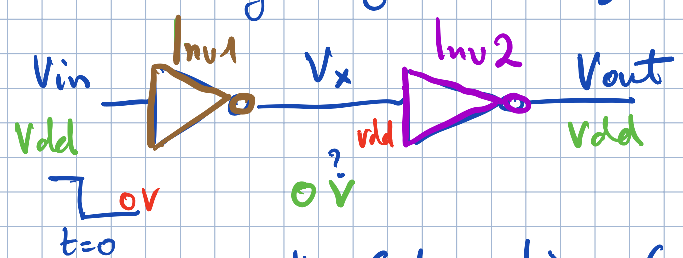
Lecture 3

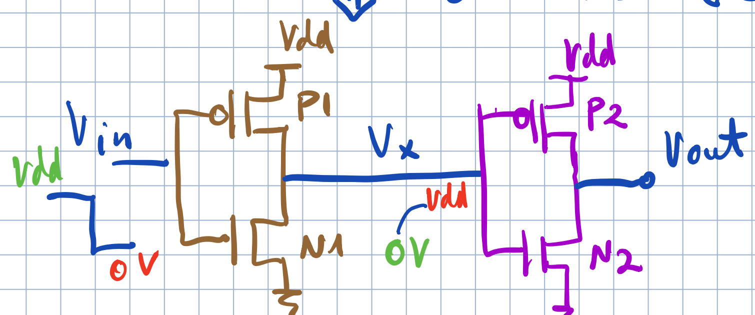
* Computing : Transistors & Logic
  * RC transients
  * Non-homogeneous diff. eqns
    * constant input
    * piece-wise constant input
    * Continuous input
* sensing

Cascading logic example:

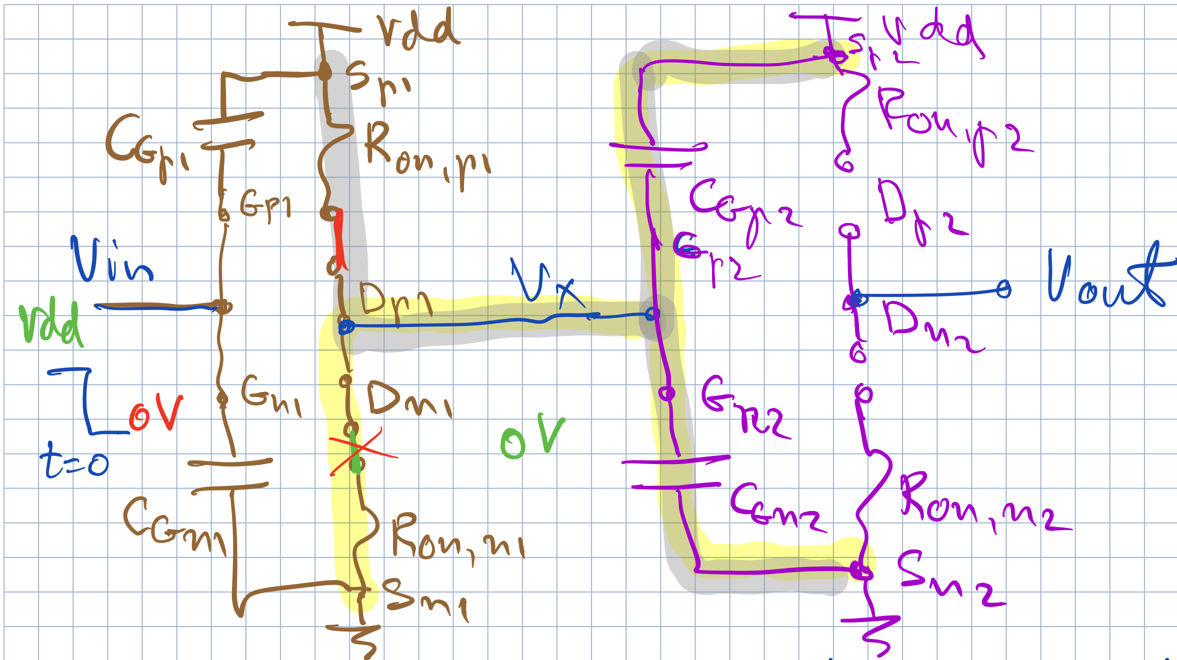


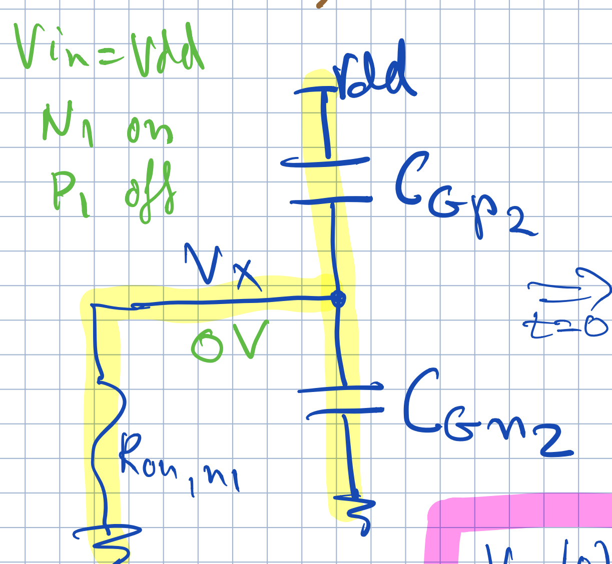
Logic view

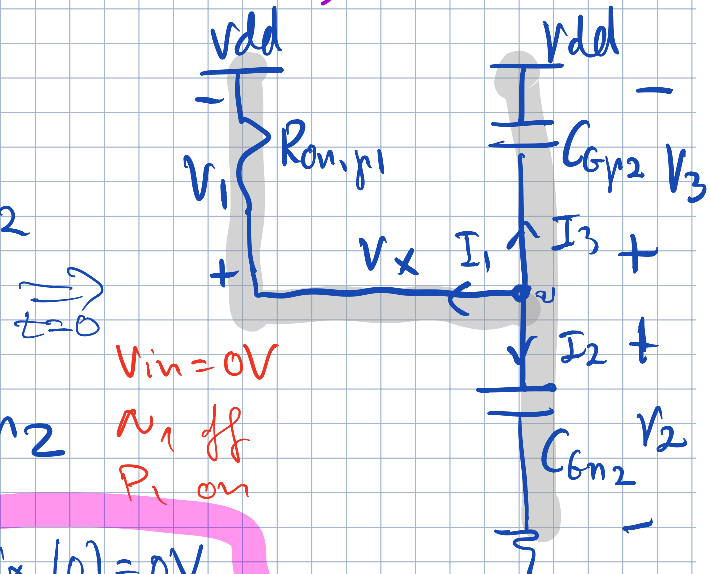
⇓ Schematic (CMOS logic)



⇓ RC model







$$\boxed{V_x(0) = 0V}$$

KCL: $I_1 + I_2 + I_3 = 0$

Elements:

$V_1 = I_1 \cdot R_{on,p1}$ $\qquad V_1 = V_x - V_{dd}$

$I_2 = C_{Gn2} \cdot \frac{dV_2}{dt}$ $\qquad V_2 = V_x$

$I_3 = C_{Gp2} \cdot \frac{dV_3}{dt}$ $\qquad V_3 = V_x - V_{dd}$

$$\underset{\uparrow \atop I_1}{\frac{V_1}{R_{on,p1}}} + \underset{\uparrow \atop I_2}{C_{Gn2}\frac{dV_2}{dt}} + \underset{\uparrow \atop I_3}{C_{Gp2}\frac{dV_3}{dt}} = 0$$

$$\frac{V_x - V_{dd}}{R_{on,p1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{d}{dt}(V_x - V_{dd}) = 0$$

$$\frac{V_x - V_{dd}}{R_{on,p1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{dV_x}{dt} = 0 \quad \left[\frac{d}{dt}V_{dd} = 0\right]$$

(1) $$\frac{V_x - V_{dd}}{R_{on,p1}} + (C_{Gn2} + C_{Gp2})\frac{dV_x}{dt} = 0$$

$$\frac{dV_x}{dt} = \underbrace{-\frac{V_x}{R_{on,p1}(C_{Gn2}+C_{Gp2})}}_{\text{homogeneous term}} + \underbrace{\frac{V_{dd}}{R_{on,p1}(C_{Gn2}+C_{Gp2})}}_{\text{non-homogenous}}$$

[Form:
$$\frac{d}{dt}x(t) = \lambda x(t) + a \quad \text{(non-homogeneous)}$$
$$\frac{d}{dt}x(t) = \lambda x(t) \quad \text{(homogeneous)}$$
]

Going back to (1)

$$\frac{V_x - V_{dd}}{R_{on,p1}} + (C_{Gn2} + C_{Gp2}) \frac{d}{dt}(V_x - V_{dd}) = 0$$

$$\hat{V}_x = V_x - V_{dd}$$

$$\Downarrow$$

$$\frac{\hat{V}_x}{R_{on,p1}} + (C_{Gn2} + C_{Gp2}) \frac{d}{dt} \hat{V}_x = 0$$

$$\boxed{\frac{d\hat{V}_x}{dt} = - \frac{\hat{V}_x}{R_{on,p1}(C_{Gn2} + C_{Gp2})}}$$

homogeneous diff. eqn.

$$\tau = R_{on,p1}(C_{Gn2} + C_{Gp2})$$

$\Rightarrow$ I know how to solve it 😊

$$\hat{V}_x(t) = \hat{V}_x(0) e^{-\frac{t}{\tau}}$$

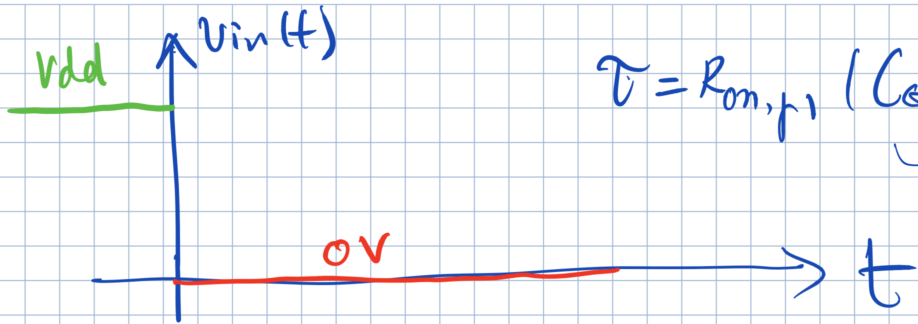
$\hat{V}_x = V_x - V_{dd}$

$$V_x(t) - V_{dd} = (\cancel{V_x(0)}^{\,0} - V_{dd}) e^{-\frac{t}{\tau}}$$

$V_x(0) =$

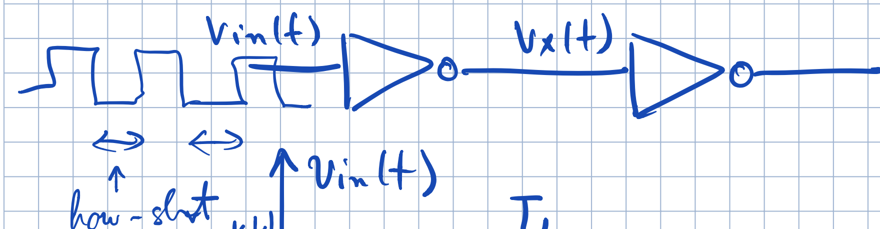
$$\boxed{V_x(t) = V_{dd}\left(1 - e^{-\frac{t}{\tau}}\right)}$$

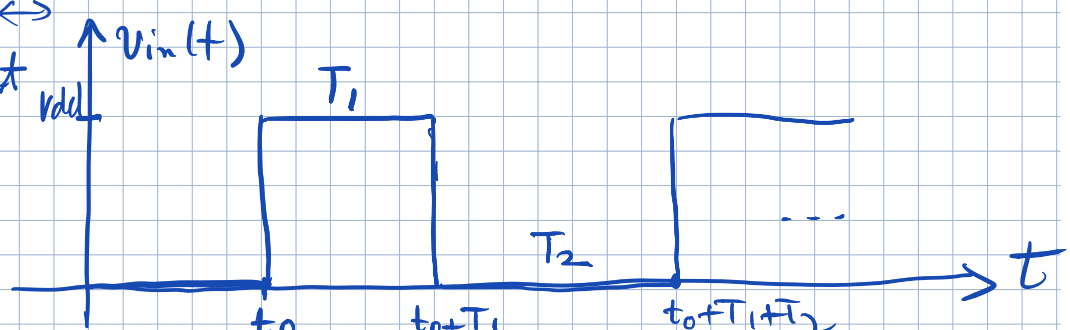
$t \geq 0$



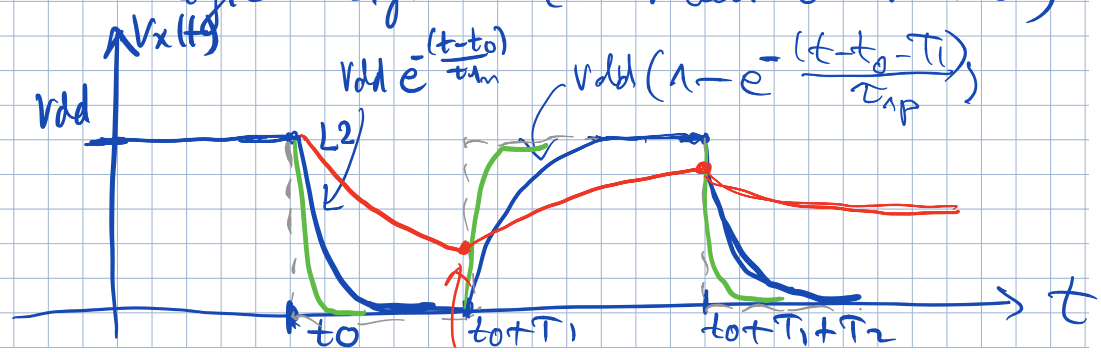
$$\tau = R_{on,p1}\,(\underbrace{C_{Gn2}+C_{Gp2}}_{\text{loading capacitance}})$$

$$\tau_1 < \tau_2 < \tau_3$$





Will $V_x(t)$ be able to follow these changes as a "logic" signal (to reach 0V or Vdd)



$\tau_{2n} < \tau_{1n} < \tau_{3n}$

$\tau_{2p} < \tau_{1p} < \tau_{3p}$

init. condition is $V_x(t_0+T_1)$ for $t \geq t_0+T_1$

$\geq t_0+T_1+T_2$

Use the previous $U_x(t)$ solution as an initial value for the next interval.

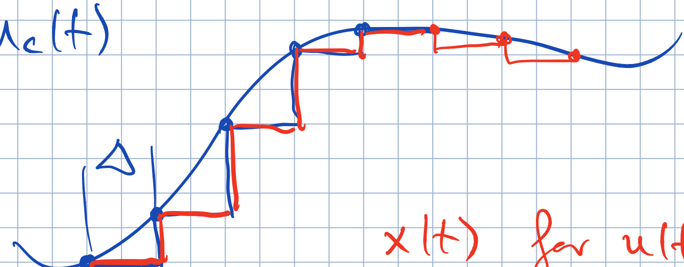
Solutions for the piece-wise constant input.

Form:

$$\frac{d}{dt}x(t) = \lambda x(t) - \lambda u(t) \Leftarrow$$ constant or piece-wise constant

want to solve for $u(t) = u_c(t)$ ↑ continuous in time

$u_c(t)$



$u(t) \neq u_c(t)$

$u(t) \rightsquigarrow u_c(t)$, $\Delta \to 0$

$x(t)$ for $u(t)$

$u(t) = const$, $t \in [t_0, t_0+\Delta)$

iterate & use the previous solution as initial condition ...

in the end $\Delta \longrightarrow 0$ (disc + lim)

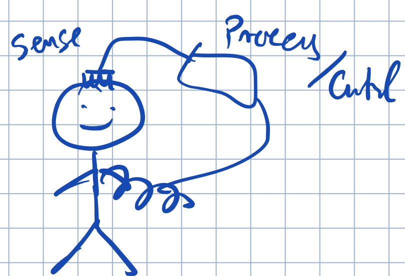
Why do we want to know

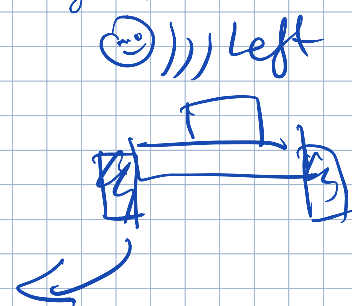
the response to continuous input ?

EECS (16A/B) Pipeline

Analog world → Sense → Process → Actuate Control → Analog world

digital

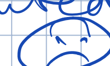




Sensing:

brain signals or voice signals

signal of interest + ~~interference (AC power, wifi/cellular RF, FM radio ...) others speaking, unwanted~~



filter this

Goal: Filter (select) signals of interest

How can a circuit become a filter / process the signal ?