Discussion 0B

1. KVL/KCL Review

Use Kirchhoff's Laws on the circuit below to find V_x in terms of $V_{\text{in}}, R_1, R_2, R_3$.

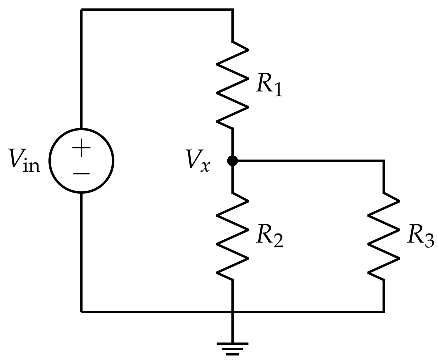


Figure 1: Example Circuit

(a) Recall Node Voltage Analysis (NVA). Determine V_x by labeling the circuit and writing equations to solve a system of equations in node voltages.

Solution:

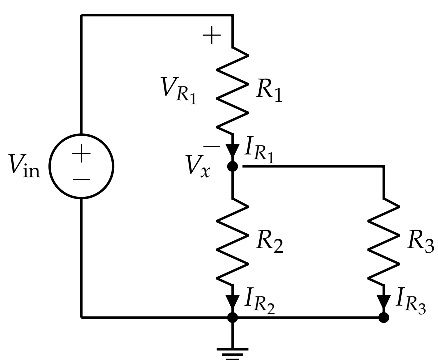


Figure 2

Applying KCL to the node at V_x, we get

$$\frac{V_{\text{in}} - V_x}{R_1} - \frac{V_x - 0}{R_2} - \frac{V_x - 0}{R_3} = 0 \tag{1}$$

Solving this equation for V_x yields

$$V_x = V_{\text{in}} \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \tag{2}$$

(b) In EECS16A, you learned you can simplify analysis by replacing series or parallel resistors with equivalents and memorizing common circuit design blocks. Determine V_x by leveraging resistor equivalence and recognition of a design block.

Solution: Observe that R_2 and R_3 are in parallel. We can replace them with a single resistor of value $R_{\text{eq}} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$ in between the ground and the node with node voltage V_x.

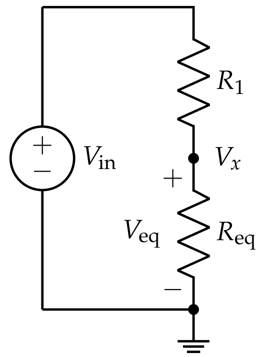


Figure 3

This circuit is a voltage divider. We can determine V_{eq} as a function of the resistances and the source, then relate V_x to V_{eq}.

$$V_{\text{eq}} = \frac{R_{\text{eq}}}{R_1 + R_{\text{eq}}} V_{\text{in}} \tag{3}$$

$$V_{\text{eq}} = V_x - 0 \tag{4}$$

$$V_x = \frac{\frac{R_2 R_3}{R_2 + R_3}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} V_{\text{in}} \tag{5}$$

$$V_x = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} V_{\text{in}} \tag{6}$$

(c) As a check, as $R_3 \to \infty$, what is V_x for what you found in (a) and (b)? The V_x's of each part should approach the same value. What is the name we used for this type of circuit?

Solution: The expressions we show above for (a) and (b) are identical. However, if they do not appear identical without simplification, the limit can be a way to verify that both solution methods inform us of the correct circuit behavior and are consistent with each other. As $R_3 \to \infty$, the $R_1 R_2$ term on the denominator will become insignificant, simplifying our expression.

$$\begin{aligned}
\lim_{R_3 \to \infty} V_x &= \lim_{R_3 \to \infty} V_{\text{in}} \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\
&= \lim_{R_3 \to \infty} V_{\text{in}} \frac{R_2}{\frac{R_1 R_2}{R_3} + R_1 + R_2} \\
&= V_{\text{in}} \frac{R_2}{R_1 + R_2}
\end{aligned}$$

When $R_3 \to \infty$, it effectively becomes an open circuit, which makes the rest of the circuit a resistive voltage divider.

2. Current Sources And Capacitors

(Adapted from EECS16A Fall 20 Disc 9A.)

(HINT: Recall charge has units of Coulombs. (C)*, and capacitance is measured in Farads.* (F)*. Also,* $1\,\text{F} = \frac{1\,\text{C}}{1\,\text{V}}$*. It may also help to note metric prefix examples:* $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$*.)*

Given the circuit in fig. 4, **find an expression for $v_{\text{out}}(t)$ in terms of I_S, C, V_0, and t**, where V_0 is the initial capacitor voltage at $t = 0$.

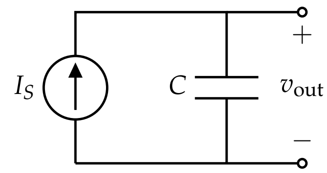


Figure 4: A current source attached to a capacitor.

Then **plot the function $v_{\text{out}}(t)$ over time on the graph below for each set of conditions, detailed below.**

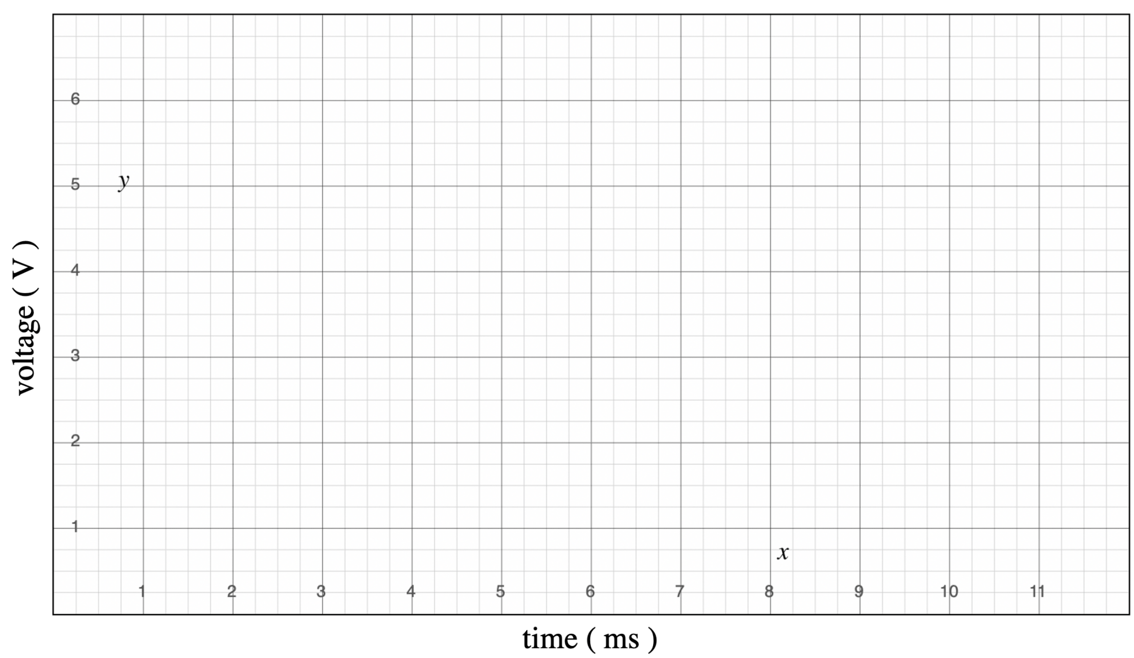
Use the values $I_S = 1\,\text{mA}$ and $C = 2\,\mu\text{F}$.

(1) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(2) Capacitor has been charged with $V_0 = 1.5\,\text{V}$ at $t = 0$.

(3) **(PRACTICE)** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

(HINT: Recall the calculus identity $\int_a^b f'(x)\,\mathrm{d}x = f(b) - f(a)$*, where* $f'(x) = \frac{\mathrm{d}f}{\mathrm{d}t}$*.)*



Solution: The key here is to exploit the capacitor equation by taking its time-derivative.

$$Q = Cv_{\text{out}} \longrightarrow \frac{\mathrm{d}Q}{\mathrm{d}t} \equiv I_C = I_s = C\frac{\mathrm{d}v_{\text{out}}}{\mathrm{d}t}. \tag{7}$$

From here we can rearrange and show that:

$$\frac{\mathrm{d}v_{\text{out}}}{\mathrm{d}t} = \frac{I_s}{C} \tag{8}$$

Thus the voltage has a constant slope!

Our solution is

$$v_{\text{out}}(t) = V_0 + \left(\frac{I_s}{C}\right)t \tag{9}$$

Formally, we are solving a differential equation that happens to return a linear function for $v_{\text{out}}(t)$:

$$\frac{\mathrm{d}v_{\text{out}}}{\mathrm{d}t} = \frac{I_s}{C} \tag{10}$$

$$\int_0^t \frac{\mathrm{d}v_{\text{out}}}{\mathrm{d}t}\,\mathrm{d}t = v_{\text{out}}(t) - v_{\text{out}}(0) \tag{11}$$

$$\int_0^t \frac{I_s}{C}\,\mathrm{d}t = \frac{I_s}{C}\int_0^t 1\,\mathrm{d}t \equiv \frac{I_s}{C}t \tag{12}$$

Thus we arrive at the same statement as seen earlier: $v_{\text{out}}(t) = v_{\text{out}}(0) + \left(\frac{I_s}{C}\right)t$.

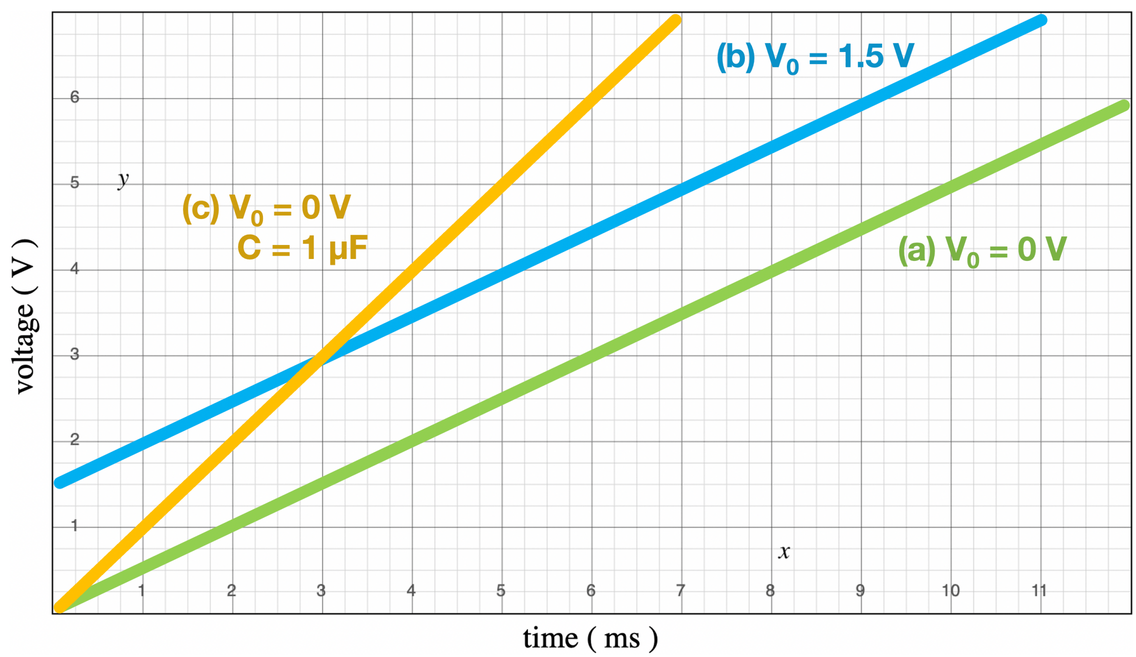
From this stage we can compute the slope of $v_{\text{out}}(t)$ in each scenario.

$$\frac{I_s}{C} = \frac{1\,\text{mA}}{2\,\mu\text{F}} = \frac{1000\,\frac{\mu\text{C}}{\text{s}}}{2\,\frac{\mu\text{C}}{\text{V}}} = 500\,\frac{\text{V}}{\text{s}} = 0.5\,\frac{\text{V}}{\text{ms}} \tag{13}$$

For part (c):

$$\frac{I_s}{C} = \frac{1\text{mA}}{1\mu\text{F}} = \frac{1000\frac{\mu\text{C}}{\text{s}}}{1\frac{\mu\text{C}}{\text{V}}} = 1000\frac{\text{V}}{\text{s}} = 1\frac{\text{V}}{\text{ms}} \tag{14}$$

When plotting, make sure to recall (a) and (c) start at the origin, while (b) has initially charged plates by $V_0 = 1.5V$. Results are shown below:



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3. Op-Amp Summer

Consider the following circuit (assume the op-amp is ideal):

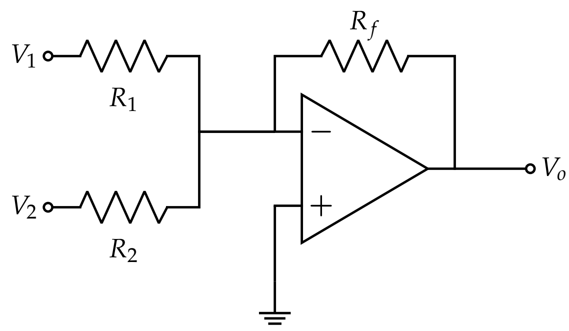

Figure 5: Op-amp Summer

What is the output V_o in terms of V_1 and V_2? You may assume that R_1, R_2, and R_f are known.

Solution:

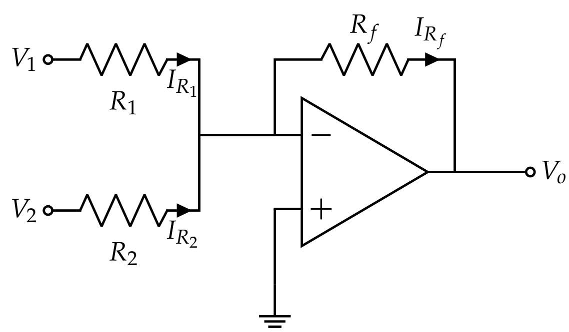

Figure 6

Let I_- be the current flowing into the (-) terminal of the op-amp

$$I_{R_f} + I_- = I_{R_1} + I_{R_2} \tag{15}$$

$$I_{R_f} + 0 = I_{R_1} + I_{R_2} \tag{16}$$

$$\frac{0V - V_o}{R_f} + 0A = \frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} \tag{17}$$

$$V_o = -\left(\frac{R_f}{R_1} \cdot V_1 + \frac{R_f}{R_2} \cdot V_2\right) \tag{18}$$

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