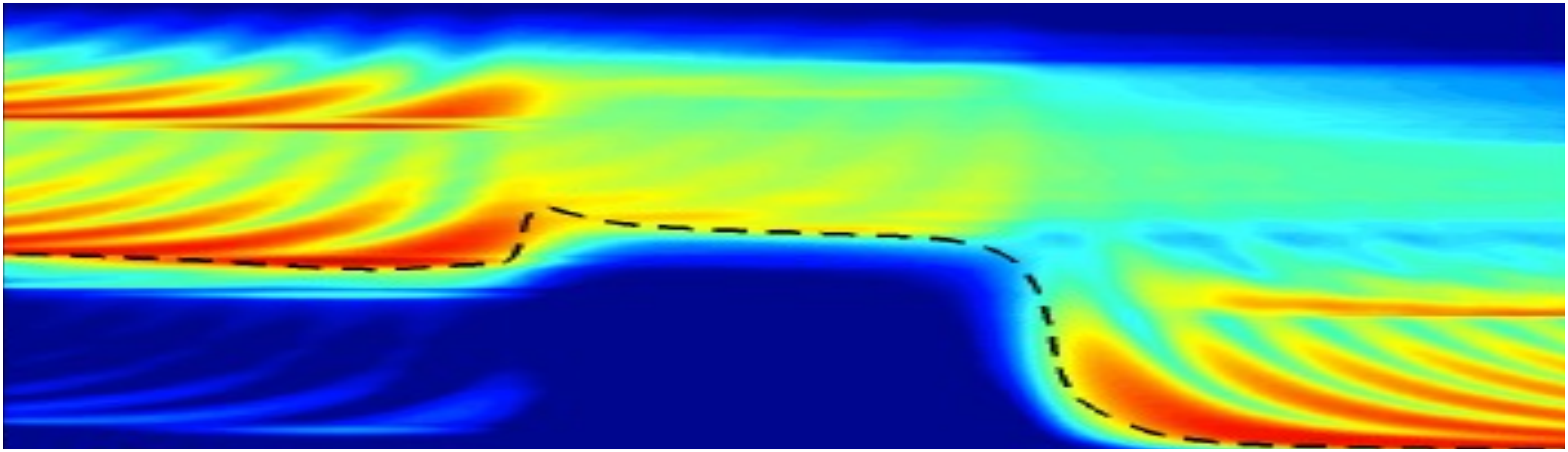

EECS 16B
Designing Information Devices and Systems II
Lecture 5

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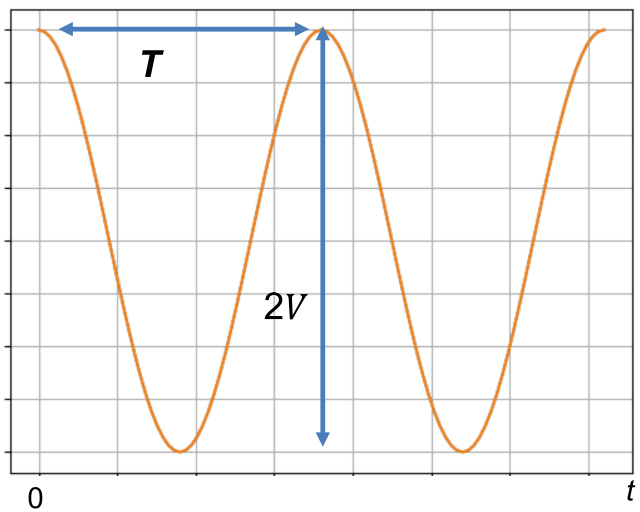
Transient Response

- Outline
  - Phasors
  - Complex Impedances
  - Solution of circuits using complex impedances

- Reading- Hambley text sections 5.2, 5.3,5.4, 5.6, 5.5, slides

Recap: Sinusoidal voltages

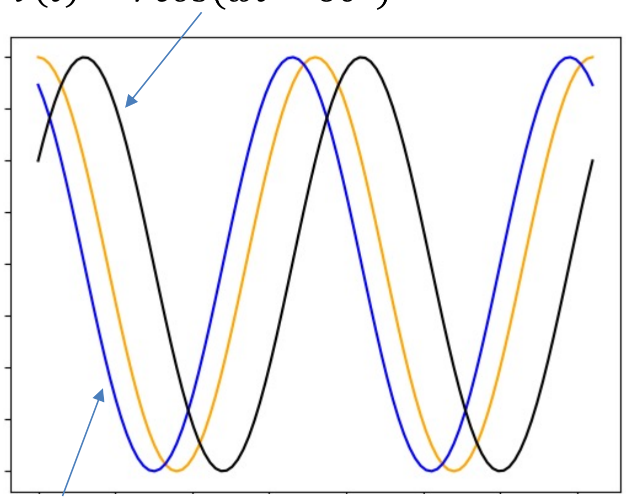
$v(t) = Vcos(\omega t)$



T : Period

$$\omega = \frac{2\pi}{T}$$

$v(t) = Vcos(\omega t - 60^0)$



$v(t) = Vcos(\omega t + 30^0)$

Recap: How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega t + 5sin\omega t - 5\cos(\omega t - 30^0)$$

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90) - 5\cos(\omega t - 30^0)$$

Remember? cos (a + b) = cos a cos b - sin a sin b

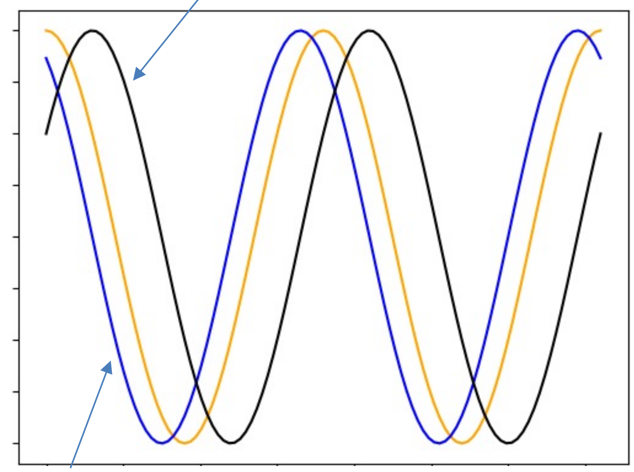
Lets do it it differently $e^{j\theta} = cos\theta + jsin\theta$

$$e^{-j\theta} = cos\theta - jsin\theta$$

Then

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \frac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$



Recap: How do we add arbitrary sinusoids?

$$v(t) = 10cos\omega\text{t} + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= \frac{1}{2}e^{j\omega t}[10] + \frac{1}{2}e^{j(\omega t-90^0)}[5] - \frac{1}{2}e^{j(\omega t-30^0)}[5]$$

$$+ \tfrac{1}{2}e^{-j\omega t}[10] + \tfrac{1}{2}e^{-j(\omega t-90^0)}[5] - \tfrac{1}{2}e^{-j(\omega t-30^0)}$$

$$cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$sin\theta = \tfrac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$= \frac{1}{2}e^{j\omega t}\left[10 + 5e^{-j90} - 5e^{-j30}\right] + \frac{1}{2}e^{-j\omega t}\left[10 + 5e^{+j90} - 5e^{+j30}\right]$$

$$= \frac{1}{2}e^{j\omega t}[10 + 5cos90 - j5sin90 - 5cos30 + j5sin30] + cc$$

$$= \frac{1}{2}e^{j\omega t}\left[10 + 0 - j5 - 5\frac{\sqrt{3}}{2} + \frac{j5}{2}\right] + cc$$

$$= \frac{1}{2}e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2}6.18e^{j(\omega t-23^0)} + cc$$

$$= 6.18\cos(\omega t - 23^0)$$

$$Ae^{-j\theta} = 5.66 - j2.5$$

$$Ae^{j\theta} = 5.66 + j2.5$$

$$A^2 = 5.66^2 - j^2 2.5^2$$

$$A^2 = 5.66^2 + 2.5^2$$

$$A = \sqrt{5.66^2 + 2.5^2} = 6.18$$

$$cos\theta = 5.66/6.18; sin\theta = 2.5/6.18$$

$$tan\theta = \frac{2.5}{5.66} = 0.44$$

$$\theta = 0.41 = 23^0$$

Recap: Some Observations

$$v(t) = \frac{1}{2} e^{j\omega t}[5.66 - j2.5] + cc$$

$$= \frac{1}{2} 6.18 e^{j(\omega t - 23^0)} + \frac{1}{2} 6.18 e^{-j(\omega t - 23^0)}$$

$$= \frac{1}{2} 6.18[\cos(\omega \mathrm{t} - 23^0) + jsin(\omega t - 23^0) \cos(\omega \mathrm{t} - 23^0) - jsin(\omega t - 23^0)]$$

$$= Real\ [6.18 e^{j(\omega t - 23^0)}]$$

Phasors

In short hand, it is represented as $6.18 \angle - 23^0$

Some Observations

$$A(t) = 5\cos(\omega t) = Real[5e^{j(\omega t)}]$$

$$B(t) = 5\cos(\omega t - 90^0) = Real[5e^{j(\omega t - 90^0)}]$$

$$C(t) = 5\cos(\omega t + 90^0) = Real[5e^{j(\omega t + 90^0)}]$$

At any given time *t*, *B(t)* is trailing or lagging behind *A(t)* by 90^0 while *C(t)* is leading *A(t)* by the same amount

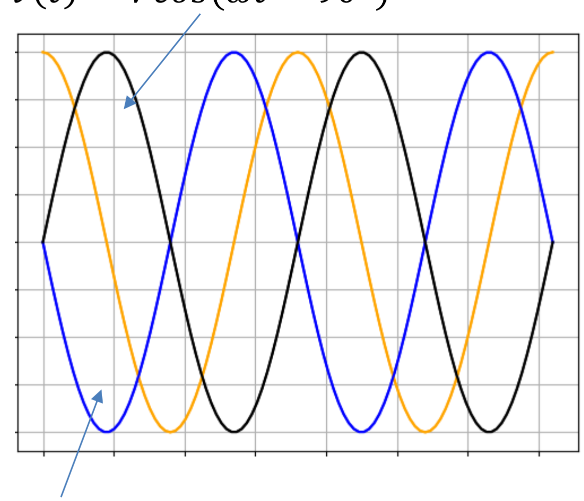
Let us now look at the phasors at *t=0*

$$5e^{j(\omega t + 0^0)} = 5$$

$$5e^{j(\omega t - 90^0)} = 5(cos90^0 - jsin90^0) = -j5$$

$$5e^{j(\omega t + 90^0)} = 5(cos90^0 + jsin90^0) = +j5$$

$v(t) = Vcos(\omega t - 90^0)$

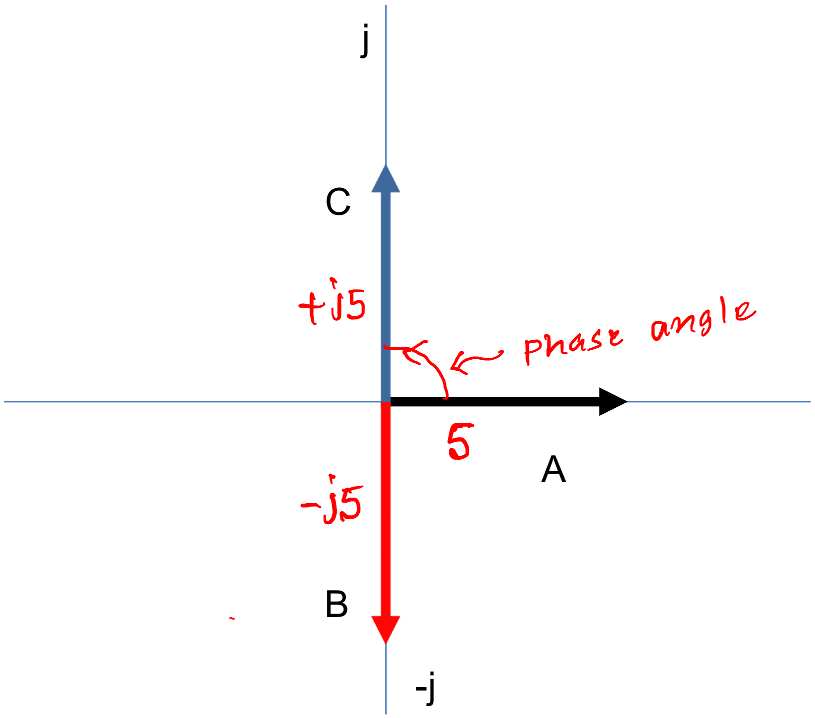


$v(t) = Vcos(\omega t + 90^0)$

Therefore *+j or –j* signifies signals having 90^0 phase lead or lag respectively.

Some Observations

+j or –j signifies signals having 90^0 phase lead or lag respectively.



$$5e^{j(\omega t-90^0)} = 5$$

$$5e^{j(\omega t-90^0)} = 5(cos90^0 - jsin90^0) = -j5$$

$$5e^{j(\omega t+90^0)} = 5(cos90^0 + jsin90^0) = +j5$$

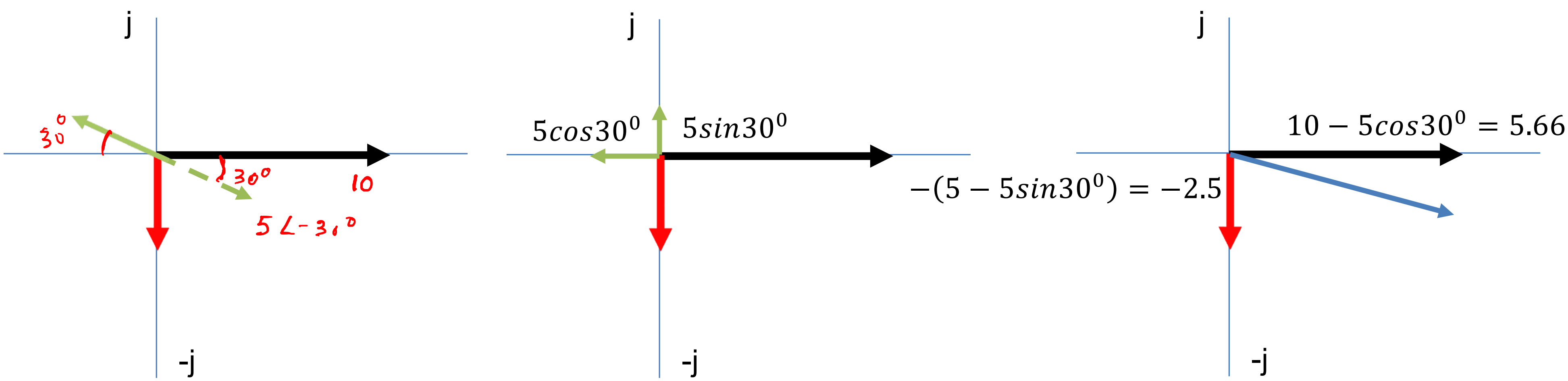
$$\Sigma = 5$$

Some Observations

$$v(t) = 10cos\omega t + 5\cos(\omega t - 90^0) - 5\cos(\omega t - 30^0)$$

$$= 10 + 5\angle - 90^0 - 5\angle - 30^0$$ In phasor notation

$$= 6.18\angle - 23^0$$



Phasors are like vectors where the phase angle denotes the angle between coordinate axes with *j* representing 90^0

Some Observatons

What is $\frac{1}{\angle\theta}$?

$1\angle\theta = 1\, e^{j(\omega t+\theta)}$

$\frac{1}{1\angle\theta} = \frac{1}{e^{j(\omega t+\theta)}} = e^{-j(\omega t+\theta)}$

$= 1\angle -\theta$

$= \frac{1}{\cos(\omega t+\theta) + j\sin(\omega t+\theta)}$

$A\angle\theta_1 \times B\angle\theta_2$

$= AB\ \angle\theta_1+\theta_2$

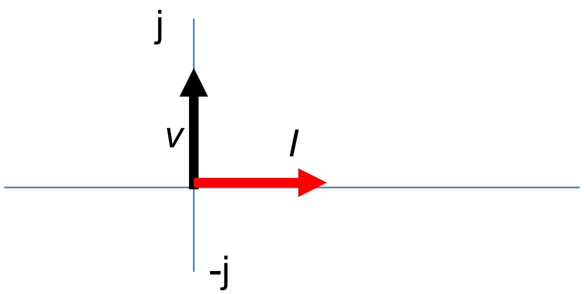
Complex Impedances

Inductance:

Say a sinusoidal current is flowing in a circuit with inductance

$$i(t) = I_0 \sin(\omega t) = I_0 \cos(\omega t - 90^0)$$

$$v_L(t) = L\frac{di}{dt} = \omega L I_0 cos\omega t$$



Therefore, the current in an inductor lags the voltage by 90^0

In the phasor notation

$$\boldsymbol{V} = \omega L I_0 \angle 0^0$$

$$\boldsymbol{I} = I_0 \angle -90^0$$

Then, inductive impedance

$$\boldsymbol{Z_L} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{\omega L}{\angle -90^0} = \frac{\omega L}{-j} = j\omega L$$

We could have obtained the same result working directly with exponentials

$$\boldsymbol{V} = L\frac{d}{dt}[I_0 e^{j(\omega t - 90^0)}]$$

$$\boldsymbol{V} = Lj\omega[I_0 e^{j(\omega t - 90^0)}]$$

$$\boldsymbol{V} = j\omega L I_0 \angle -90^0$$

$$\boldsymbol{V} = j\omega L \boldsymbol{I}$$

Complex Impedances

$\hat{j} = \angle 90^\circ$

Capacitance:

$i = C \frac{dv}{dt}$

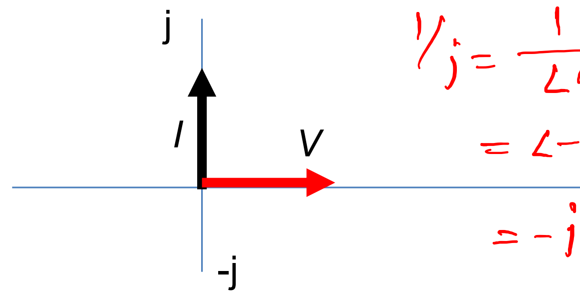
$\tilde{I} = C \frac{d}{dt} \underbrace{V_0 e^{j\omega t}}_{\tilde{V}}$

$= C j\omega V_0 e^{j\omega t}$

$\tilde{I} = (j\omega C)\tilde{V}$

capacitive Impedance: $\frac{\tilde{V}}{\tilde{I}} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$\tilde{V} = \left(\frac{-j}{\omega C}\right)\tilde{I}$



$1/j = \frac{1}{\angle 90} = \angle -90 = -j$

Voltage lags the current by 90°

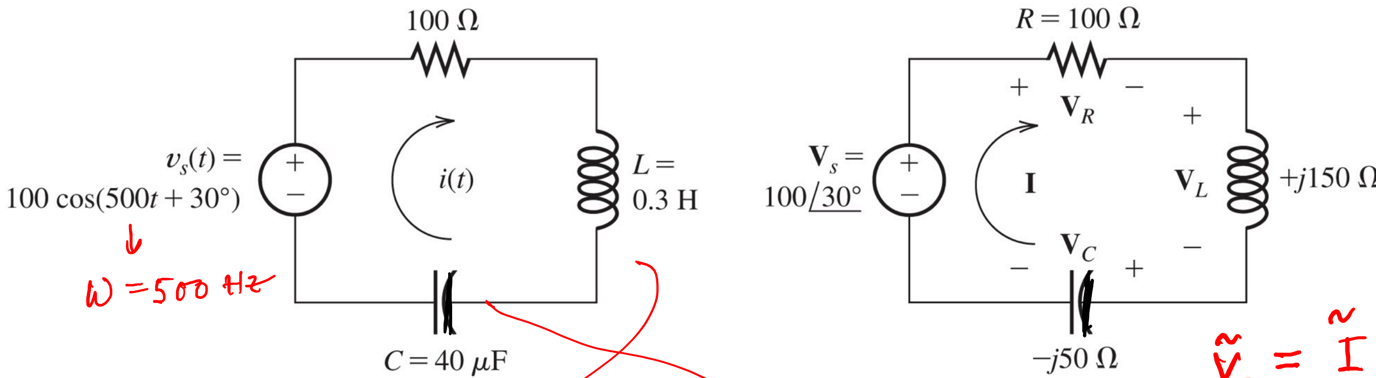
Complex Impedances

- AC steady-state analysis using phasors allows us to express the relationship between current and voltage using a formula that looks likes Ohm's law:

$$\boldsymbol{V} = \boldsymbol{I}\boldsymbol{Z}$$

- Impedance depends on the frequency ω.
- Impedance is a complex number.
- Impedance allows us to use the same solution techniques for AC steady state as

Circuit Solution with sinusoidal sources



(a) (b)

$\omega = 500$ Hz

$\tilde{V}_s = \tilde{I}R + \tilde{I}Z_L + \tilde{I}Z_C$

$\tilde{V}_s = 100\angle 30^\circ$

$Z_L = j\omega L = j\,500 \times 0.3$

$= j\,150\,\Omega$

$Z_C = \frac{-j}{\omega C} = \frac{-j}{500 \times 40 \times 10^{-6}}$

$= \frac{-j}{2 \times 100} = -j50\,\Omega$

Circuit Solution with sinusoidal sources

$$100\angle 30 = (100 + j150 - j50)\tilde{I}$$
$$= (100 + j100)\tilde{I}$$

$$\tilde{I} = \frac{100\angle 30}{100 + j100}$$
$$= \frac{100\angle 30}{\sqrt{100^2+100^2}\angle 45}$$
$$= \frac{100}{100\sqrt{2}}\frac{\angle 30}{\angle 45}$$
$$\tilde{I} = \frac{1}{\sqrt{2}}\angle -15^\circ$$

$$\tilde{V}_R = \tilde{I}R = \frac{1}{\sqrt{2}}\angle -15 \times 100$$
$$\tilde{V}_R = 0.71 \times 100 \angle -15 = 71\angle -15^\circ$$

$$\tilde{V}_L = \tilde{Z}_L\tilde{I} = j\omega L \times \frac{1}{\sqrt{2}}\angle -15$$
$$= 150\angle 90^\circ \times \frac{1}{\sqrt{2}}\angle -15^\circ$$
$$= A\angle 75^\circ$$

$$V_c = \tilde{Z}_c\tilde{I} = -j50 \times 0.71\angle -15$$
$$\simeq 35\angle -90\angle -15$$
$$\simeq 35\angle -105^\circ$$

$V_R = 71 \angle -15$

$V_L = A \angle 75^\circ$

$V_C = 35 \angle -105$

$V_s = 100 \angle 30^\circ$