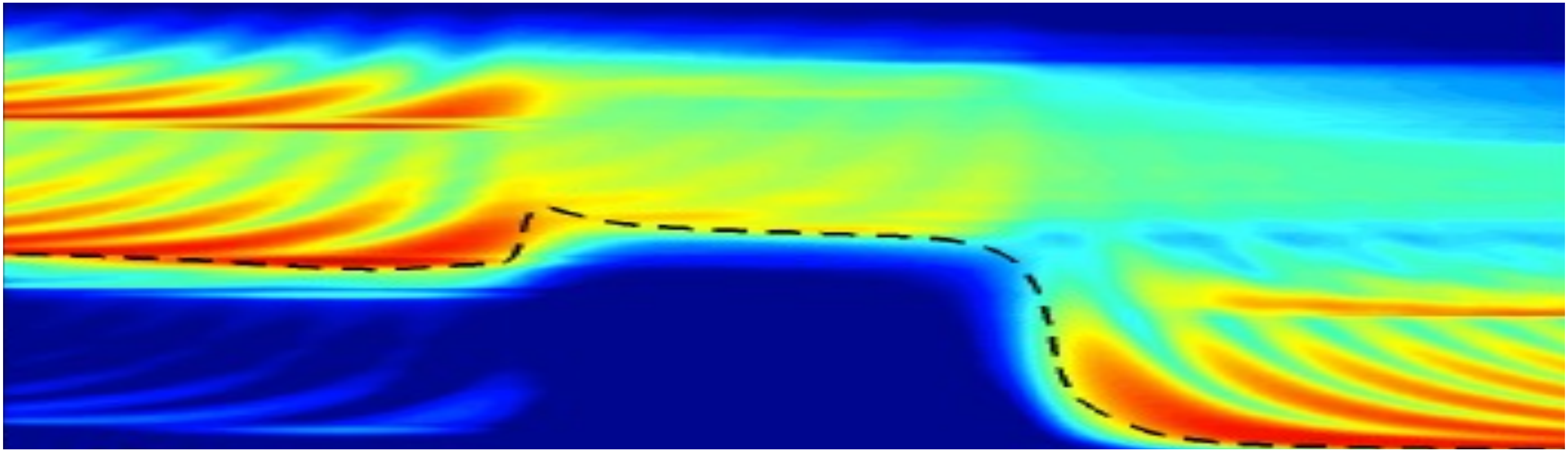

EECS 16B
Designing Information Devices and Systems II
Lecture 6

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Transient Response

- Outline
  - Power in the AC circuits
  - Transfer Function and Filters

- Reading- Hambley text sections 5.6, 6.1, 6.2, slides

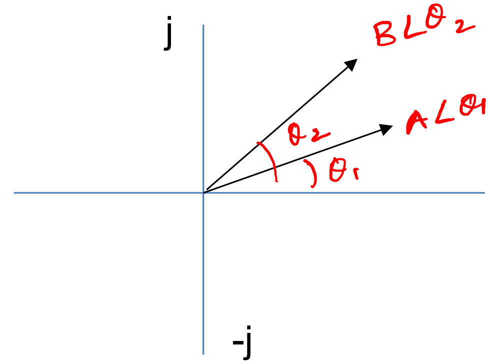
Recap: Phasors and sinusoidal steady state

$$\tilde{V}_1 = A\angle\theta_1$$

$$\tilde{V}_2 = B\angle\theta_2$$

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{A\,e^{j\omega\theta_1}}{B\,e^{j\omega\theta_2}} = \frac{A}{B}\,e^{j\omega(\theta_1-\theta_2)}$$

$$= \frac{A}{B}\angle\theta_1-\theta_2$$



$$\tilde{V}_1\,\tilde{V}_2 = AB\,e^{j\omega(\theta_1+\theta_2)} = AB\angle\theta_1+\theta_2$$

$$\downarrow$$

$$AB\cos\{\omega t + (\theta_1+\theta_2)\}$$

Phasors define phase relationships at an instant of time, i.e., with respect to ωt

Recap: Solving circuits with R, L and C

$$v_s(t) = iR + v_c + L\frac{di}{dt}$$

$$= RC\frac{dv_c}{dt} + v_c + LC\frac{d^2v_c}{dt^2}$$

$$\Rightarrow \frac{d^2v_c}{dt^2} + \frac{dv_c/dt}{L/R} + \frac{v_c}{LC} = \frac{v_s}{LC}$$

$$\Rightarrow \frac{d^2v_c}{dt^2} + 2\alpha\frac{dv_c}{dt} + \omega_0^2 v_c = \omega_0^2 v_s$$

Homogeneous solⁿ:

$$\frac{d^2v_c}{dt^2} + 2\alpha\frac{dv_c}{dt} + \omega_0^2 v_c = 0$$

Try: $e^{st} \rightarrow s^2 + 2\alpha s + \omega_0^2 = 0 \Rightarrow s_1, s_2$

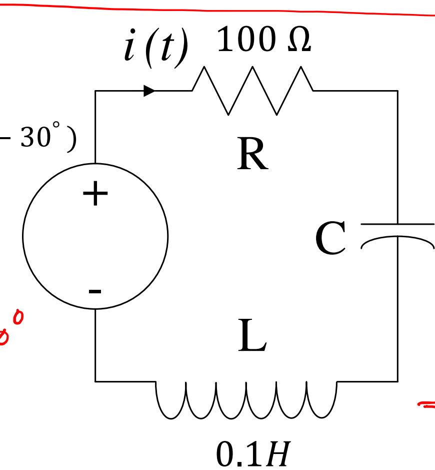
$$v_c^h(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\boxed{\begin{aligned} Z &= R + j\omega L + \frac{1}{j\omega C} \\ &= 100 + j200 - j100 = 100 + j100 \end{aligned}}$$

$v_s(t)$
$= 5\sqrt{2}\cos(2000\omega t - 30^\circ)$

$\omega = 2000$

$\tilde{V}_s = 5\sqrt{2}\angle -30^\circ$



$$\frac{1}{\omega C} = \frac{1}{2000\times 5\times 10^{-6}} = \frac{1}{1/100} = 100$$

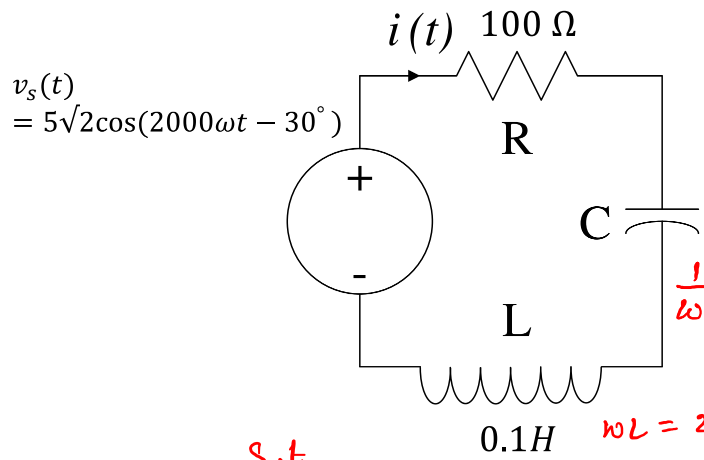
$$\omega L = 2000 \times 0.1 = 200$$

Recap: Solving circuits with R, L and C

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{5\sqrt{2}\angle -30^\circ}{100 + j100}$$

$$= \frac{5\sqrt{2}\angle -30^\circ}{100\sqrt{2}\angle 45^\circ}$$

$$= \frac{1}{20}\angle -75^\circ$$



$$\frac{1}{\omega C} = 100$$

$$\omega L = 200$$

$$\tilde{V}_c = \tilde{I}\,\overset{Z_C}{\frac{1}{j\omega C}} = -j\,\frac{1}{20}\angle -75\,(100)$$

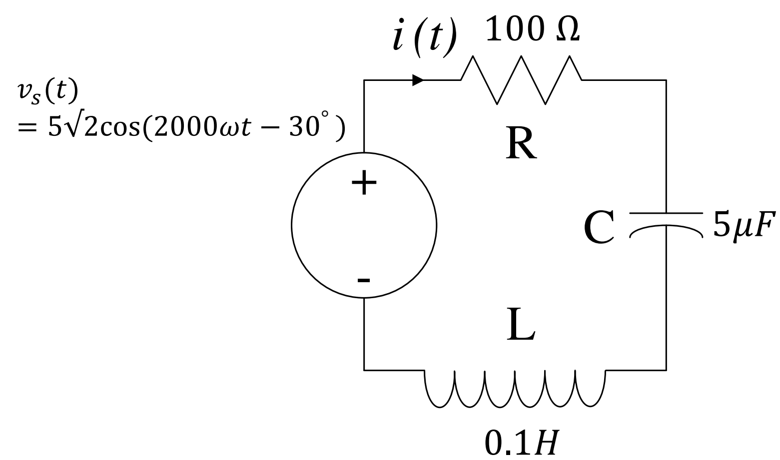
$$= 5\angle -90^\circ \angle -75^\circ$$

$$= 5\angle -165^\circ$$

$$v_c^{p}(t) = 5\cos(2000t - 165^\circ)$$

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + 5\cos(2000t - 165^\circ)$$

Recap: Solving circuits with R, L and C



Recap: Power in AC Circuits

Purely resistive Circuit

$$v_{rms} = \frac{V_m}{\sqrt{2}}$$

$$i_{rms} = \frac{I_m}{\sqrt{2}}$$

$v(t) = V_m cos\omega t$

$i(t) = I_m cos\omega t$

$$P_{avg} = \frac{1}{T}\int_0^T dt\ v(t)i(t) = \frac{V_m I_m}{T}\underbrace{\int_0^T dt\cos^2\omega t}_{} = \frac{V_m I_m}{T}\times\frac{T}{2}$$

$$P_{avg} = \frac{V_m I_m}{2}$$

$$= (v_{rms}\ i_{rms})$$

$$\frac{V_m I_m}{2T}\int_0^T 2\cos^2\omega t\, dt$$

$$= \frac{V_m I_m}{2T}\int_0^T (1+\cos 2\omega t)\, dt$$

$$= \frac{V_m I_m}{2T}\left[T - 0 + \frac{1}{2\omega}(\sin 2\omega t)_0^T\right]$$

$$= \frac{V_m I_m}{2T}\left[T + \frac{1}{2\omega}(\cancel{\sin 4\pi}^{\,0} - \sin 0)\right] = \frac{V_m I_m}{2}$$

Power in AC Circuits

Purely Inductive Circuit

$v(t) = V_m cos\omega t$

$i(t) = I_m \cos(\omega t - 90^{\circ})$

$$P_{avg} = \frac{1}{T}\int_0^T dt\ v(t)i(t) = \frac{V_m I_m}{T}\int_0^T dt\ cos\omega t sin\omega t$$

$$= \frac{V_m I_m}{2T}\int_0^T dt\ sin2\omega t$$

$= 0$

Purely Capacitive Circuit

$v(t) = V_m \cos(\omega t - 90^{\circ})$

$i(t) = I_m \cos(\omega t)$

$$P_{avg} = \frac{1}{T}\int_0^T dt\ v(t)i(t) = \frac{V_m I_m}{T}\int_0^T dt\ cos\omega t sin\omega t$$

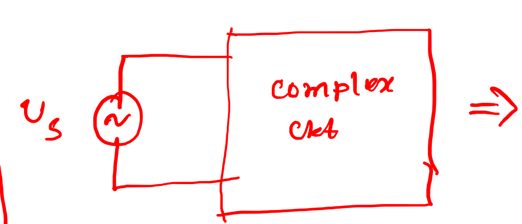
$$= \frac{V_m I_m}{2T}\int_0^T dt\ sin2\omega t = 0$$

Power in AC Circuits

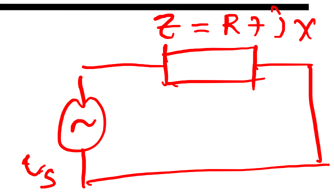
General case

$$v(t) = V_m \cos(\omega t + \theta_1)$$
$$i(t) = I_m \cos(\omega t + \theta_2)$$

$$P(t) = v(t)\, i(t)$$
$$= V_m \cos(\omega t + \theta_1)\, I_m \cos(\omega t + \theta_2)$$
$$= Re\{\underbrace{V_m e^{j(\omega t + \theta_1)}}_{\tilde{V}}\}\, Re\{\underbrace{I_m e^{j(\omega t + \theta_2)}}_{\tilde{I}}\}$$
$$= \frac{1}{2}(\tilde{V} + c.c_{\tilde{V}}) \times \frac{1}{2}(\tilde{I} + c.c_{\tilde{I}})$$
$$= \frac{1}{2}(\tilde{V} + \tilde{V}^*) \times \frac{1}{2}(\tilde{I} + \tilde{I}^*)$$



$\Rightarrow$



$* \equiv$ complex conjugate

$$= \frac{1}{4}\left[(\tilde{V}\tilde{I} + \tilde{V}^*\tilde{I}^*) + (\tilde{V}\tilde{I}^* + \tilde{V}^*\tilde{I})\right]$$
$$= \frac{1}{4}\left[(\tilde{V}\tilde{I}) + (\tilde{V}\tilde{I})^*\right] + \left[\tilde{V}\tilde{I}^* + (\tilde{V}\tilde{I}^*)^*\right]$$
$$= \frac{1}{2}\left[Re(\tilde{V}\tilde{I}) + Re(\tilde{V}\tilde{I}^*)\right]$$
$$= \frac{1}{2}\left[Re\{V_m I_m e^{j(\omega t + \theta_1 + \omega t + \theta_2)}\} + Re\{V_m I_m^* e^{j(\omega t + \theta_1 - \omega t - \theta_2)}\}\right]$$

$$p(t) = \frac{1}{2} \operatorname{Re}\left\{ V_m I_m \, e^{j(2\omega t + \theta_1 + \theta_2)} \right\} + \frac{1}{2} \operatorname{Re}\left\{ V_m I_m^* \, e^{j(\theta_1 - \theta_2)} \right\}$$

$$= \frac{V_m I_m}{2} \cos(2\omega t + \theta_1 + \theta_2) + \frac{V_m I_m}{2} \cos(\theta_1 - \theta_2)$$

$V_m I_m$ both are real numbers

$$P_{avg} = \frac{1}{T} \int_0^T dt \; p(t)$$

$$= \frac{V_m I_m}{2t} \underbrace{\int_0^T dt \cos(2\omega t + \theta_1 + \theta_2)}_{0} + \frac{V_m I_m}{2T} \Big[\underbrace{\int_0^T dt}_{T} \Big] \cos(\theta_1 - \theta_2)$$

$$= \frac{V_m I_m}{2} \cos \theta_d$$

$\theta_d = \theta_1 - \theta_2 =$ Difference in the phase angle between v and i

$$\boxed{P_{avg} = (v_{rms}\, i_{rms}) \cos \theta_d}$$

power factor

Power in AC Circuits

General case

$A = a + jb$

$B = c + jd$

$AB = (a + jb)(c + jd)$

$= ac - bd + j(bc + da)$

$A^* B^* = (a^* - jb^*)(c^* - jd^*)$

$= a^*c^* - b^*d^* - j(b^*c^* + d^*a^*)$

$(AB)^* = a^*c^* - b^*d^* - j(b^*c^* + d^*a^*) = A^* B^*$

Power in AC Circuits: Power Triangle

$Z = |Z| \angle \theta$

$I = \dfrac{\tilde{V}_s}{Z} = \underbrace{\dfrac{\tilde{V}_s}{|Z|}}_{I_m} \angle -\theta$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos\theta_d}$$

$$= \frac{V_m I_m}{2} \frac{R}{|Z|}$$

$$= \frac{1}{2} I_m \underbrace{\frac{V_m}{|Z|}}_{I_m} R$$

$$\boxed{P_{arg} = \frac{1}{2} I_m^2 R} \equiv P$$

$\theta \equiv \theta_d$

$\tilde{V}_s = V_m \angle 0^\circ$

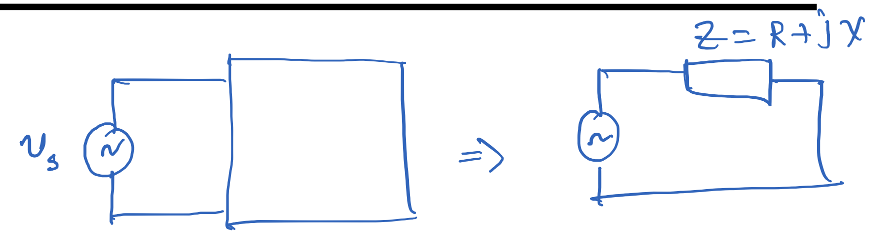
$\dfrac{V_m}{|Z|} = \dfrac{|\tilde{V}_s|}{|Z|} = I_m$

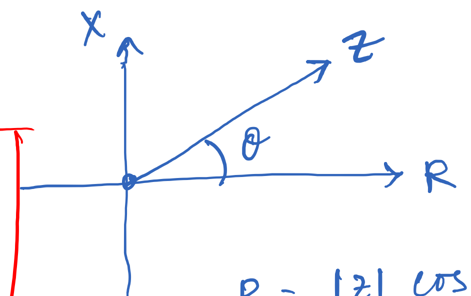
$$Q = \frac{1}{2} I_m^2 X$$

$$= \frac{1}{2} I_m \frac{V_m}{|Z|} X$$

$$\boxed{Q = \frac{1}{2} V_m I_m \sin\theta}$$

$$P^2 + Q^2 = \left(\frac{V_m I_m}{2}\right)^2$$

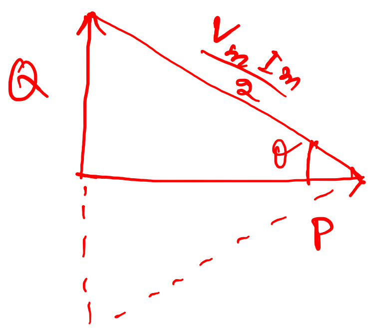




$R = |Z| \cos\theta$

$X = |Z| \sin\theta$

Power in AC Circuits: Power Triangle



complex power:

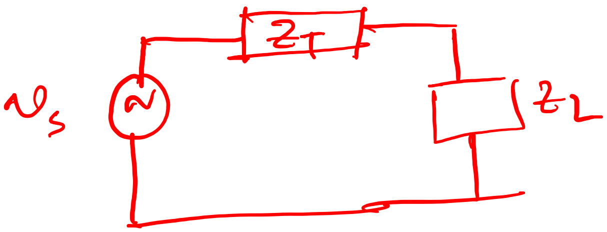
$$\tilde{V} = V_m e^{j\omega t}$$
$$\tilde{I} = I_m e^{j(\omega t + \theta)}$$

$$S = \frac{1}{2} \tilde{V} \tilde{I}^*$$

$$= \frac{1}{2} V_m I_m \left[\cos\theta + j \sin\theta\right]$$

$$\boxed{S = P + jQ}$$

Power in AC Circuits: Maximum Power Transfer



$$Z = Z_T + Z_L$$
$$= R + jX$$

$$I = \frac{\tilde{V}_s}{R + jX}$$

$$= \frac{\tilde{V}_s}{\sqrt{R^2 + X^2} \angle \tan^{-1} X/R}$$

max power gets transferred when $X = 0$

$$Z_T = Z_L^*$$

if $Z_T = a + jb$
$Z_L = a - jb$

$Z = 2a$