16B Make-up lecture due to zoom recording glitch ☹

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Today: Finish Uppertriangularization story: eigenvalues
Understand consequences for stability
Symmetric Matrix Case

Recall: $\tilde{A} = U^T A U$ where $U^T U = I$ since U has n orthonormal columns that form a basis for n-dim

Upper-triangular:

$$\tilde{A} = \begin{bmatrix} \lambda_1 & \cdots & \beta_{i,j} & \cdots \\ & \lambda_2 & \cdots & \vdots \\ & 0 & \lambda_3 & \cdots \\ & & & \ddots \; \lambda_n \end{bmatrix}$$

"stuff"

Upper-triangular

Let's just play: Curiosity

lower-triangular

$$\tilde{A}^T = (U^T A U)^T = (U^T A^T U) = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & 0 \\ \text{Stuff} & & \lambda_3 & \\ \beta_{j,i} & & \ddots & \lambda_n \end{bmatrix}$$

What if A were symmetric: $A = A^T$

$$\Longrightarrow \tilde{A} = \tilde{A}^T \Longrightarrow \beta_{i,j} = 0$$

$$\Longrightarrow \tilde{A} \text{ is actually diagonal } \Lambda$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix}$$

$$\tilde{A} = \Lambda = U^T A U$$

$$A = U \Lambda U^T$$

inverses of each other.

$\Longrightarrow$ $\vec{u}_i$ ← i^{th} column in U

$$A \vec{u}_i = U \Lambda U^T \vec{u}_i = U \Lambda \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$ ← i^{th} position

$$= U \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \lambda_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i^{th} \text{ position}$$

$$= \lambda_i \vec{u}_i$$

$\Longrightarrow$ $\vec{u}_i$ is an eigenvector of A with e-val λ_i

A has a full complement of eigenvectors that are all orthogonal to each other.

A has an orthonormal eigenbasis.

Any real symmetric $n \times n$ matrix with all real eigenvalues has an orthonormal eigenbasis that has n vectors in it.

Question: Must real symmetric matrices have real eigenvalues?

Next time.