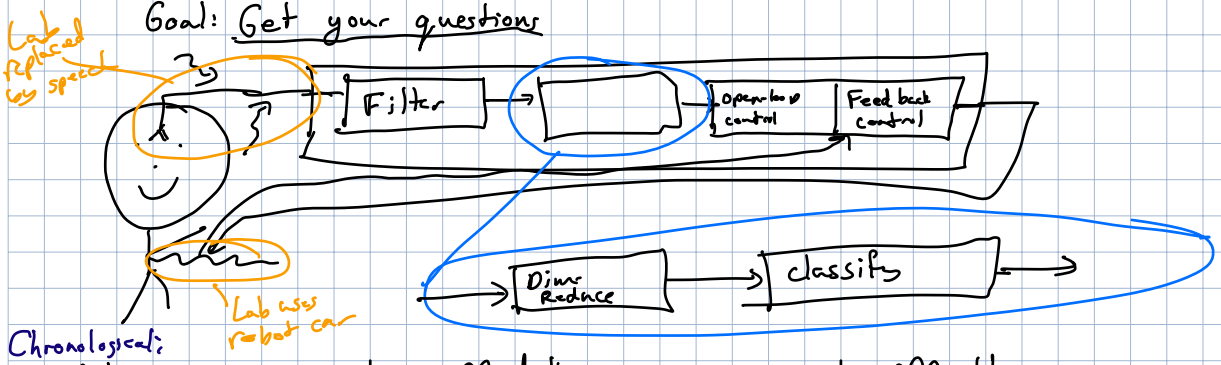
16B Prof. Anant Sahai

Announcements: HW 14 Released
Extra HW Org Given
Final Dec 17th Friday 8 AM

Today: Review: Three Views
Stick-man Cyborg + Chronological
Themes / Patterns

Goal: Get your questions



Motivation	Modeling	Math
Why don't computers work infinitely fast?	Transistors: NMOS / PMOS ↳ As switches + resistance + capacitor RC time constants.	Diff Eqs. $\frac{d}{dt}x(t) = \lambda x(t) + u(t)$ Solve by Guess/Check Backed up by uniqueness proofs Heuristics: 1) Separation of Variables ↳ Discretize & take limits. Change of variables for nonhomogenous case.
How can we filter out interference and keep the desired signals?	Idea of a filter circuit as analog information processing. Inductor as "dual" to capacitor. impedance idea: reduces circuits with cap. & inductors to what looks like resistive circuits. ← complex	Systems of Differential Eqns. Vector Change-of-variables ie. coordinate changes Eigenbasis Complex eigenvalues Complex exponentials s-phasors s-impedances phasors & impedances (jω) ⇒ Reduced System of Diff Eqs to systems of linear equations

Low pass filters
High pass filters

Magnitude plot
Phase plot

Composition of filters by cascading them through buffers.

Transfer functions

Bode Plots as approximations

How to control a robot?

Disturbances / Imperfections.
"Nature's vote"

How to get models from data?

System Identification
Model Order Selection.

How to pick using held-out data. Validation
Data not used to fit model

Controlability

Feedback to set eigenvalues.

STABILITY
(BIBO)

Discretization (Already Had)

Orthogonality
Least Squares (From 16A)

Gram-Schmidt Orthonormalization

--> Why do matrices have eigenvalues? (Fundamental Theorem of Algebra)

Upper-triangularization
↳ Solve general systems of linear diff eqs.
↳ Prove stability

Spectral Thm for Real Sym Matrices
↳ Why do Real Sym. Matrices have real eigenvalues?

How to iteratively solve least squares by treating it like a dynamical system.

Motivation	Modeling	Math.
	Minimum-energy control	Minimum Norm Solution for $Ax = b$ ↑ wide
		SVD → outer-product form; → Compact Form; Full Form
How to interpret brain signals as commands?	Mean-based classification.	Frobenius Norm
	Dimensionality Reduction.	Low-rank approximation.
	Linear models for patterns. ← PCA	
	How to validate? → Use held-out data.	Eckhart-Young-Mirsky Thm. → SVD gives low-rank approximations implicitly.
	Binary Classification by Least-Squares	

Beyond Linear Models

	$\frac{d}{dt}\vec{x}(t) = \vec{F}(\vec{x}, \vec{w}) + \vec{w}(t)$	Partial Derivatives.
How to control a nonlinear system?	Equilibrium Points ←	Jacobians
	Local Approximation.	Hessians
	Linear / Quadratic	Need to solve $\vec{F}(\vec{x}, \vec{w}) = \vec{0}$
		Newtons Method. (Pretend locally linear)
How to understand nonlinear circuit elements?	small-signal gain.	Iterated Least Squares
		Gradient Descent.
How to do classification better?	Loss function selection	

exponential loss
Logistic loss.

How to make a filter given examples?

Automatic Differentiation.

Complex Inner Product

$\vec{x}^*$ ← conjugate transpose

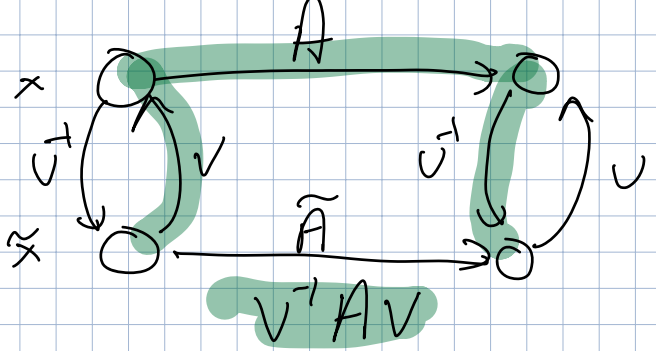
Another Perspective: By Themes & Patterns instead of chronological!

(1) Reductions: Turn a problem you don't yet know how to solve into a problem you know how to solve.

(2) Changing Coordinates/Variables: In what coordinates is this problem the most transparent? (or easy to solve)

Additive Shifts e.g. Let $\tilde{x} = x + 7$

Coordinate Changes



Eigenbasis: Reduces to parallel scalar problems. (when it exists)

Orthonormal bases: Preserve lengths, "angles",
Easy inverses (just conjugate transpose)
Easy Projections

Upper-triangularization: Orthonormal + Always exists.
(For square matrices)
Reduces to chained scalar problems.

SVD : Works of non-square matrices.
Orthonormality.
Reveals nested seq of low-rank approximations.

"Local Coordinates" — Possibly valid in a neighborhood
↳ "small errors"
↓ ⟶ Allow problems to be viewed as linear
Also what is done in trajectory tracking.

Phasors are what mix (spiritually?)
⟶ Coordinate Change
⟶ Eigenbasis-like.
(Out-of-scope) ⟶ Orthonormal.
Spiritually Local.

(3) Iteration : Two types.
a) "Induction Style" ⟶ Makes problems smaller in a counting sense

e.s. Gaussian Elimination
Gram-Schmidt.
Upper triangularization.

b) "Convergence style" — Moves us closer (← continuous sense.) to a solution at each iteration.

e.s. Iteration to solve least-squares
(Reduces problem into a control-type dynamical system)

Newtons Iterations
Iterated Least Squares to solve nonlinear nonquadratic optimization
Gradient Descent.

(4) Approximation:

Critical idea if "disturbances" in models.

a) Global (Valid everywhere)
e.s. Bode Plots
Low-rank Approximation.
System-Id.

b) Local: Valid in a neighborhood
Linearization.

(5) Learning From Data. System-ID.
PCA & dimensionality reduction
Classification

(6) Stability: rejects disturbances
converging 

(7) Optimization View: Least-Squares Min-Norm
Low-rank approx.
Classification using general loss