# Lecture 2

* Computing: Transistors & Logic
  * RC models
  * Solving RC circuits
  * RC transients

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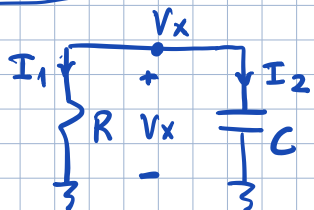


Elements:

$I_2 = C \cdot \frac{dV_x}{dt}$

$V_x = I_1 \cdot R$

KCL:

$I_1 + I_2 = 0$

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$$I_1 = \frac{V_x}{R}, \quad I_2 = C\frac{dV_x}{dt} \underset{KCL}{\Rightarrow} \frac{V_x}{R} + C\frac{dV_x}{dt} = 0$$

$t \geq 0$

$$\Rightarrow \quad \frac{dV_x}{dt} = -\frac{V_x}{RC}$$ (differential equation)

① Guess: $V_x(t) = a \cdot e^{bt}$ (educated guess based on the behavior / properties of the $\frac{d}{dt} V_x$)

$$V_x(0) = a \, \cancel{e^{b \cdot 0}}^{\,1} = a$$

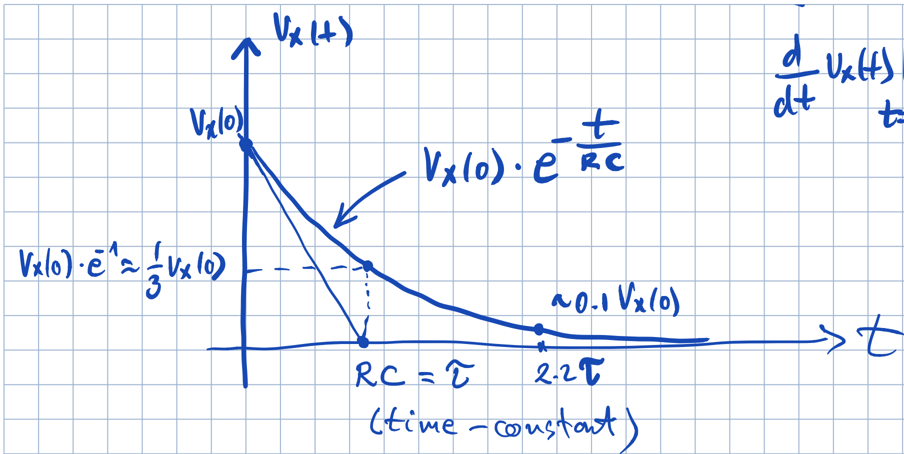
↑ initial state

$$\frac{d}{dt} V_x(t) = \frac{d}{dt}\left(a \cdot e^{bt}\right) = a \cdot b \cdot e^{bt} = b \cdot V_x(t)$$

$$\Rightarrow \quad \frac{d}{dt} V_x(t) = -\frac{V_x(t)}{RC} \text{ and } \left.\begin{matrix} \\ \frac{d}{dt} V_x(t) = b \cdot V_x(t) \end{matrix}\right\} \Rightarrow \boxed{b = -\frac{1}{RC}}$$

$$\boxed{V_x(t) = V_x(0) \cdot e^{-\frac{t}{RC}}}$$

$$\frac{d}{dt} V_x(t)\Big|_{t=0} = -\frac{V_x(0)}{RC}$$




② <u>Check for uniqness of the guess:</u>

Suppose $y(t)$ which also solves.

Shorter notation : $x(0) = X_0 \quad (1)$

$$\frac{d}{dt} x(t) = \lambda \cdot x(t) \quad (2)$$

In ① Guessed & checked that $x_d(t) = x_0 \cdot e^{\lambda t}, \quad t \geq 0$ satisfies (1) & (2).

<u>Need to show $y(t) = x_d(t)$.</u>

Either prove $\frac{y(t)}{x_d(t)} = 1$ or $y(t) - x_d(t) = 0$

$$\frac{y(t)}{x_d(t)} = \frac{y(t)}{x_0 \cdot e^{\lambda t}}$$

$\left\lceil \frac{d}{dt} y(t) = \lambda \cdot y(t) \right\rfloor$ since $y(t)$ is also a solution

$$\frac{d}{dt}\left(\frac{y(t)}{x_d(t)}\right) = \frac{d}{dt}\left(\frac{y(t)}{x_0 e^{\lambda t}}\right) = \frac{1}{x_0} \frac{d}{dt}\left(y(t)\, e^{-\lambda t}\right) =$$

$$= \frac{1}{x_0}\left(\frac{d}{dt} y(t) \cdot e^{-\lambda t} + y(t)\,(-\lambda) \cdot e^{-\lambda t}\right)$$

$$= \frac{1}{x_0}\left(\lambda\, y(t)\, e^{-\lambda t} - \lambda \cdot y(t) \cdot e^{-\lambda t}\right) = 0$$

$$\frac{d}{dt}\left(\frac{y(t)}{x_d(t)}\right) = 0 \Rightarrow \boxed{\frac{y(t)}{x_d(t)} = a} \quad (\text{constant}) \quad t \geq 0$$

From (1) $\quad x(0) = X_0$

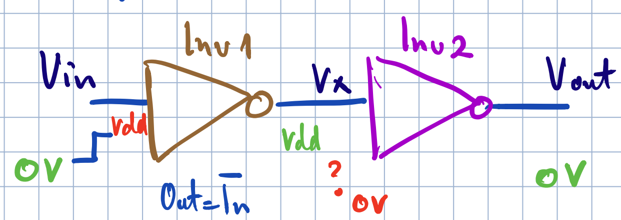
$y(0) = X_0$ since $y$ is also a solution

$$\frac{y(0)}{x_d(0)} = \frac{\cancel{y(0)} \to X_0}{X_0 \cdot \cancel{e^{\lambda 0}}} = \underbrace{1 = a}$$

$$\frac{y(t)}{x_d(t)} = 1 \qquad x_d(t) = y(t)$$

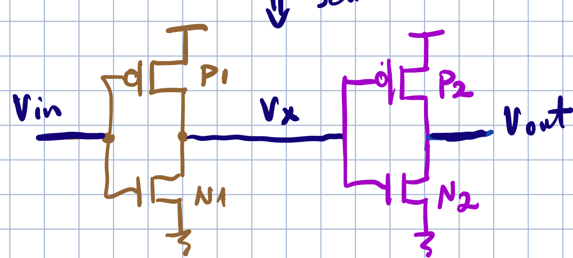
($x_d(t)$ is unique)

More complete proof in disc / hw.



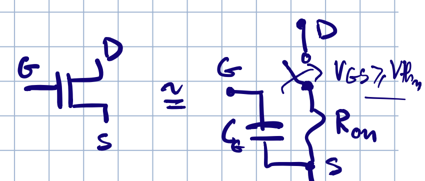

Logic view

⇓ Schematic

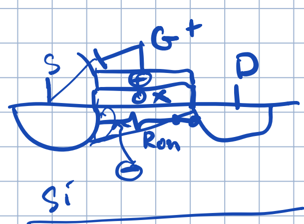


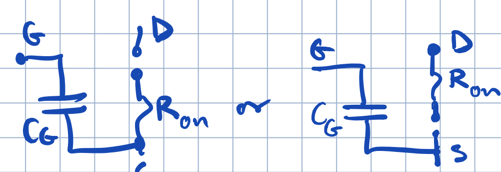

Implementation Circuit (CMOS view)

more in 151



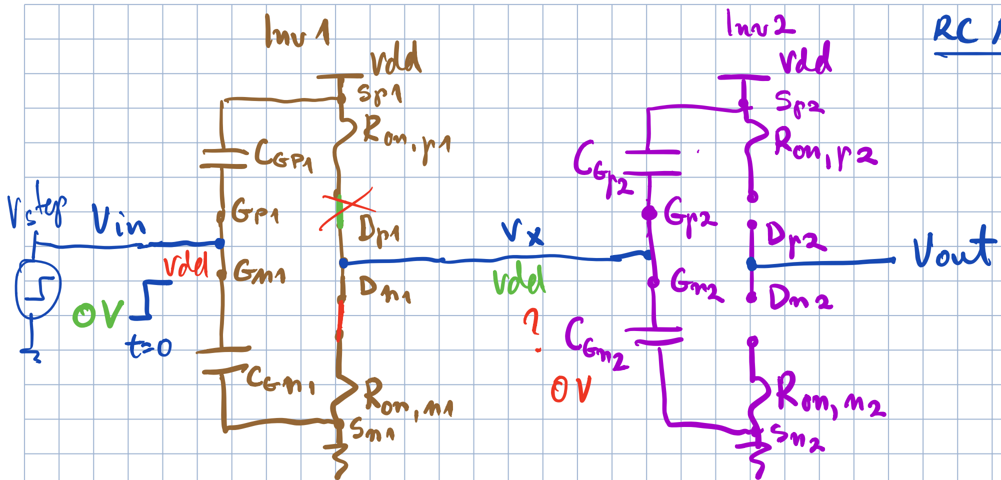

⇓ Model (RC)

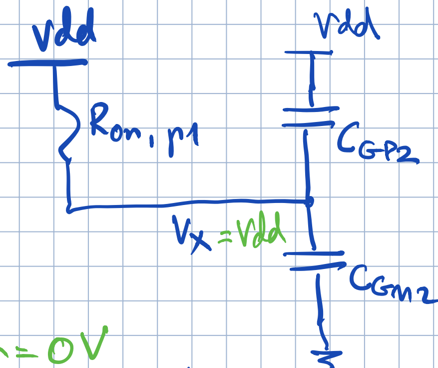



equivalent models

## RC Model view:

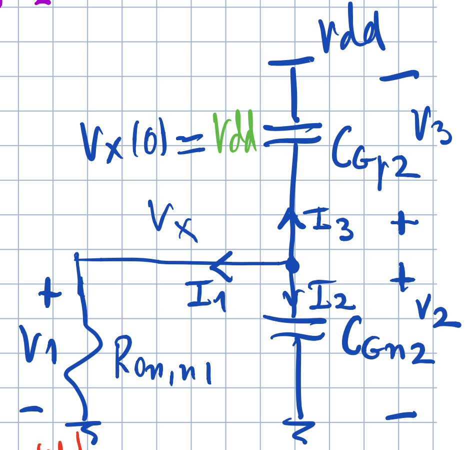



$V_{in} = 0V$

$t < 0$

$\Rightarrow$ $t = 0$




$V_{in} = Vdd$

KCL: $I_1 + I_2 + I_3 = 0$

$V_1 = V_x$

Elements: $V_1 = I_1 \cdot R_{on,n1}$

$V_2 = V_x$

$I_2 = C_{Gn2} \cdot \frac{dV_2}{dt}$

$V_3 = V_x - Vdd$

$I_3 = C_{Gp2} \frac{dV_3}{dt}$

$V_x(0) = Vdd$

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KCL: $$\underbrace{\frac{V_1}{R_{on,n1}}}_{I_1} + \underbrace{C_{Gn2} \frac{dV_2}{dt}}_{I_2} + \underbrace{C_{Gp2} \frac{dV_3}{dt}}_{I_3} = 0$$

$$\frac{V_x}{R_{on,n1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\cdot\underbrace{\frac{d}{dt}(V_x - V_{dd})}_{= \frac{d}{dt}V_x} = 0$$

$$\frac{V_x}{R_{on,n1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{dV_x}{dt} = 0$$

$$\frac{V_x}{R_{on,n1}} + (C_{Gn2} + C_{Gp2})\frac{dV_x}{dt} = 0$$

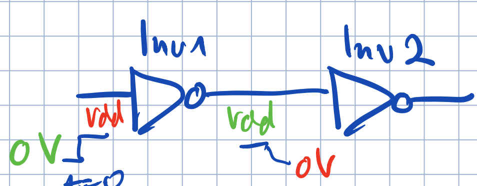
$$\frac{dV_x}{dt} = -\frac{V_x}{\underbrace{R_{on,n1}\cdot(C_{Gn2} + C_{Gp2})}_{\tau}}$$

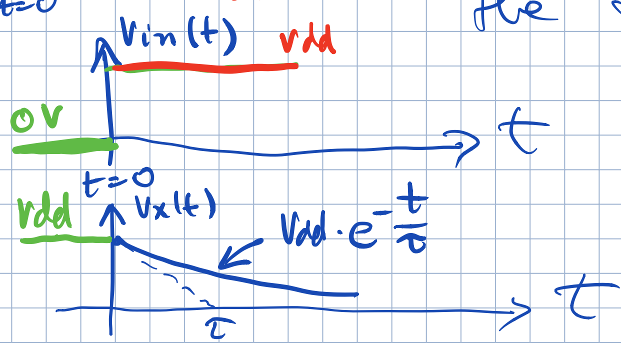
$$V_x(t) = V_x(0)\cdot e^{-\frac{t}{\tau}}, \quad \boxed{\tau = R_{on,n1}\cdot(C_{Gn2} + C_{Gp2})}$$

$$V_x(0) = V_{dd} \quad \Rightarrow \quad \boxed{V_x(t) = V_{dd}\cdot e^{-\frac{t}{\tau}}}$$

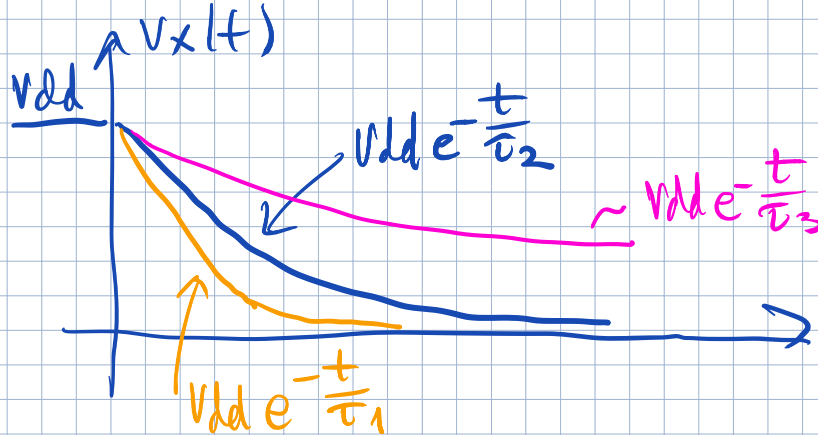
$$t \geq 0$$

determines the speed of the transition



$\tau_1 < \tau_2 < \tau_3$




$\tau = R_{on} \cdot (C_{Gn} + C_{Gp})$

For computer to be faster $\tau \downarrow$

Moore's law transistor scaling.