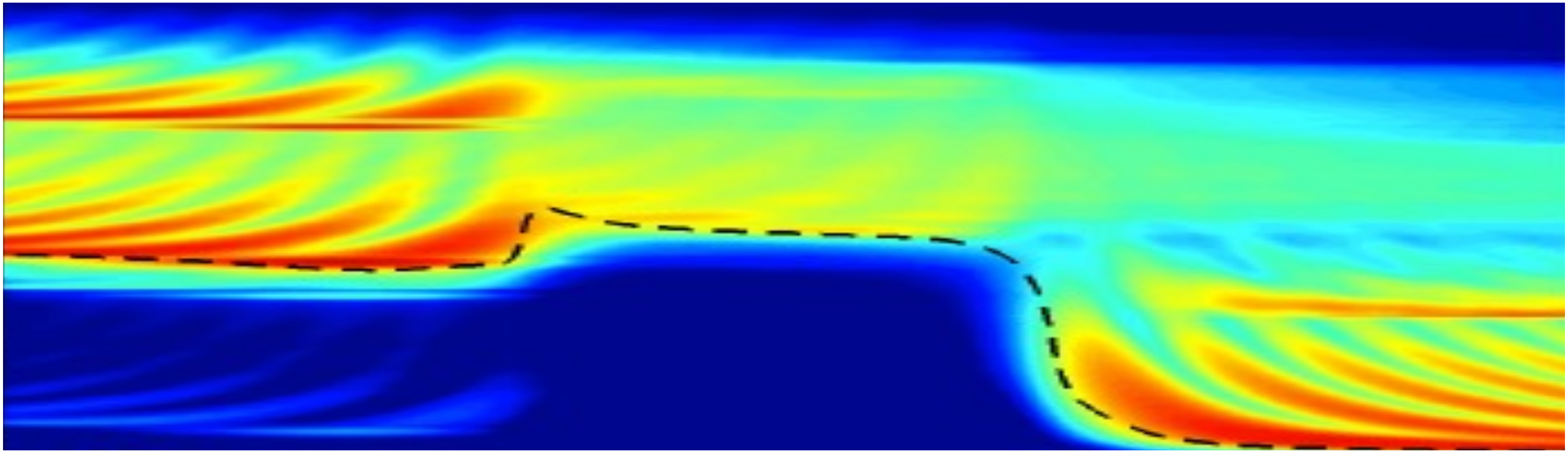

EECS 16B
Designing Information Devices and Systems II
Lecture 2

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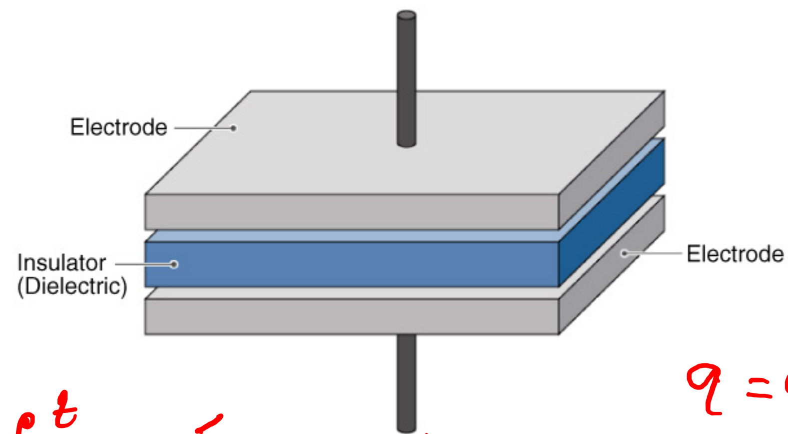
Reacp:Current flow in a Capacitor

Current flow through a capacitor is proportional to the rate of change in potential difference of the plates

$$i = C\frac{dv}{dt}$$



Time Varying Voltage

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\ dt + v(t_0)$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

$$q = Cv$$

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$$i = C\frac{dv}{dt}$$

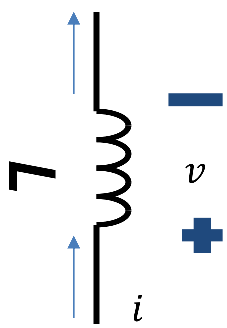
Also, from the definition of current

$$i = \frac{dq}{dt}$$

Time Varying Charge

$$q(t) = \int_{t_0}^{t} i\ dt + q(t_0)$$

Recap: Current in an Inductor



$$v(t) = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v \ dt + i(t_0)$$

Remember the capacitors

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i \ dt + v(t_0)$$

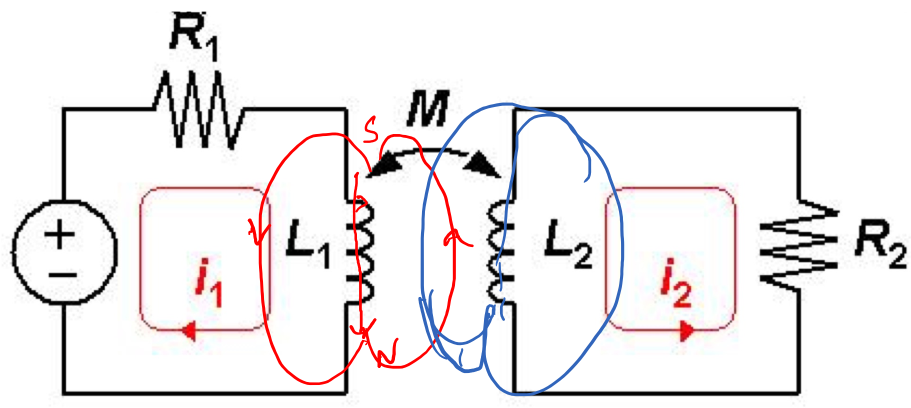
Outline

- Outline
  - Mutual Inductance
  - DAC and ADC
  - R-C circuits
  - R-L circuits
  - Steady State

- Reading: Section 3.6, 4.1-4.4, Slides

Mutual Inductance

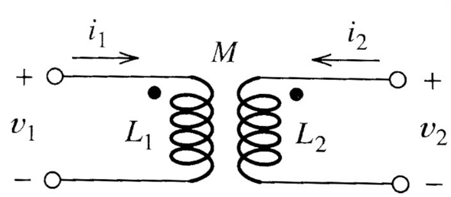
- Mutual inductance occurs when two windings are arranged so that they have a mutual flux linkage
- The change in current in one winding causes a voltage drop to be induced in the other



Transformers (adapters), motors, generators (electric cars)

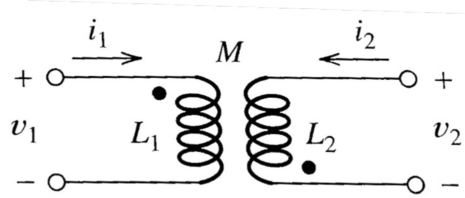
The Dot Convention

- If a current enters the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is positive at its dotted terminal.
- If a current leaves the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is negative at its dotted terminal.
- Total voltage induced in a coil is a summation of its own induced voltage and the mutually induced voltage



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Summary

Capacitors:

$$i = C\frac{dv}{dt}$$

$$w = \frac{1}{2}Cv^2$$

- v cannot charge instantaneously
- i **can** charge instantaneously (do not short circuit a charged capacitor)
- N capacitors in series $\frac{1}{C_{eq}} = \sum_{i=1}^{N}\frac{1}{C_i}$
- N capacitors in parallel $C_{eq} = \sum_{i=1}^{N}C_i$

Inductors:

$$v = L\frac{di}{dt}$$

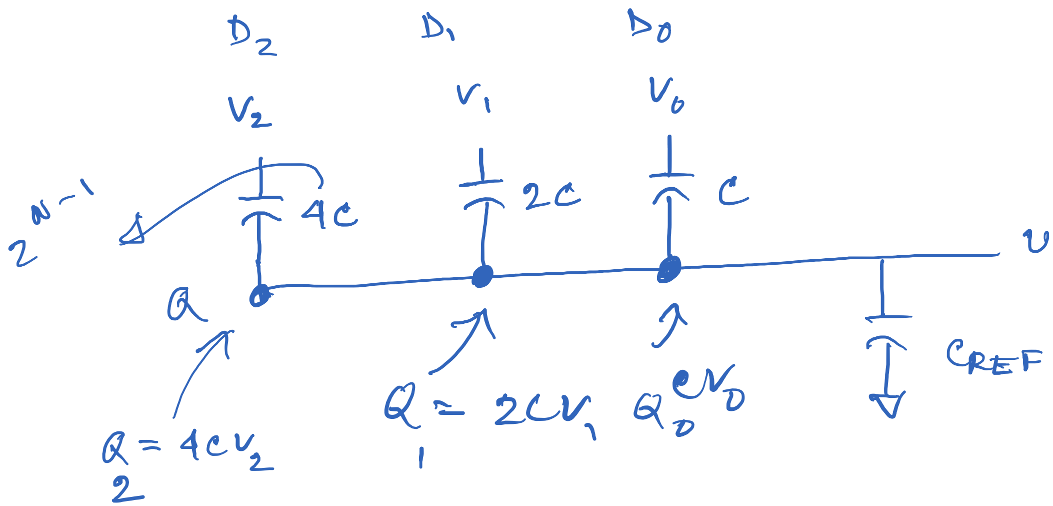
$$w = \frac{1}{2}Li^2$$

- i cannot charge instantaneously
- v **can** charge instantaneously (do not open an inductor with current)
- N inductors in series $L_{eq} = \sum_{i=1}^{N}L_i$
- N inductors in parallel $\frac{1}{L_{eq}} = \sum_{i=1}^{N}\frac{1}{L_i}$

Simple Circuits

[3 bits]

CAP DAC



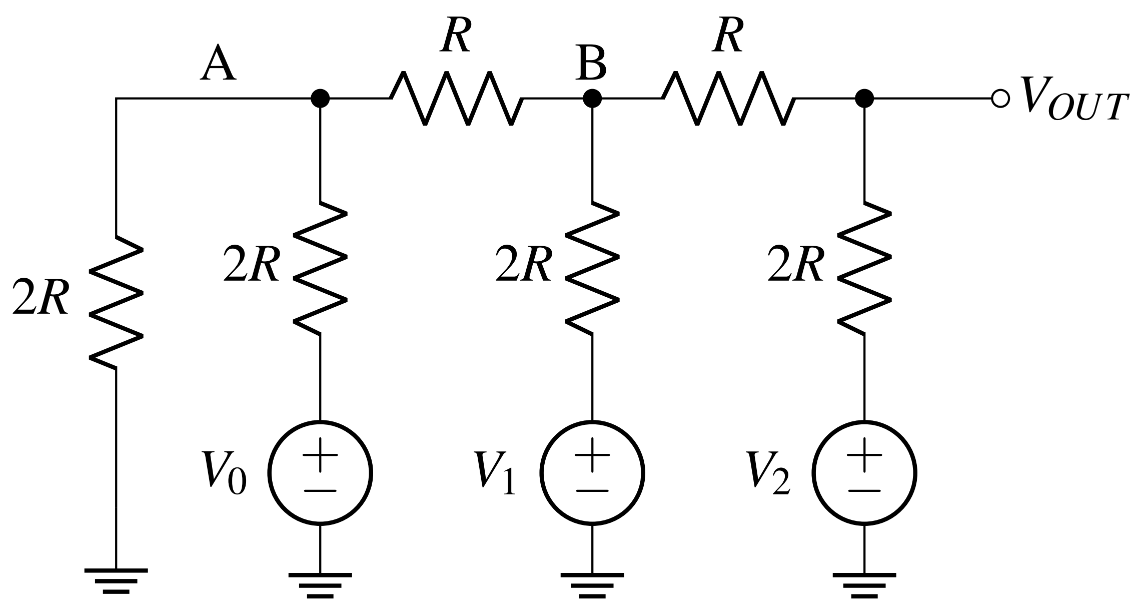
$$V_{out} = \frac{Q}{C_{REF}} = \frac{Q_2 + Q_1 + Q_0}{C_{REF}}$$

$$= \frac{4CV_2 + 2CV_1 + V_0 C}{C_{REF}}$$

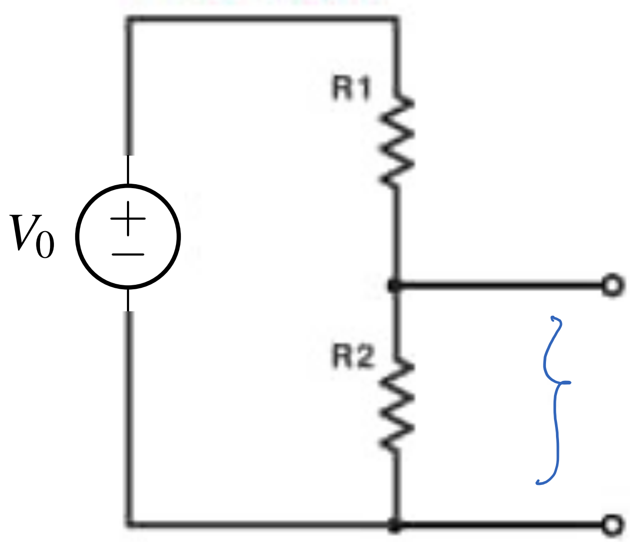
$$= \frac{C}{C_{REF}} \underbrace{\left(4\overset{1}{V_2} + 2\overset{0}{V_1} + \overset{1}{V_0}\right)}_{5}$$

101 ≈ 5

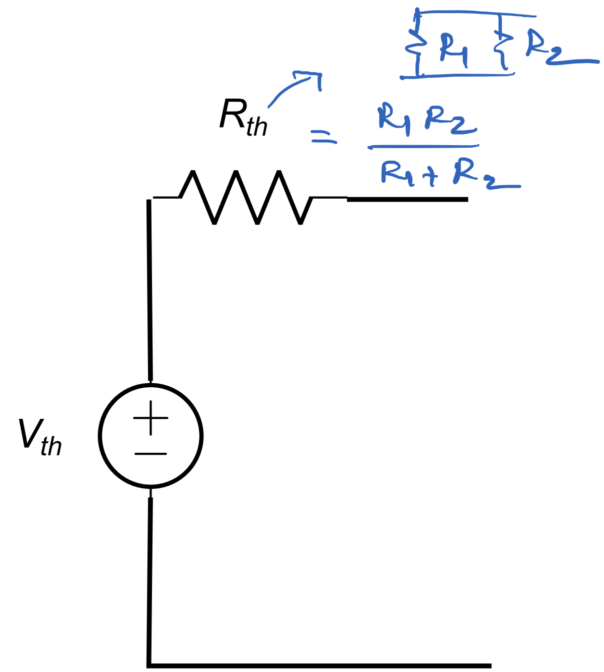
R-2R Ladder Digital-to-Analog Converter



Remember Superposition and Equivalence?



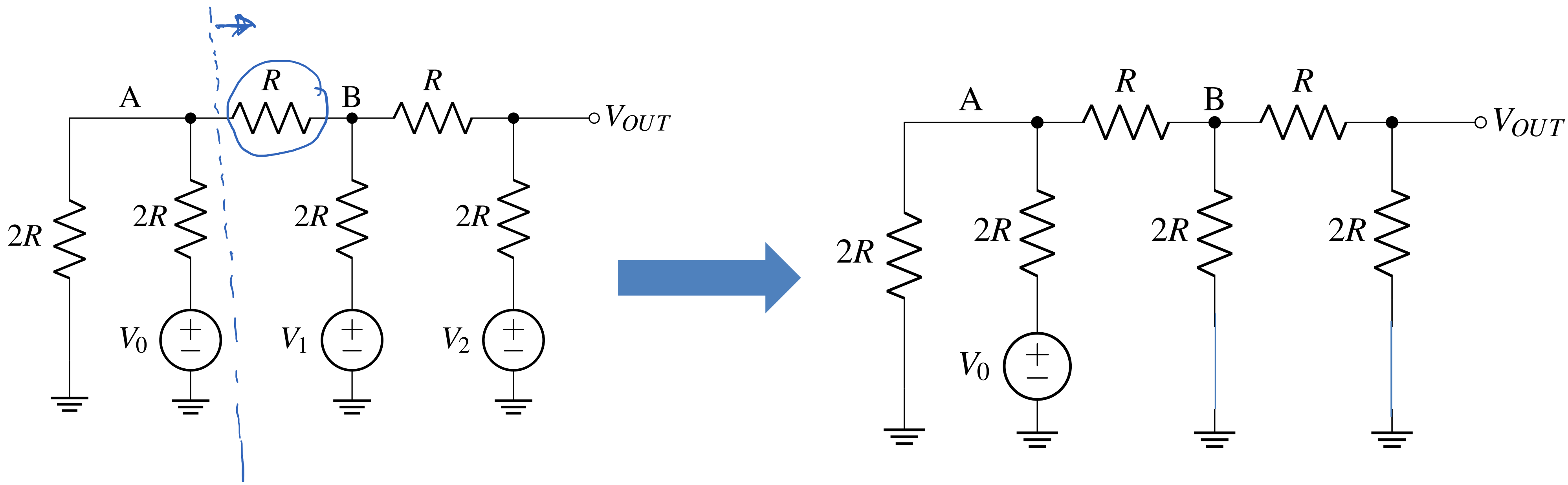
$$V_{th} = \frac{R_2}{R_1 + R_2} V_0$$



Thevenin Equivalent

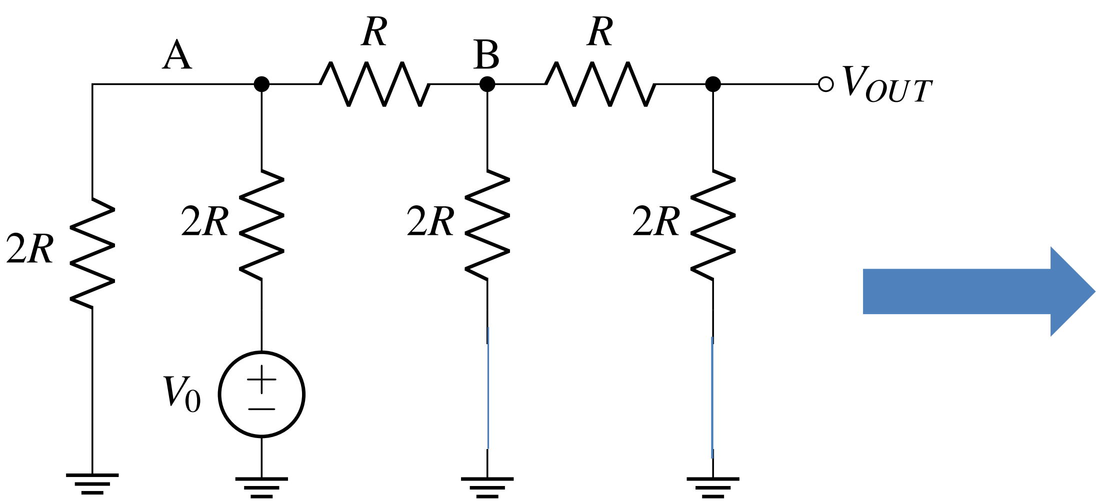
R-2R Ladder Digital-to-Analog Converter

Use superposition: Start with V_0



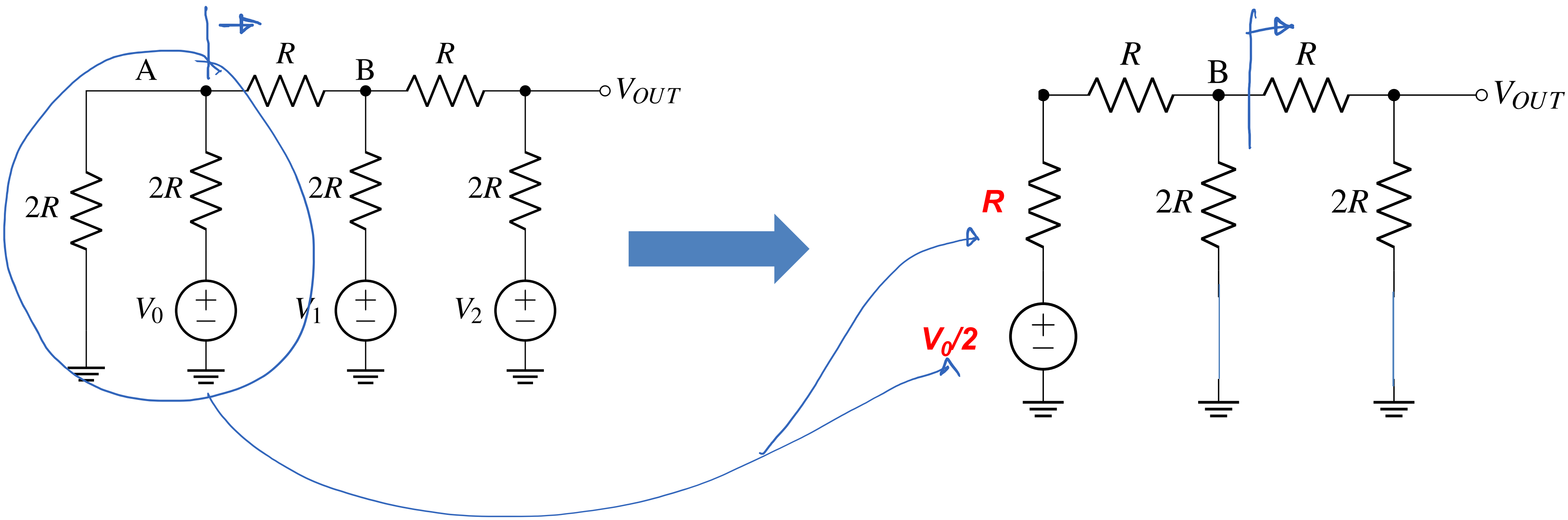
R-2R Ladder Digital-to-Analog Converter

Use superposition: Start with V_0



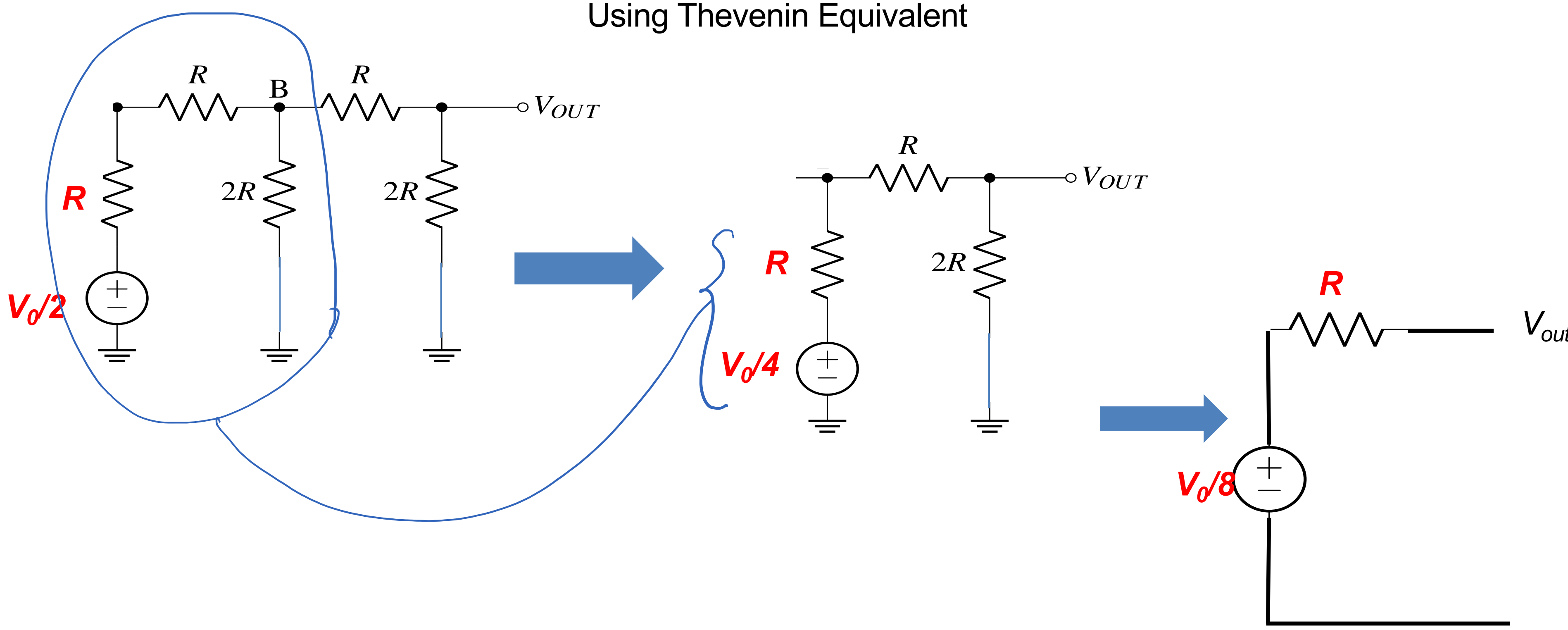
R-2R Ladder Digital-to-Analog Converter

Using Thevenin Equivalent



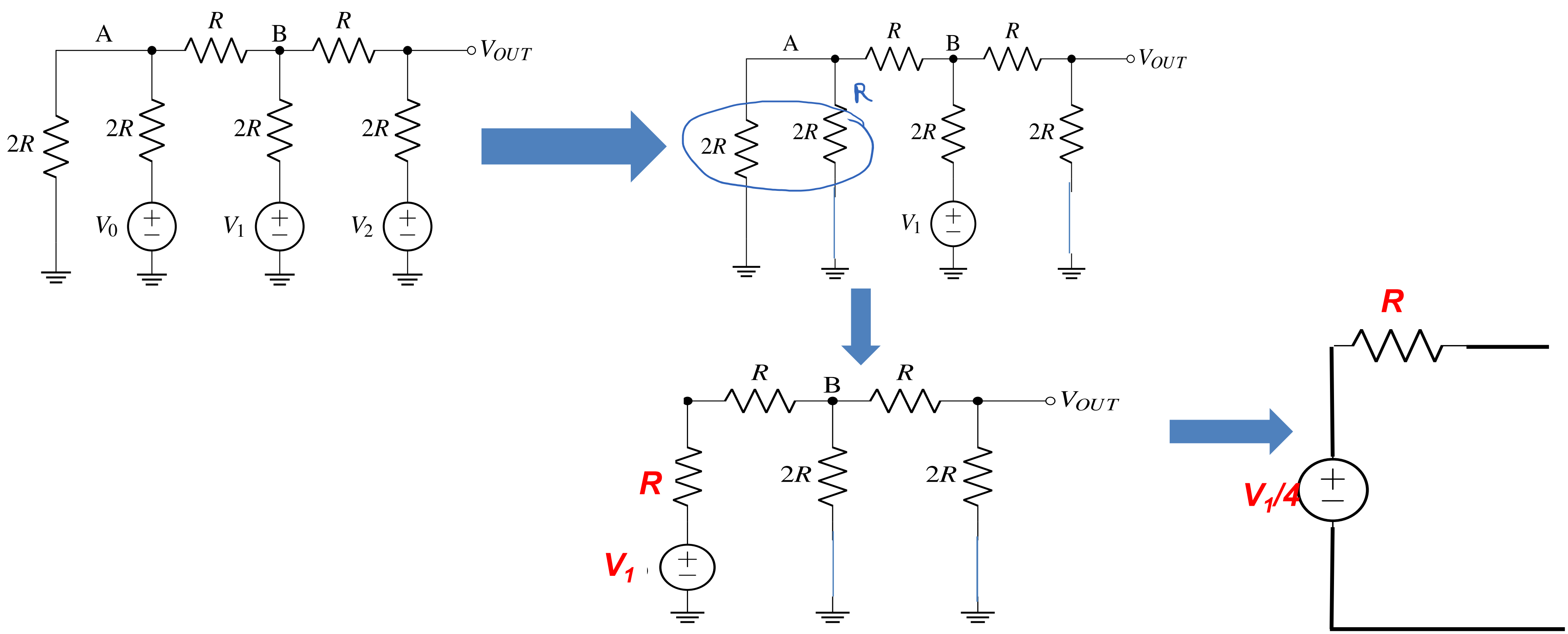
R-2R Ladder Digital-to-Analog Converter

Using Thevenin Equivalent



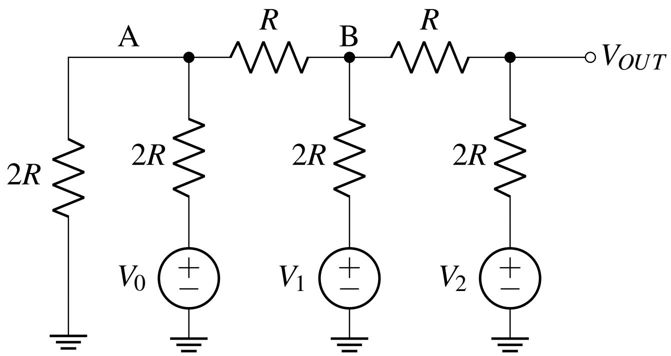
R-2R Ladder Digital-to-Analog Converter

Now lets look at V_1



R-2R Ladder Digital-to-Analog Converter

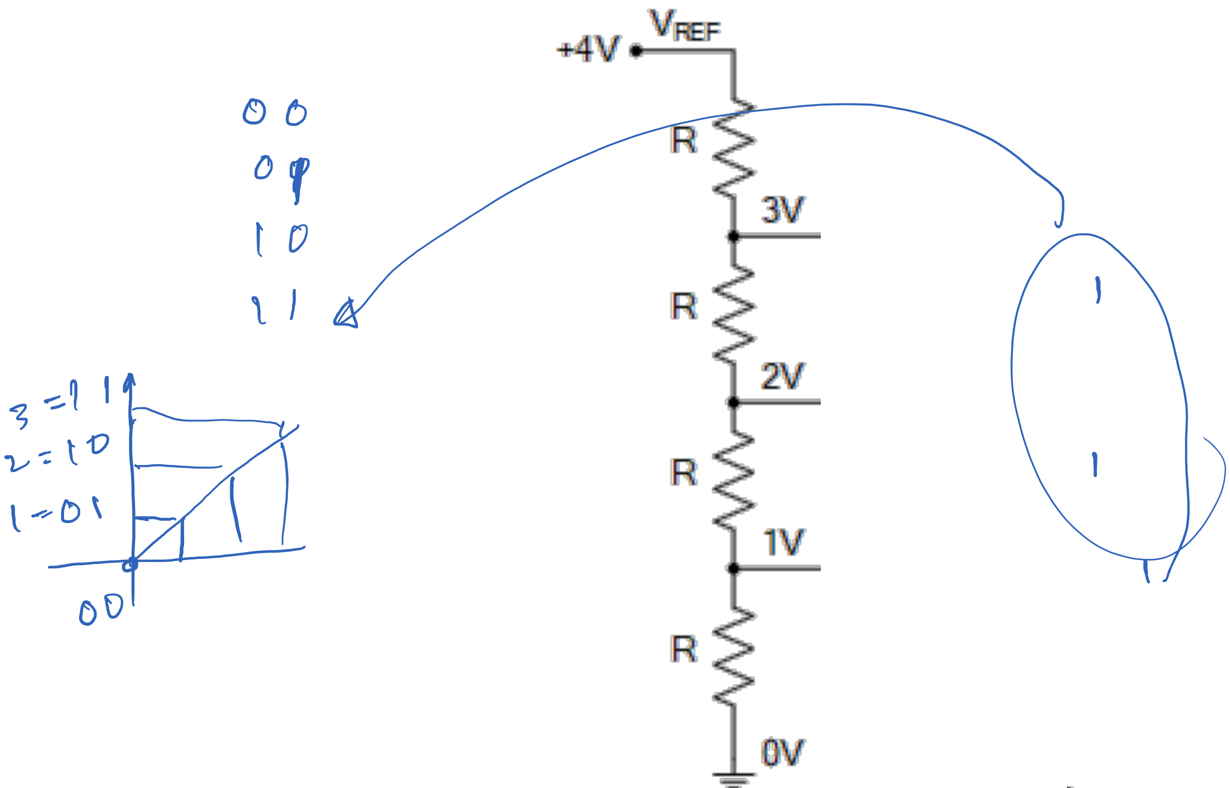
Adding all contributions from the sources



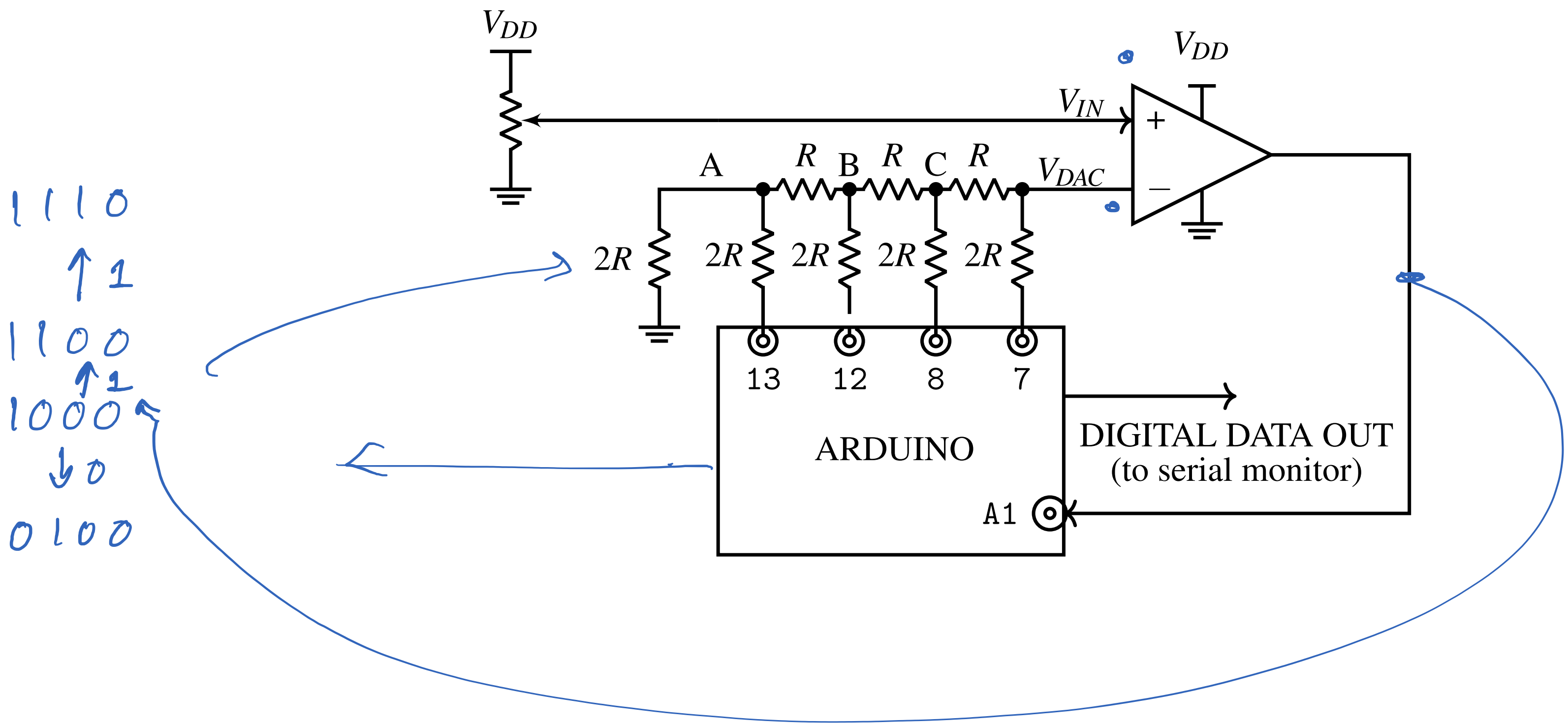
$$V_{out} = \frac{V_0}{8} + \frac{V_1}{4} + \frac{V_2}{2}$$

Analog to Digital Conversion

Say we want to convert an analog signal to a 2 bit digital signal → 4 levels



Lab 2: SAR (Successive Approximation Resistor) ADC



Transience

R-C circuits: Response in time

$\int \frac{dy}{y} = \log y$

Say a capacitor C had a stored charge of Q so that it held a voltage of V_i across it. At t=0 a switch connects it to a resistor completing the circuit.

$\tau = RC$

$Q = C v_i$

$v_c + v_R = 0$

$\Rightarrow v_c + iR = 0$

$\Rightarrow v_c + RC \frac{dv_c}{dt} = 0$

divide by RC

$\frac{dv_c}{dt} + \frac{v_c}{RC} = 0$

$\Rightarrow \frac{dv_c}{dt} = -\frac{v_c}{RC}$

$\frac{dv_c}{v_c} = -\frac{dt}{\tau}$

$\int \frac{dv_c}{v_c} = -\frac{1}{\tau} \int_0^t dt'$

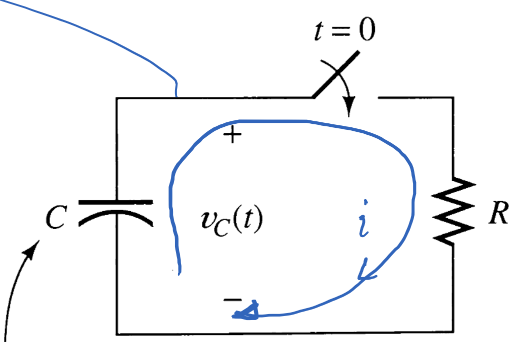
$\log v_c = -\frac{1}{\tau} t' \Big|_0^t + c$

$\log v_c = -t/\tau + c$

$v_c = e^{(-t/\tau + c)}$

$= e^{-t/\tau} e^{c} = A e^{-t/\tau}$

(e^{c}: constant)



Capacitance charged to V_i prior to $t = 0$

R-C circuits: Response in time

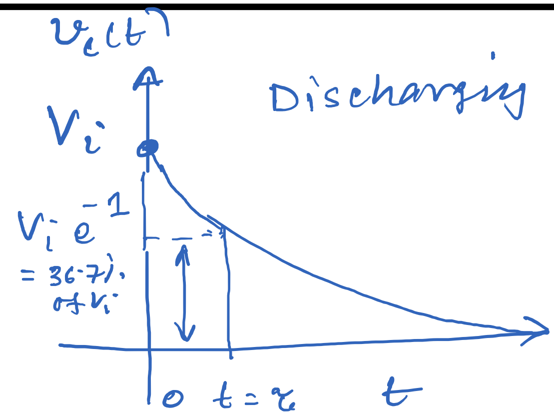
$v_c(t) = A\, e^{-t/\tau}$

given, at $t = 0$, $v_c(0) = V_i$

$v_c(0) = V_i = A\, e^{0} = A$

$v_c(t) = V_i\, e^{-t/\tau}$

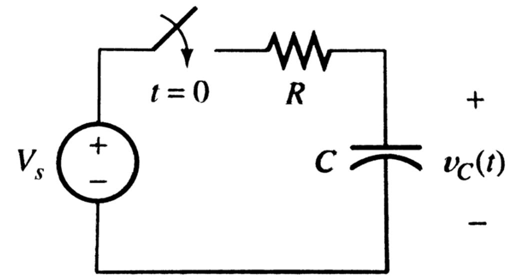
Discharging



$= RC$

$=$ characteristic time / time constant

$e^{-1} = 0.367$



R-C circuits: Response in time

We now ask a slightly different question. What happens if a capacitor that had initially no charge is connected to a constant voltage at t=0

$v_R + v_c = V_s$

$iR + v_c = V_s$

$RC \frac{dv_c}{dt} + v_c = V_s$

$\Rightarrow \frac{dv_c}{dt} + \frac{v_c}{\tau} = \frac{V_s}{\tau}$ Divide by RC

$RC = \tau$

Put, $V_s = 0 \rightarrow$ find v_c

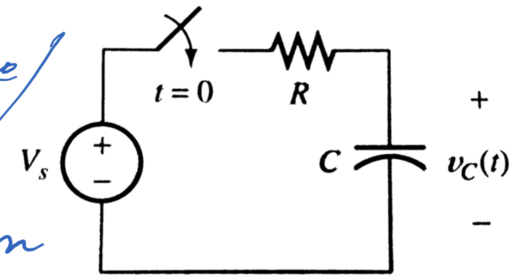
Put $V_s \rightarrow$ find v_c

add those two results

$V_s = 0$

natural response

homogeneous solution



$\frac{dv_c}{dt} + \frac{v_c}{\tau} = 0$

$v_c^h(t) = A e^{-t/\tau}$