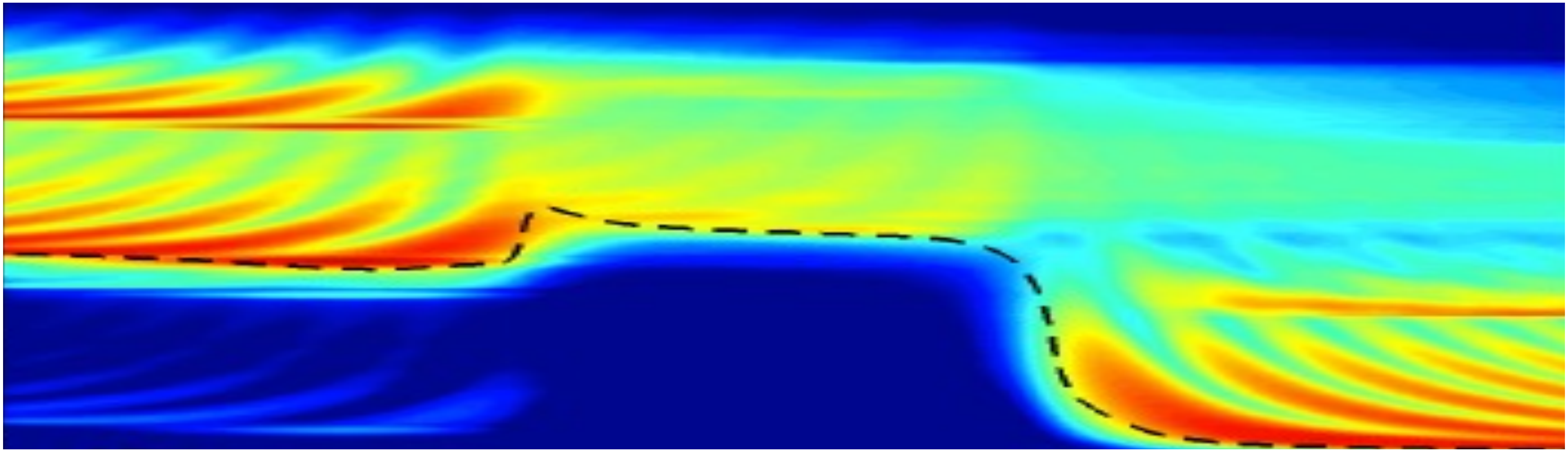


EECS 16B
Designing Information Devices and Systems II
Lecture 3

Prof. Sayeef Salahuddin
Department of Electrical Engineering and Computer Sciences, UC Berkeley,
sayeef@eecs.berkeley.edu

Transient Response

- Outline
  - R-C circuits
  - R-L circuits
  - R-L-C Circuits

- Reading- Hambley text sections 4.1-4.5, slides

Recap: R-C circuits: Response in time

We now ask a slightly different question. What happens if a capacitor that had initially no charge is connected to a constant voltage at t=0

$\boxed{Rc = \tau}$

$v_R + v_c = V_s$

$iR + v_c = V_s$

$Rc \frac{dv_c}{dt} + v_c = V_s$

$\Rightarrow \frac{dv_c}{dt} + \frac{v_c}{\tau} = \frac{V_s}{\tau}$ Divide by Rc

put, $V_s = 0 \rightarrow$ find v_c }

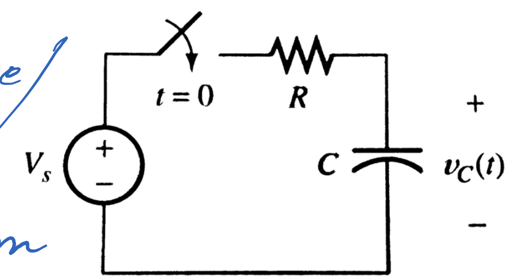
put $V_s \rightarrow$ find v_c

add those two results

$V_s = 0$

natural response

homogeneous solution

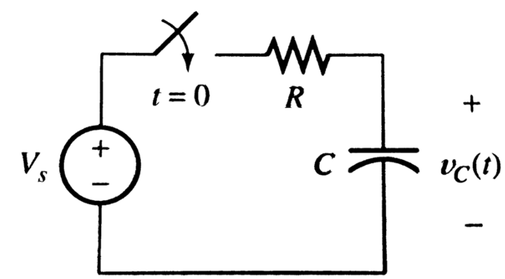


$$\frac{dv_c}{dt} + \frac{v_c}{\tau} = 0$$

$$v_c^h(t) = A e^{-t/\tau}$$

R-C circuits: Response in time

We now ask a slightly different question. What happens if a capacitor that had initially no charge is connected to a constant voltage at t=0



$$\frac{dv_c}{dt} + \frac{v_c}{\tau} = \frac{V_s}{\tau} \quad — (1)$$

First order, linear differential equation

if we can convert eq^n (1) to look like

$$\frac{dy}{dt} = g(t) \Rightarrow y(t) = \int_0^t g(t')\,dt'$$

$y = f(t)\,v_c$

if

$\frac{dy}{dt}$

Multiply eq^n (1) by an unknown function $f(t)$

$$f(t)\frac{dv_c}{dt} + \frac{f(t)}{\tau}v_c = V_s\frac{f(t)}{\tau}$$

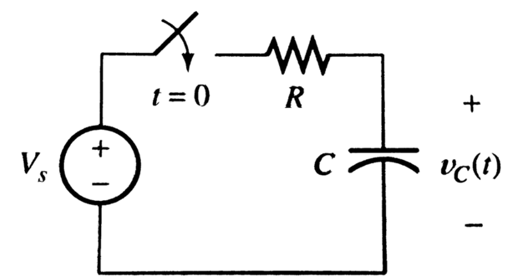
$$\left\{ \frac{d}{dt}(mn) = m\frac{dn}{dt} + n\frac{dm}{dt} \right.$$

$$f(t)\frac{dv_c}{dt} + v_c\frac{df(t)}{dt} = V_s\frac{f(t)}{\tau} \Rightarrow \text{if } \frac{df(t)}{dt} = \frac{f(t)}{\tau}$$

R-C circuits: Response in time

We now ask a slightly different question. What happens if a capacitor that had initially no charge is connected to a constant voltage at t=0

$\tau = RC$



$$\frac{dy}{dt} = \frac{V_s f(t)}{\tau} \quad \text{where,} \quad \boxed{\frac{df(t)}{dt} = \frac{f(t)}{\tau}}$$

$$\int \frac{df}{f} = \int \frac{dt}{\tau}$$

$$\ln f = \frac{t}{\tau}$$

$$f = e^{t/\tau}$$

$$\Rightarrow y(t) = \int_0^t \frac{V_s}{\tau} f(t')\,dt'$$

$$f(t)\,v_c(t) = \int_0^t \frac{V_s}{\tau} e^{t'/\tau}\,dt'$$

$$\Rightarrow v_c(t) = \frac{1}{f(t)} \int_0^t \frac{V_s}{\tau} e^{t'/\tau}\,dt'$$

$$\boxed{v_c^p(t) = e^{-t/\tau} \int_0^t \frac{V_s}{\tau} e^{t'/\tau}\,dt'}$$

$$V_c(t) = V_c^h(t) + v_c^p(t)$$

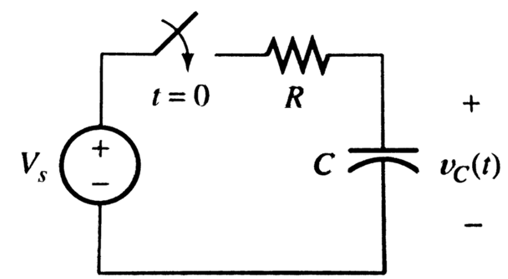
$$\boxed{v_c(t) = A e^{-t/\tau} + e^{-t/\tau} \int_0^t \frac{V_s}{\tau} e^{t'/\tau}\,dt'}$$

Charging and Discharging a Capacitor

$v_c = 0$ at $t = 0$, $V_s = V_0 \rightarrow$ constant voltage

$$\int e^{ax}\,dx = \frac{1}{a} e^{ax}$$

$$v_c(t) = A e^{-t/\tau} + e^{-t/\tau} \underbrace{\int_0^t dt' \frac{V_s}{\tau} e^{+t'/\tau}}$$



$$\int_0^t dt' \frac{V_0}{\tau} e^{-t'/\tau} = \frac{V_0}{\tau} \cancel{e^{-t/\tau}} \int_0^t dt' e^{+t'/\tau}$$

$$= \frac{V_0}{\cancel{\tau}} \cancel{\tau} \; e^{+t'/\tau} \Big|_0^t \; e^{-t/\tau}$$

$$= V_0 \left[e^{t/\tau} - 1\right] e^{-t/\tau}$$

$$v_c(t) = A e^{-t/\tau} + V_0 \left[e^{t/\tau} - 1\right] e^{-t/\tau}$$

$$\boxed{v_c(t) = A e^{-t/\tau} + V_0 \left[1 - e^{-t/\tau}\right]}$$

initial condition

$v_c = 0$, $t = 0$

$$0 = A e^{0} + V_0 \left[1 - e^{0}\right]$$

$$\Rightarrow A = 0$$

$$\boxed{v_c(t) = V_0 \left[1 - e^{-t/\tau}\right]}$$

General Solution of the First order, Linear, Differential Equation

For a first order, linear differential equation of the form

$$\frac{dy}{dt} + ay(t) = b(t)$$ where we assume a to be a constant

Homogeneous/Complementary solution

$$\frac{dy}{dt} + ay(t) = 0$$
$$\Rightarrow \frac{dy}{y} = -a$$
$$\Rightarrow \ln(y) = -at + C$$
$$\Rightarrow y(t) = Ke^{-at}$$

Particular Solution (Integrating Factor Method):

$$\frac{dy}{dt} + ay(t) = b(t)$$

We want to find a multiplier function *f(t)* such that

$$f(t)\frac{dy}{dt} + af(t)y(t) = b(t)f(t)$$

can be written as

$$\frac{d}{dt}[y(t)f(t)] = b(t)f(t) \quad \text{--(A)}$$

For equation (A) to hold

$$\frac{df(t)}{dt} = af(t)$$
$$\Rightarrow f(t) = e^{at}$$

Then from (A)

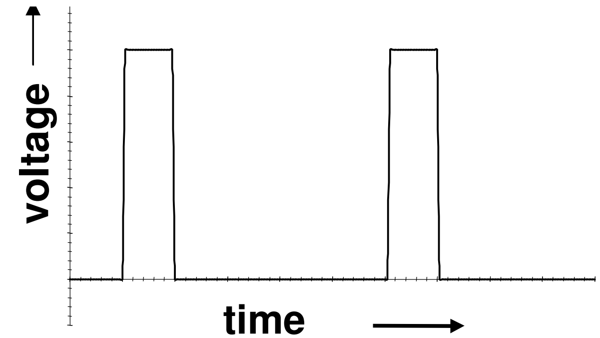
$$y(t) = \frac{1}{f(t)}\int b(t)f(t)dt$$
$$\Rightarrow y_p(t) = e^{-at}\int e^{at}b(t)dt$$

$$\boxed{y(t) = Ke^{-at} + e^{-at}\int e^{at}b(t)dt}$$

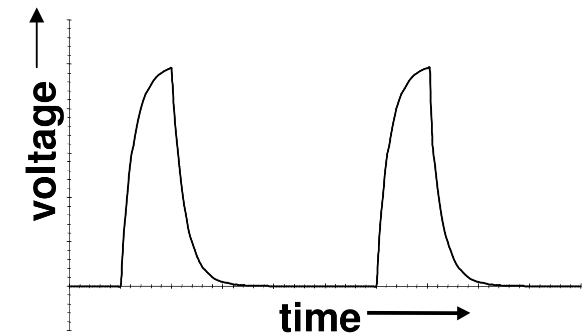
K is determined using initial condition

Digital Signals to a RC circuit

- Every node in a real circuit has capacitances

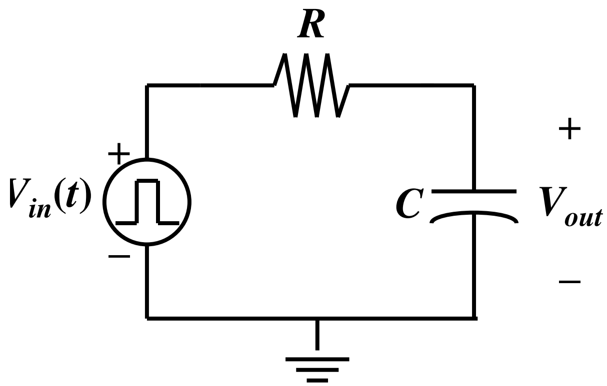


  - Even if we send in very 'pure' square looking pulses what we actually get is how it looks in the right due to capacitor charging and discharging <u>unless we go very very slow</u>

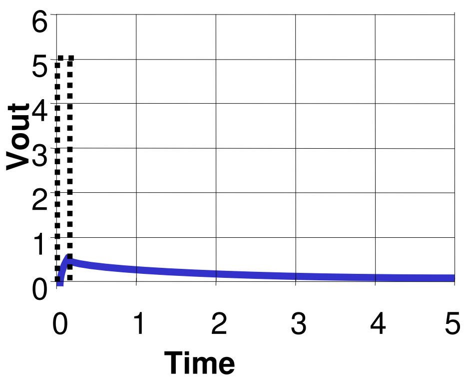


Pulse Distortion

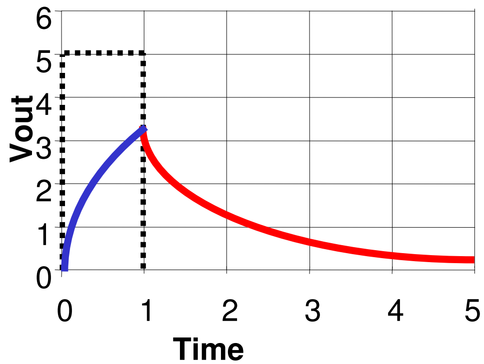
The input voltage pulse width must be large enough; otherwise the pulse is distorted



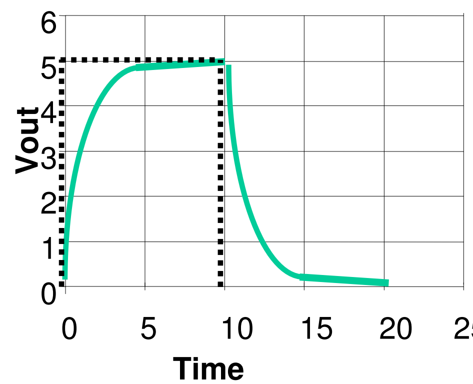
Pulse width = 0.1*RC*



Pulse width = *RC*

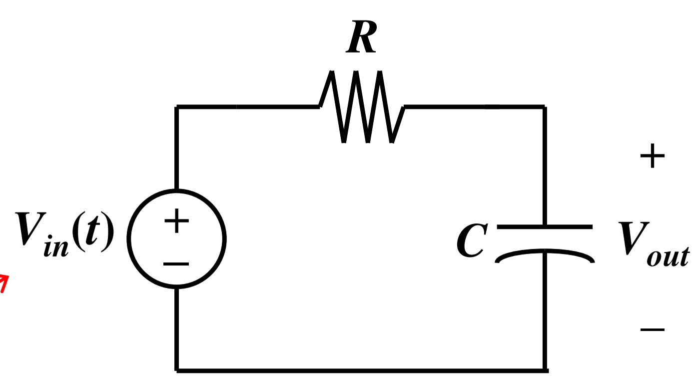


Pulse width = 10*RC*



Computers are RC circuits (almost)

- Digital circuits are predominantly RC circuits (other than the communication part)
- Simplistically a logic gate can be model as a RC circuit
- The speed of the computer is limited by the RC time constant



switches between "low" (logic 0) and "high" (logic 1) voltage states

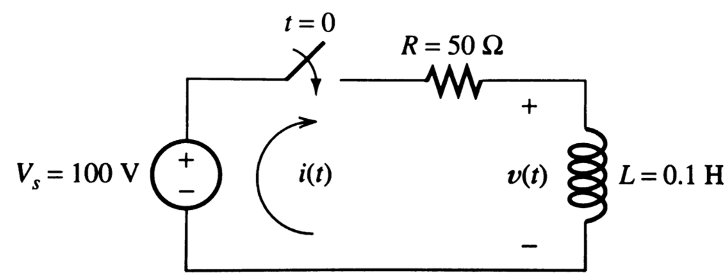
R-L Circuits

$$V_s = iR + L\frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{i}{(L/R)} = \frac{V_s}{L}$$

$$\therefore i = A e^{-\frac{t}{(L/R)}} + \underbrace{e^{-t/(L/R)} \int_0^t \frac{V_s}{L} e^{t'/(L/R)} dt'}$$

$$V_s = V_0$$



$$= \frac{V_0}{L} \cdot \frac{L}{R} e^{-t/(L/R)} e^{t/(L/R)} \Big|_0^t$$

$$\underbrace{\frac{V_0}{R}\left[1 - e^{-t/(L/R)}\right]}_{i_p(t)}$$

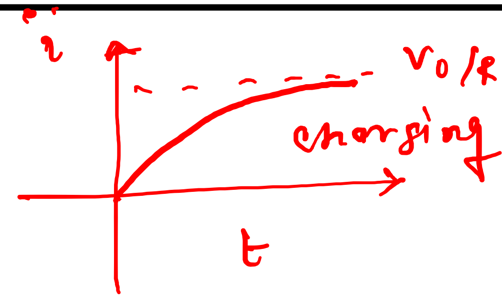
$$\boxed{\therefore i = A e^{-t/(L/R)} + \frac{V_0}{R}\left[1 - e^{-t/(L/R)}\right]}$$

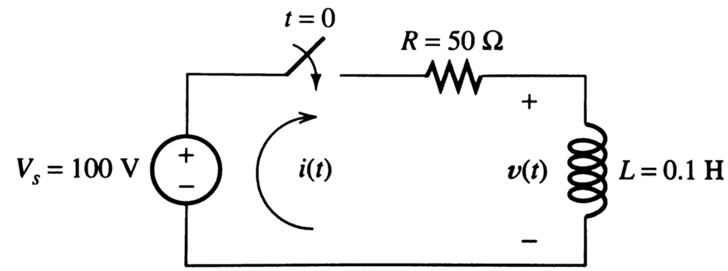
R-L Circuits

$i = 0$, at $t = 0$

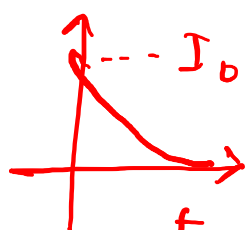
$\therefore A = 0$

$$i = \frac{V_0}{R}\left[1 - e^{-t/(L/R)}\right]$$



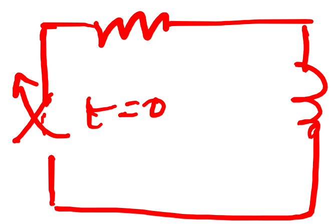


$$\boxed{i = I_0 e^{-t/(L/R)}}$$



$i = I_0$ at $t = 0$

$\Leftarrow$



Circuits with a constant source

$$V_c(t) = A e^{-t/\tau} + V_0\left[1 - e^{-t/\tau}\right]$$

$$= \underbrace{(A - V_0)}_{B} e^{-t/\tau} + V_0$$

$$V_c(t) = \underbrace{B e^{-t/\tau}}_{V_c^h} + \underbrace{V_0}_{V_c^p}$$

check, if $V_c = 0$, at $t = 0$

$$B + V_0 = 0 \Rightarrow B = -V_0$$

$$\therefore V_c(t) = V_0\left[1 - e^{-t/\tau}\right]$$

Same as before

Particular Solution: Observations

$$\frac{dy}{dt} + ay = b$$

y follows the same form as b

if b is constant y is constant

if b is $\sin \omega t$ $\quad y = A \sin \omega t + B \cos \omega t$

if $b = At$ $\quad y = Bt + C$

Complex Numbers

- $e^{i\theta} = \cos(\theta) + isin(\theta)$

- Read the note j