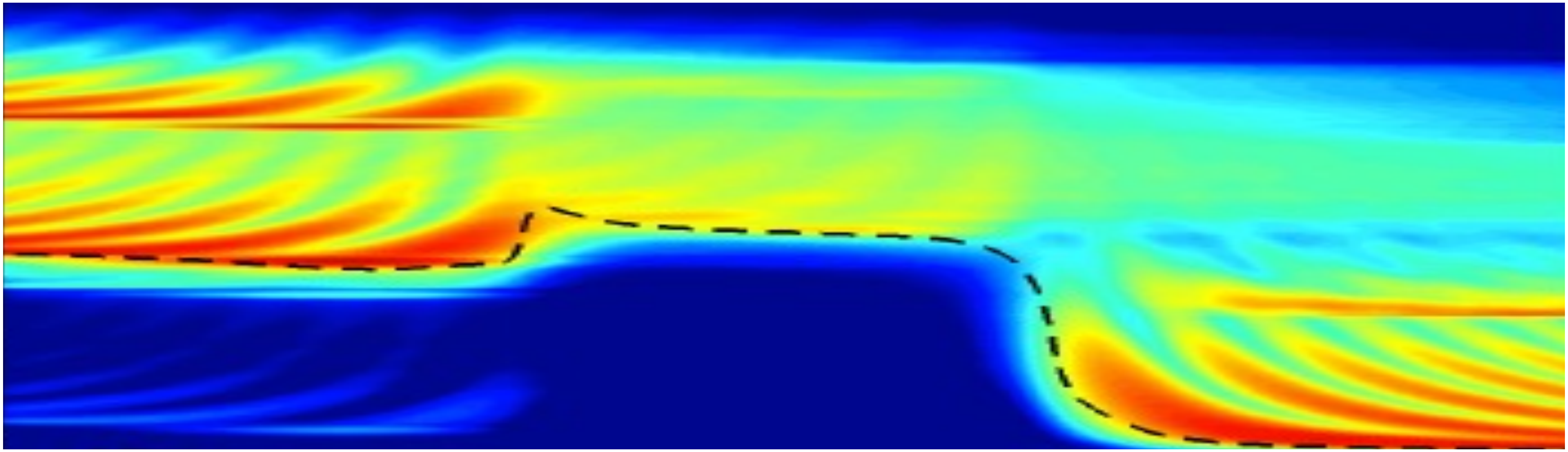

EECS 16B
Designing Information Devices and Systems II
Lecture 4

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Transient Response

- Outline
  - R-L-C Circuits
  - Phasors

- Reading- Hambley text sections 4.5, 5.1,5.2slides

R-L-C circuits: Response in time

$$v_s = iR + v_c + v_L$$

$$v_s = RC\frac{dv_c}{dt} + v_c + L\frac{di}{dt}$$

$$\Rightarrow v_s = RC\frac{dv_c}{dt} + v_c + LC\frac{d}{dt}\left(\frac{dv_c}{dt}\right)$$

$$\Rightarrow v_s = RC\frac{dv_c}{dt} + v_c + LC\frac{d^2v_c}{dt^2}$$

$$\Rightarrow \boxed{\frac{d^2v_c}{dt^2} + \frac{dv_c/dt}{(L/R)} + \frac{v_c}{LC} = \frac{v_s}{LC}}$$

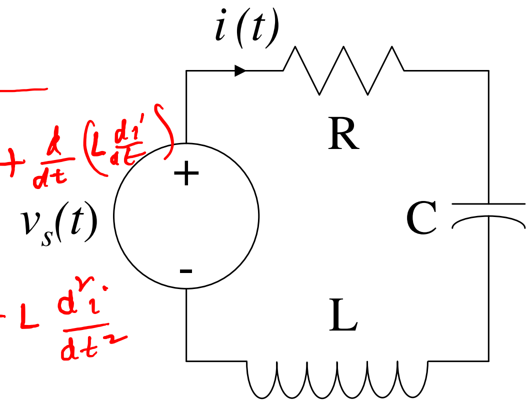
$$i = C\frac{dv_c}{dt}$$

$$\frac{dv_s}{dt} = R\frac{di}{dt} + \frac{dv_c}{dt} + \frac{d}{dt}\left(L\frac{di}{dt}\right)$$

$$\Rightarrow \frac{dv_s}{dt} = R\frac{di}{dt} + \frac{i}{C} + L\frac{d^2i}{dt^2}$$

$$\downarrow$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{di/dt}{L/R} + \frac{i}{LC} = \frac{1}{L}\frac{dv_s}{dt}}$$



Filters, antennas, resonances

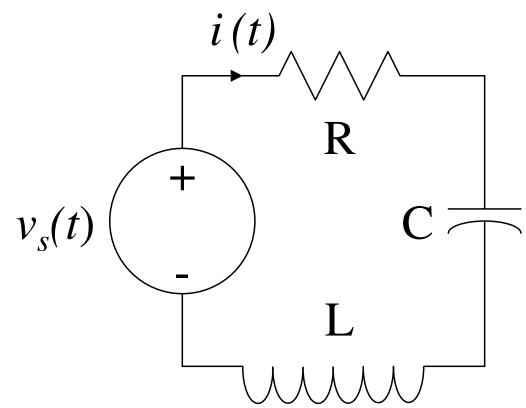
R-L-C circuits: Response in time

$$\frac{d^2 v_c}{dt^2} + \frac{1}{(L/R)}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$2\alpha = \frac{1}{L/R} \Rightarrow \alpha = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\frac{d^2 v_c}{dt^2} + 2\alpha\frac{dv_c}{dt} + \omega_0^2 v_c = \frac{v_s}{LC}$$



Homogeneous solution

$$\frac{d^2 v_c}{dt^2} + 2\alpha\frac{dv_c}{dt} + \omega_0^2 v_c = 0$$

From previous discussions we have seen that an exponential solution works

Lets try: $v_c(t) = Ae^{st}$

$$As^2e^{st} + 2\alpha A s e^{st} + \omega_0^2 A e^{st} = 0$$

$$\Rightarrow \boxed{s^2 + 2\alpha s + \omega_0^2 = 0}$$

R-L-C circuits: Response in time

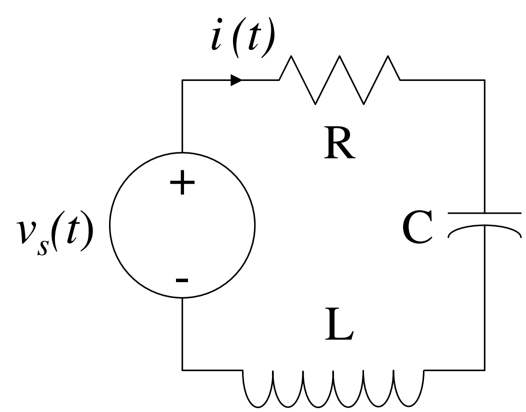
$s^2 + 2\alpha s + \omega_0^2 = 0$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s = \dfrac{-2\alpha \pm \sqrt{4\alpha^2 - 4\omega_0^2}}{2\cancel{\alpha}}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$\Rightarrow s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$



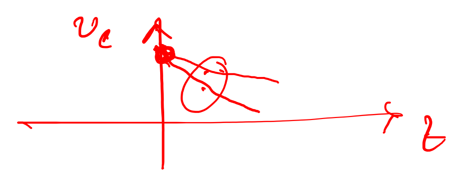
$\boxed{v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$

$\boxed{\alpha > \omega_0} \Rightarrow$ both s_1 and s_2 are negative

$a = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$b = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$v_c(t) = A_1 e^{-|a|t} + A_2 e^{-|b|t}$



over damped system

$\boxed{\alpha = \omega_0}$ $a = b = -\alpha$; $v_c(t) = \underbrace{(A_1 + A_2)}_{K} e^{-\alpha t} \rightarrow$ critically damped

R-L-C circuits: Response in time

$\boxed{\alpha < \omega_0}$

under damped

$$\begin{cases} S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \end{cases}$$

$$s_1 = -\alpha + \sqrt{-\underbrace{(\omega_0^2 - \alpha^2)}_{\beta^2}}$$

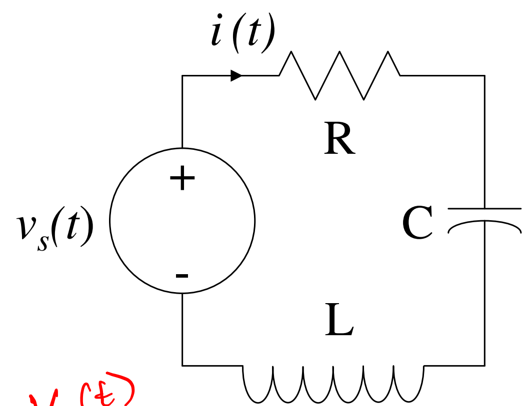
$$\sqrt{-1} = j$$

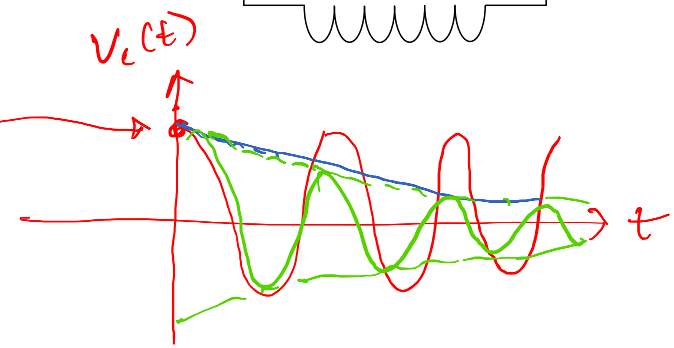
$$= -\alpha + j\beta$$

$$S_2 = -\alpha - j\beta$$

$$V_c(t) = A_1 e^{-\alpha t + j\beta t} + A_2 e^{-\alpha t - j\beta t}$$

$$= A_1 e^{-\alpha t} e^{j\beta t} + A_2 e^{-\alpha t} e^{-j\beta t}$$





R-L-C circuits: Response in time

$\boxed{R=0}$ $\quad \alpha = \frac{R}{2L} = 0$

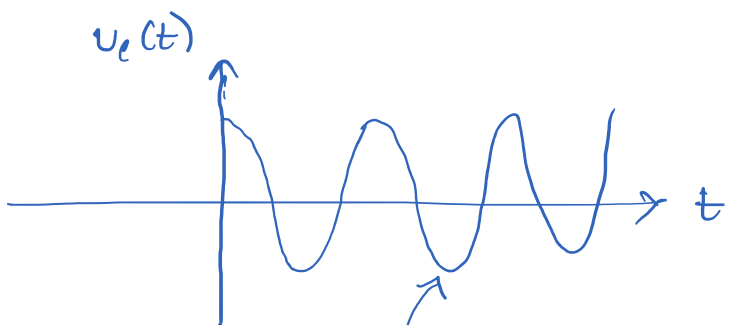
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = j\beta = j\omega_0, \quad \beta = \omega_0$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -j\beta = -j\omega_0$$

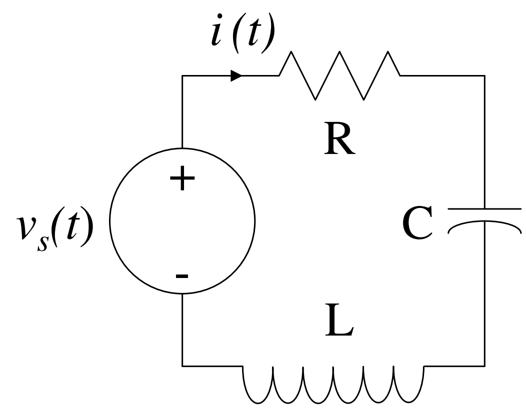
$$v_c(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t}$$

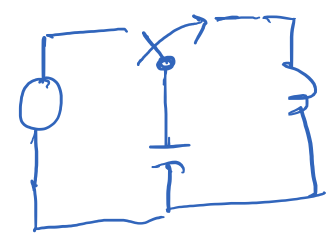
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$\equiv$ resonance frequency



$$f = \frac{\omega_0}{2\pi}$$





Summary

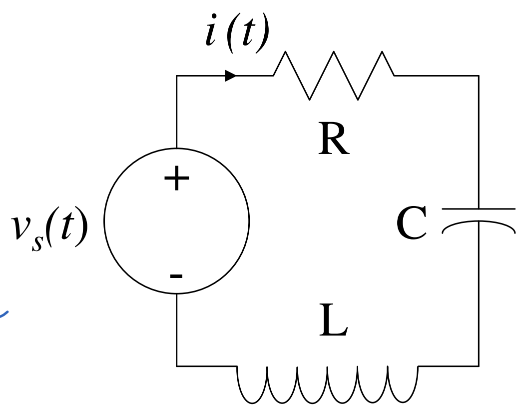
- Overdamped – real unequal roots
- Critically damped – Real Equal roots
- Underdamped – Complex roots

R-L-C circuits: Response in time

Particular solution:

$$\frac{d^2v_c}{dt^2} + \frac{1}{(L/R)}\frac{dv_c}{dt} + \frac{1}{LC}v_c = v_s$$

$$\frac{d^2i}{dt^2} + \frac{1}{(L/R)}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{L}\frac{dv_s}{dt}$$

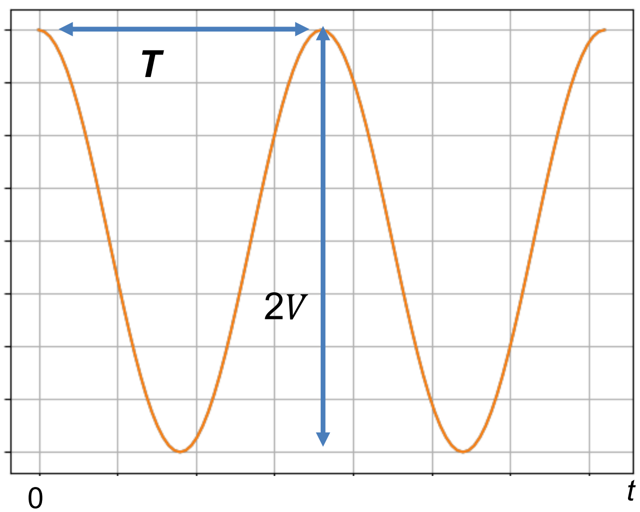


Constant source: $v_s = V_0$ → particular solution is simply the steady state solution

Sinusoidal voltages

Degree → R : Deg × $\frac{\pi}{180}$

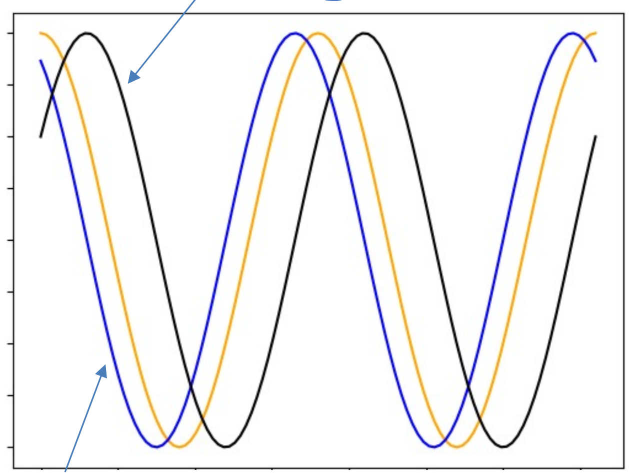
$v(t) = V cos(\omega t)$



T : Period

$\omega = \frac{2\pi}{T}$

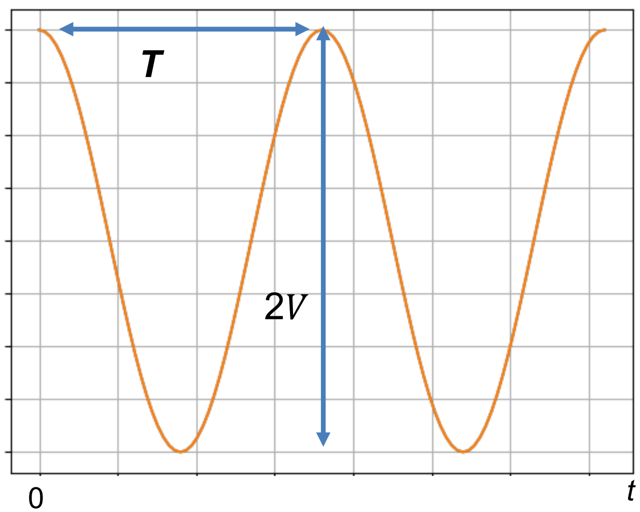
$v(t) = V cos(\omega t - 60^0)$



$v(t) = V cos(\omega t + 30^0)$

Root Mean Square Values

$$v(t) = V cos(\omega t)$$



T : Period

$$\omega = \frac{2\pi}{T}$$

Average Power over one period:

$$P = \frac{1}{T}\int_0^T \frac{v^2}{R} dt$$

$$P = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2 dt}\right]^2}{R}$$

Comparing with conventional equation: P=voltage^2/R

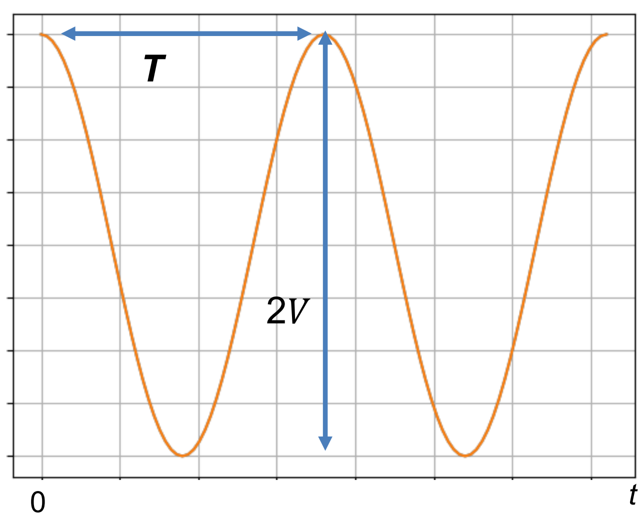
A new quantity is defined for time-varying voltages known as the root-mean-square voltage

$$v_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

Root Mean Square Value for Sinusoidal Voltage

$$2\cos^2\omega t = 1 + \cos(2\omega t)$$

$$v(t) = V cos(\omega t)$$



T : Period

$$\omega = \frac{2\pi}{T}$$

$$v_{rms}^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{T}\int_0^T V^2 \cos^2(\omega t) dt = \frac{1}{2}\frac{1}{T} V^2 \int_0^T (1 + cos2\omega t) dt$$

$$v_{rms}^2 = \frac{V^2}{2T}(t + 2sin2\omega t)_0^T = \frac{V^2}{2T}[T - 0 + sin2\omega T - 0] = \frac{V^2}{2}$$

$$\sin 2\omega \cdot \frac{2\pi}{\omega} = \sin 2\pi = 0$$

$$v_{rms} = \frac{V}{\sqrt{2}}$$

How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega\mathrm{t} + 5sin\omega t - 5\cos(\omega t - 30^0)$$

$$v(t) = 10cos\omega\mathrm{t} + 5\cos(\omega t - 90) - 5\cos(\omega t - 30^0)$$

Remember? cos (a + b) = cos a cos b - sin a sin b

Lets do it it differently $e^{j\theta} = cos\theta + jsin\theta$

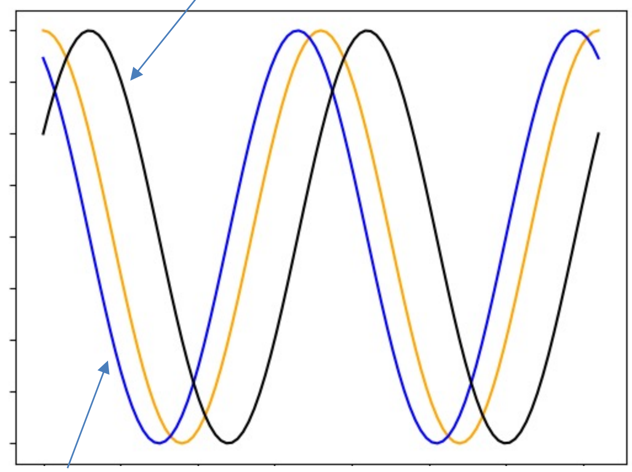
$$e^{-j\theta} = cos\theta - jsin\theta$$

Then

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$

$v(t) = Vcos(\omega t - 60^0)$



$v(t) = Vcos(\omega t + 30^0)$

How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= \frac{1}{2}e^{j\omega t}[10] + \frac{1}{2}e^{j(\omega t - 90^0)}[5] - \frac{1}{2}e^{j(\omega t - 30^0)}[5]$$

$$+ \frac{1}{2}e^{-j\omega t}[10] + \frac{1}{2}e^{-j(\omega t - 90^0)}[5] - \frac{1}{2}e^{-j(\omega t - 30^0)}$$

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$= \frac{1}{2}e^{j\omega t}\left[10 + 5e^{-j90} - 5e^{-j30}\right] + \frac{1}{2}e^{-j\omega t}\left[10 + 5e^{+j90} - 5e^{+j30}\right]$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5cos90 - j5sin90 - 5cos30 + j5sin30] + cc$$

complex conjugate
flip the sign of j

$$= \frac{1}{2}e^{j\omega t}\left[10 + 0 - j5 - 5\frac{\sqrt{3}}{2} + \frac{j5}{2}\right] + cc$$

$$= \frac{1}{2}e^{j\omega t}[\underbrace{5.66 - j2.5}_{Ae^{j\theta}}] + cc$$

$$= \frac{1}{2}6.18e^{j(\omega t - 23^0)} + cc$$

$$= 6.18\cos(\omega t - 23^0)$$

multiply

$$Ae^{-j\theta} = 5.66 - j2.5$$

$$Ae^{j\theta} = 5.66 + j2.5$$

$$A^2 = 5.66^2 - j^2 2.5^2$$

$$A^2 = 5.66^2 + 2.5^2$$

$$A = \sqrt{5.66^2 + 2.5^2} = 6.18$$

$$cos\theta = 5.66/6.18; sin\theta = 2.5/6.18$$

$$tan\theta = \frac{2.5}{5.66} = 0.44$$

$$\theta = 0.41 = 23^0$$

Some Observations

$$v(t) = \frac{1}{2}e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \underbrace{\frac{1}{2}6.18e^{j(\omega t - 23^0)}} + \frac{1}{2}6.18e^{-j(\omega t - 23^0)}$$

$$= \frac{1}{2}6.18[\cos(\omega \mathrm{t} - 23^0) + \cancel{jsin(\omega t - 23^0)} + \cos(\omega \mathrm{t} - 23^0) - \cancel{jsin(\omega t - 23^0)}] \quad = \quad 6.18\cos(\omega t - 23^0)$$

$$= Real\ [\underbrace{6.18e^{j(\omega t - 23^0)}}]$$

Phasors

In short hand, it is represented as $\boxed{6.18\ \angle - 23^0}$