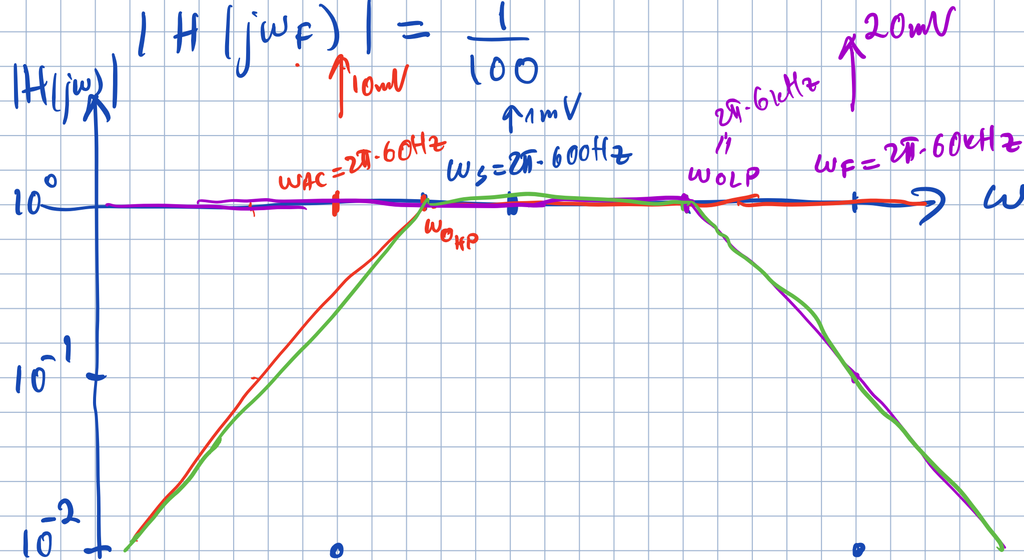
$$V_{out}(t) = |H(j\omega_{AC})| \cdot V_{AC} \cdot \cos(\omega_{AC}t + \phi_{AC} + \measuredangle H(j\omega_{AC}))$$
$$+ \ |H(j\omega_S)| \cdot V_S \cdot \cos(\omega_S t + \phi_S + \measuredangle H(j\omega_S))$$
$$+ \ |H(j\omega_F)| \cdot V_F \cdot \cos(\omega_F t + \phi_F + \measuredangle H(j\omega_F))$$

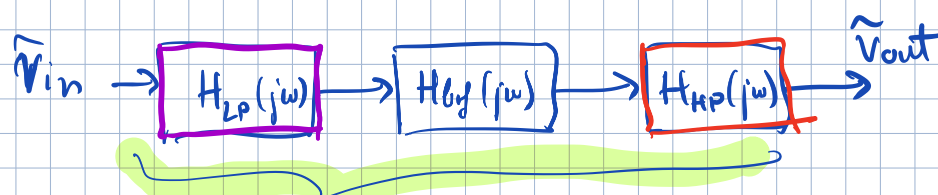
Design goal is to attenuate AC & Fluorescent by 100x :

$$|H(j\omega_{AC})| = \frac{1}{100}$$

$$|H(j\omega_S)| = 1$$

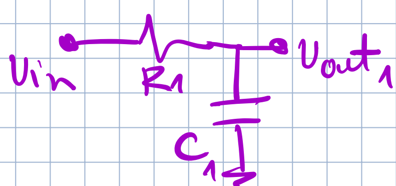
$$|H(j\omega_F)| = \frac{1}{100}$$




$$H(j\omega) = H_{LP}(j\omega) \cdot H_{bf}(j\omega) \cdot H_{HP}(j\omega)$$

$H_{LP}(j\omega)$ – a "low-pass" filter – already know how to do this (L9)

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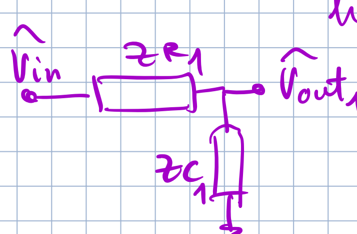
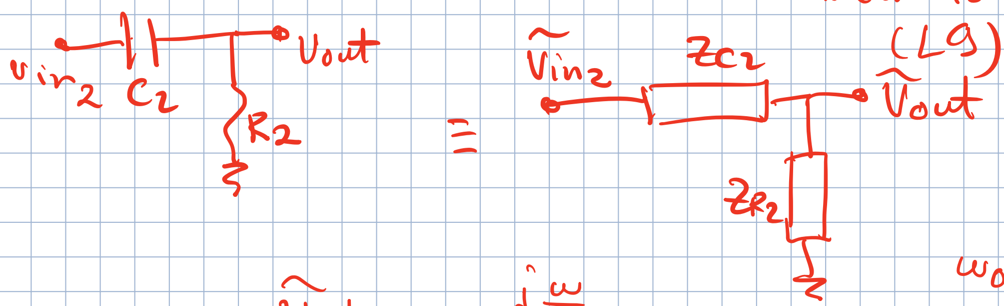


$$H_{LP}(j\omega) = \frac{\hat{v}_{out1}}{\hat{v}_{in}} = \frac{1}{1 + j\frac{\omega}{\omega_{0LP}}}, \quad \omega_{0LP} = \frac{1}{R_1 C_1}$$

$H_{HP}(j\omega)$ – a "high-pass" filter – already know how to do this




$$\omega_{0HP} = \frac{1}{R_2 C_2}$$

$$H_{HP}(j\omega) = \frac{\hat{V}_{out}}{\hat{V}_{in2}} = \frac{j\frac{\omega}{\omega_{0HP}}}{1 + j\frac{\omega}{\omega_{0HP}}} = \frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}$$

$$H(j\omega) = \underbrace{\frac{\hat{V}_{out}}{\hat{V}_{in2}}}_{H_{HP}(j\omega)} \cdot \underbrace{\frac{\hat{V}_{in2}}{\hat{V}_{out1}}}_{H_{buf}(j\omega)} \cdot \underbrace{\frac{\hat{V}_{out1}}{\hat{V}_{in}}}_{H_{LP}(j\omega)} = \boxed{\frac{1}{1 + j\frac{\omega}{\omega_{0LP}}} \cdot \frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}}$$

$$\hat{V}_{in} \rightarrow \boxed{H_{LP}(j\omega)} \xrightarrow{\hat{V}_{out1}} \boxed{H_{buf}(j\omega)} \xrightarrow{\hat{V}_{in2}} \boxed{H_{HP}(j\omega)} \rightarrow \hat{V}_{out}$$

Need to choose $\omega_{0LP}$ and $\omega_{0HP}$.

Compromise – want to attenuate the interference but not the signal.

$$\omega_{0HP} = \sqrt{\omega_{AC} \cdot \omega_S} \qquad \log \omega_{0HP} = \frac{1}{2}\log \omega_{AC} + \frac{1}{2}\log \omega_S$$

$$\omega_{OLP} = \sqrt{\omega_S \cdot \omega_F}$$

$$\log \omega_{OHP} = \log(\omega_{AC} \cdot \omega_S)^{\frac{1}{2}} = \frac{1}{2}\log(\omega_{AC} \cdot \omega_S) = \frac{1}{2}\log \omega_{AC} + \frac{1}{2}\log \omega_S$$

$$\omega_{OHP} = \frac{1}{R_2 C_2}, \quad \omega_{OLP} = \frac{1}{R_1 C_1}$$

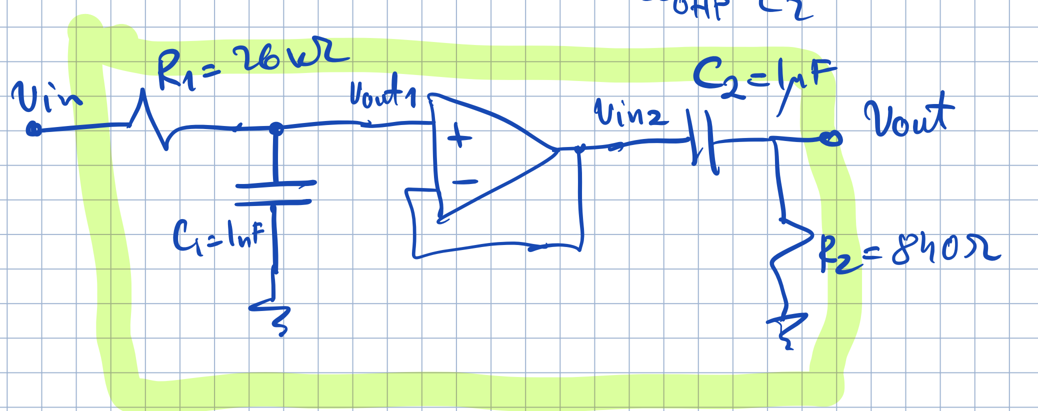
$$\omega_{OHP} = \sqrt{2\pi\, 60\,Hz \cdot 2\pi\, 600\,Hz} = 2\pi \cdot 190\,Hz$$

$$\omega_{OLP} = \sqrt{2\pi \cdot 600\,Hz \cdot 2\pi \cdot 60\,kHz} = 2\pi \cdot 6000\,Hz$$

Pick a reasonable C:

$$C_1 = 1\,nF \Rightarrow R_1 = \frac{1}{\omega_{OLP} \cdot C_1} = 26\,k\Omega$$

$$C_2 = 1\,\mu F \Rightarrow R_2 = \frac{1}{\omega_{OHP} \cdot C_2} = 840\,\Omega$$




$H_{LP}(j\omega)$ $\quad H_{buf}(j\omega)$ $\quad H_{HP}(j\omega)$

Let's see how we did:

| $\omega$ | $\lvert H_{LP}(j\omega)\rvert$ | $\lvert H_{HP}(j\omega)\rvert$ | $\lvert H(j\omega)\rvert$ | $V_{out} = V_{in}\cdot\lvert H(j\omega)\rvert$ |
| --- | --- | --- | --- | --- |
| $2\pi\cdot 60Hz$ | $\frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 60Hz}{2\pi\cdot 6kHz}\right)^2}} \simeq 1$ | $\frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 190Hz}{2\pi\, 60Hz}\right)^2}} \simeq 0.3$ | $1\cdot 0.3 = 0.3$ | $10mV\cdot 0.3 = 3mV$ |
| $2\pi\cdot 600Hz$ | $\frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 600Hz}{2\pi\cdot 6kHz}\right)^2}} \simeq 1$ | $\simeq 0.95$ | $1\cdot 0.95 = 0.95$ | $1mV\cdot 0.95 = 0.95mV$ |
| $2\pi\cdot 60kHz$ | $\frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 60kHz}{2\pi\, 6kHz}\right)^2}} = 0.1$ | $\simeq 1$ | $0.1\cdot 1 = 0.1$ | $20mV\cdot 0.1 = 2mV$ |

Wanted $\frac{1}{100}$ on $\lvert H(j\omega_{AC})\rvert$ & $\lvert H(j\omega_F)\rvert$

but only get $0.3$ & $0.1$.

Not quite enough.

Can keep going ...

$$\to \underbrace{\boxed{H_{LP}(j\omega)} \to \boxed{H_{buf}} \to \boxed{H_{HP}(j\omega)} \to \boxed{H_{buf}} \to \boxed{H_{LP}} \to \boxed{H_{buf}} \to \ldots}$$

$$H^*(j\omega) = H_{LP}^{n}(j\omega)\cdot H_{HP}^{m}(j\omega)$$

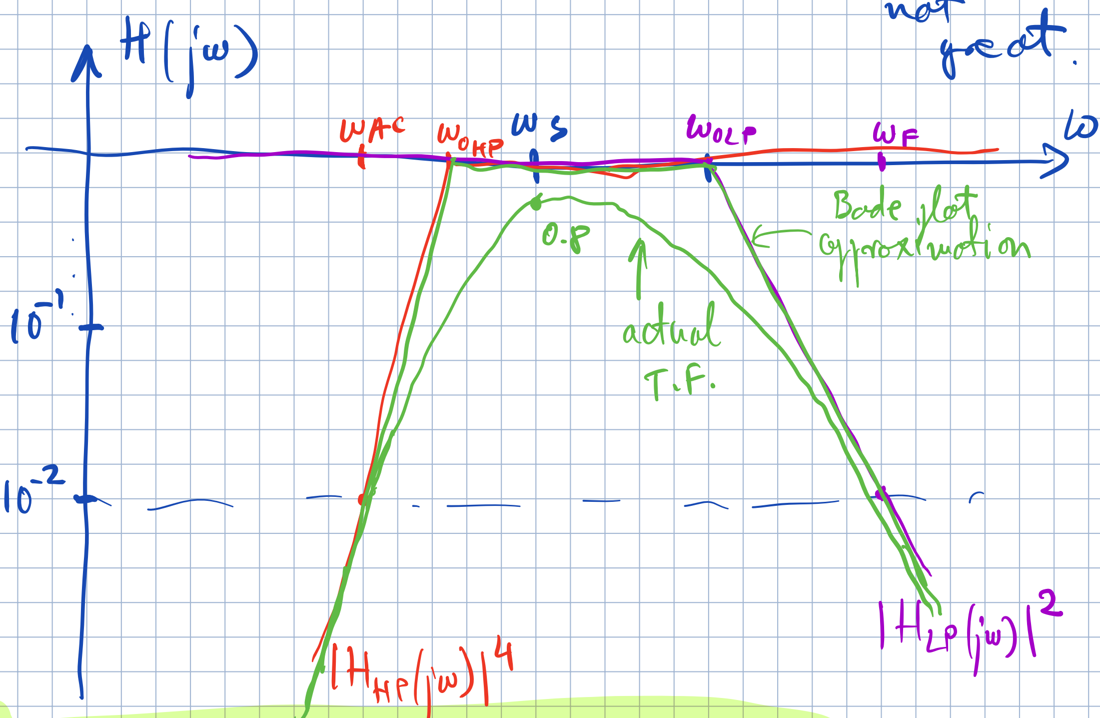
$$\lvert H^*(j\omega_F)\rvert = \frac{1}{100} = 0.1^n \cdot 1^m \Rightarrow n=2$$

$$|H^*(j\omega_{AC})| = \frac{1}{100} = 1^n \cdot 0.3^m \Rightarrow m = 4$$

check:

$$|H^*(j\omega_S)| = 1^n \cdot 0.95^m = 0.8 \quad \text{o.k.}$$

but not great.




$$\boxed{H^*(j\omega) = \frac{\left(j\frac{\omega}{\omega_{OHP}}\right)^4}{\left(1+j\frac{\omega}{\omega_{OHP}}\right)^4\left(1+j\frac{\omega}{\omega_{OLP}}\right)^2}}$$

In general:

$$H(j\omega) = K \cdot \frac{(j\omega)^{N_{zo}} \cdot \left(1+j\frac{\omega}{\omega_{z1}}\right) \cdots \left(1+j\frac{\omega}{\omega_{zn}}\right)}{(j\omega)^{N_{po}} \left(1+j\frac{\omega}{\omega_{p1}}\right) \cdots \left(1+j\frac{\omega}{\omega_{pm}}\right)}$$

# of origin zeroes

origin poles

$\omega_{zn}$ – zeroes
$\omega_{pm}$ – poles

control sys terminology 105, 120...

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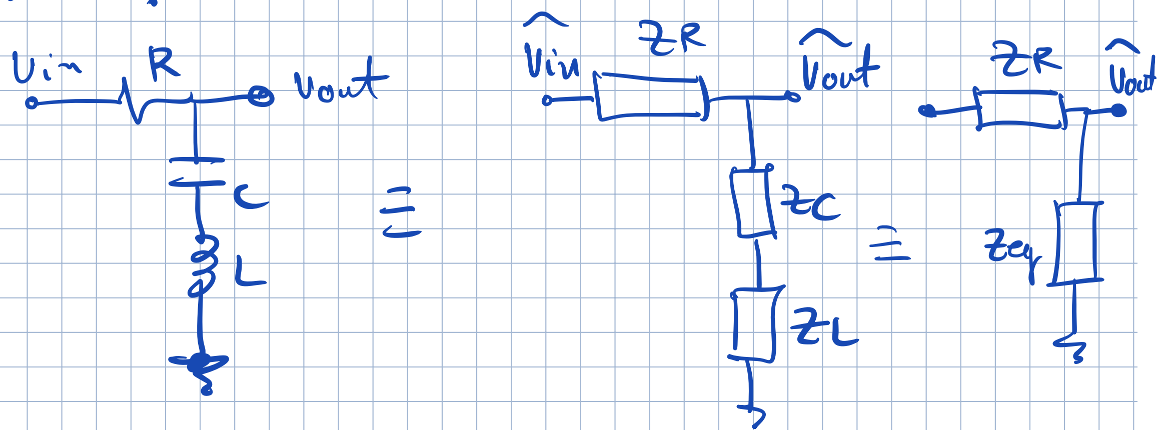
What if our desired signal is at 100 Hz? Our previous design won't work. (too close to $\omega_{AC}$)

Need a different filter!

Consider:




$$Z_{eq} = Z_C + Z_L$$

$$= \frac{1}{j\omega C} + j\omega L$$

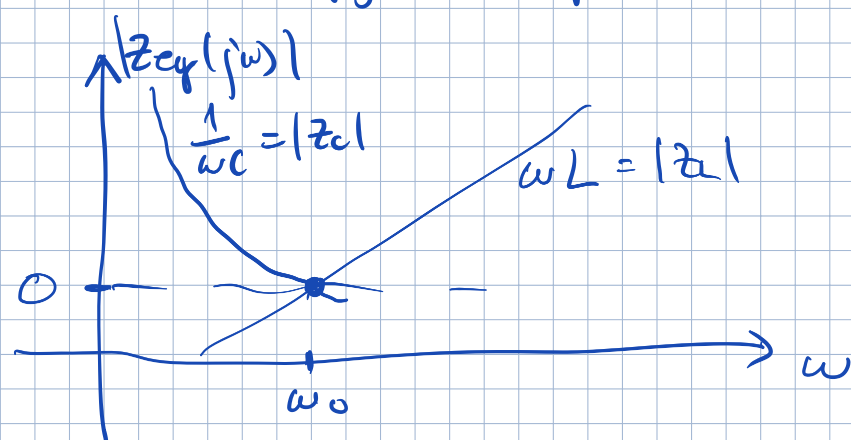
$$Z_{eq} = j\left(\omega L - \frac{1}{\omega C}\right)$$ Fantastic!

$$H(j\omega) = \frac{\tilde{V}_{out}}{\hat{V}_{in}} = \frac{Z_{eq}}{Z_{eq} + Z_R}$$

Say $Z_{eq}(j\omega_0) = 0 = j\left(\omega_0 L - \frac{1}{\omega_0 C}\right)$

$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ - resonance frequency

$$|H(j\omega_0)| = \frac{Z_{eq}(j\omega_0)}{Z_{eq}(j\omega_0) + Z_R(j\omega_0)} = 0$$




$$H(j\omega) = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

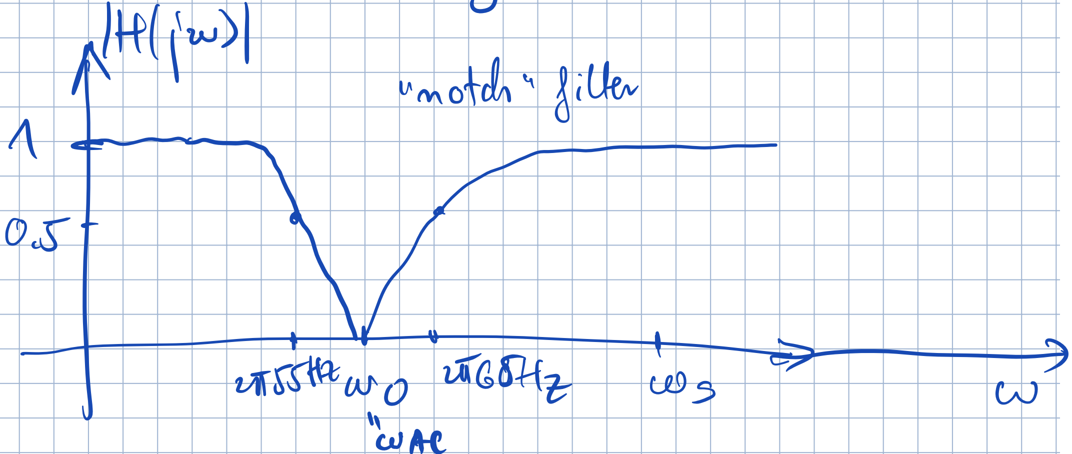
$\omega_0 = \omega_{AC} = 2\pi\, 60\,Hz$

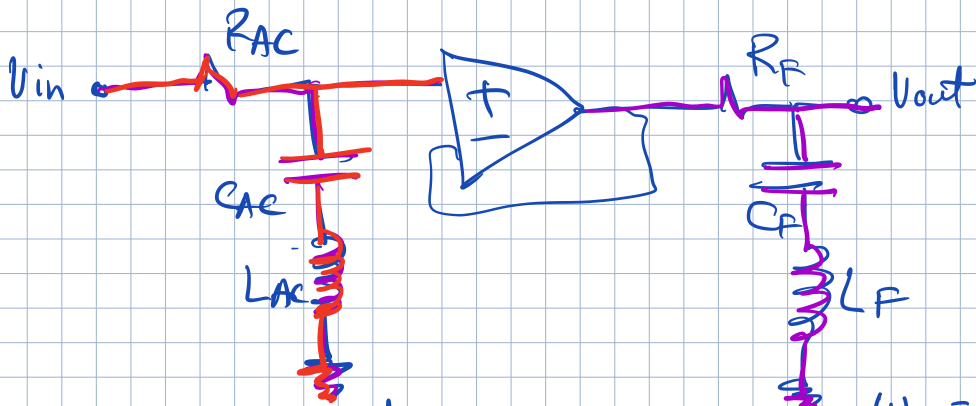
$C = 100\,\mu F$, $L = 70\,mH$

$$H(j\underbrace{\omega_0}_{2\pi\,60\,Hz}) = 0$$

$$H(j2\pi\cdot 55\,Hz) = \frac{j\cdot 3.5\,\Omega}{R + j\,3.5\,\Omega} \qquad R = 3\,\Omega$$

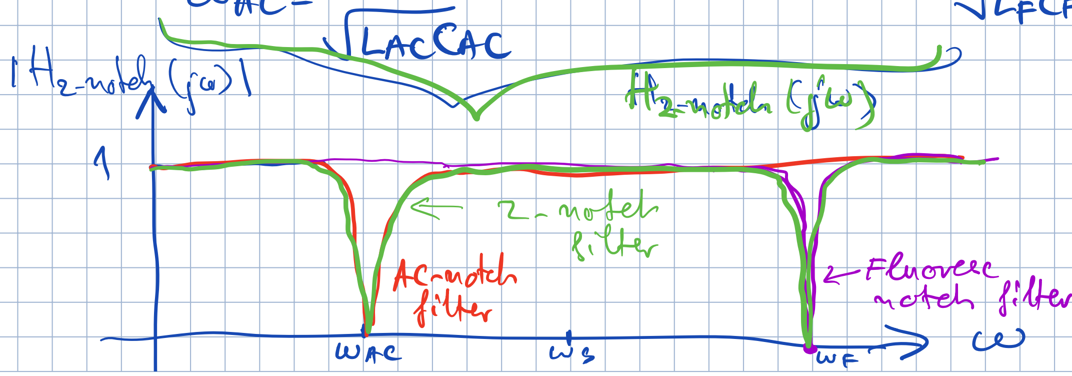
$$H(j\,2\pi\,65\,Hz) = \frac{j\,4.7\,\Omega}{R + j\,4.7\,\Omega} \qquad \approx 0.5$$




$$\omega_F = \frac{1}{\sqrt{L_F C_F}}$$

$$\omega_{AC} = \frac{1}{\sqrt{L_{AC} C_{AC}}}$$



Band-pass :