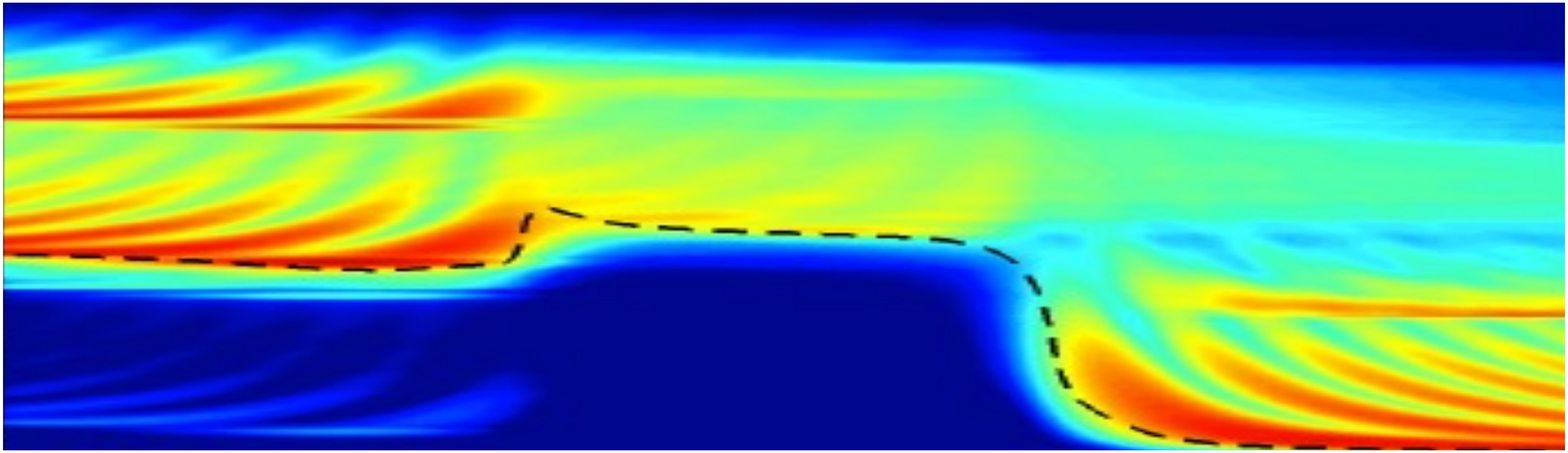


EECS 16B
Designing Information Devices and Systems II
Lecture 7

Prof. Sayeef Salahuddin
Department of Electrical Engineering and Computer Sciences, UC Berkeley,
sayeef@eecs.berkeley.edu

Transient Response

- Outline
  - Power in the AC circuits
  - Transfer Function and Filters

- Reading- Hambley text sections 5.6, 6.1, 6.2, 6.3 slides

Maximum Power Transfer

$$P_{R_2} = I^2 R_2$$

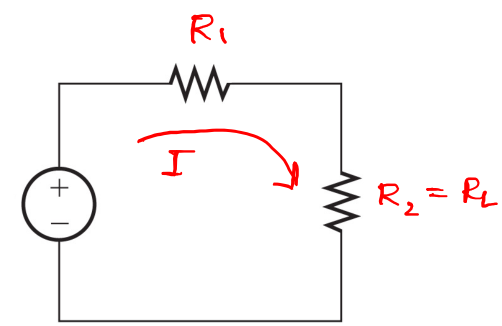
$$I = \frac{V}{R_1 + R_2}$$

$$= \frac{V^2}{(R_1+R_2)^2} \cdot R_2$$

$$= \frac{V^2}{R_1} \cdot \frac{R_1 R_2}{(R_1+R_2)^2}$$

$$= \frac{1}{4} \frac{V^2}{R_1} \frac{4 R_1 R_2}{(R_1+R_2)^2}$$

$$= \frac{1}{4} \frac{V^2}{R_1} \frac{(R_1+R_2)^2 - (R_1 - R_2)^2}{(R_1+R_2)^2}$$

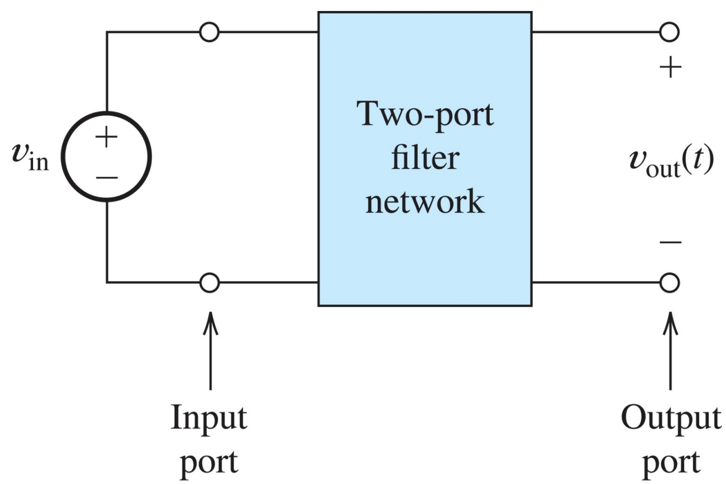


$$P_{R_2} = \frac{1}{4} \frac{V^2}{R_1} \left[1 - \left(\frac{R_1 - R_2}{R_1 + R_2} \right)^2 \right]$$

$P_{R_2}^{max}$ happens at $R_1 = R_2 = R$

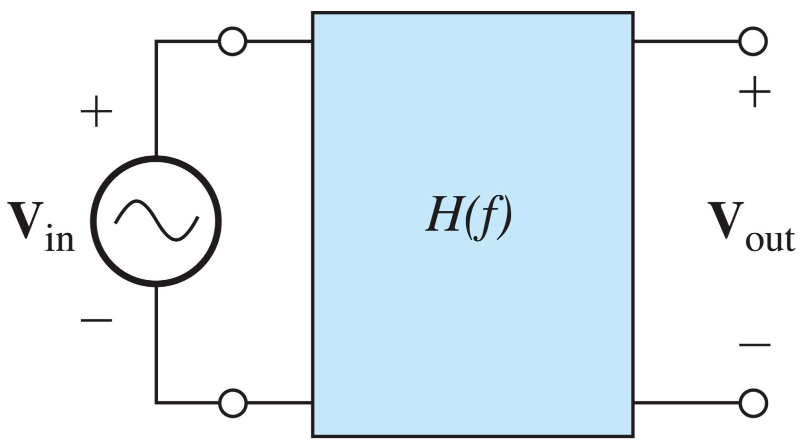
$$\boxed{P_{R_2} = \frac{1}{4} \frac{V^2}{R}}$$

Concept of Transfer Function



Two port Filter Network or more generally Two port network

Concept of Transfer Function

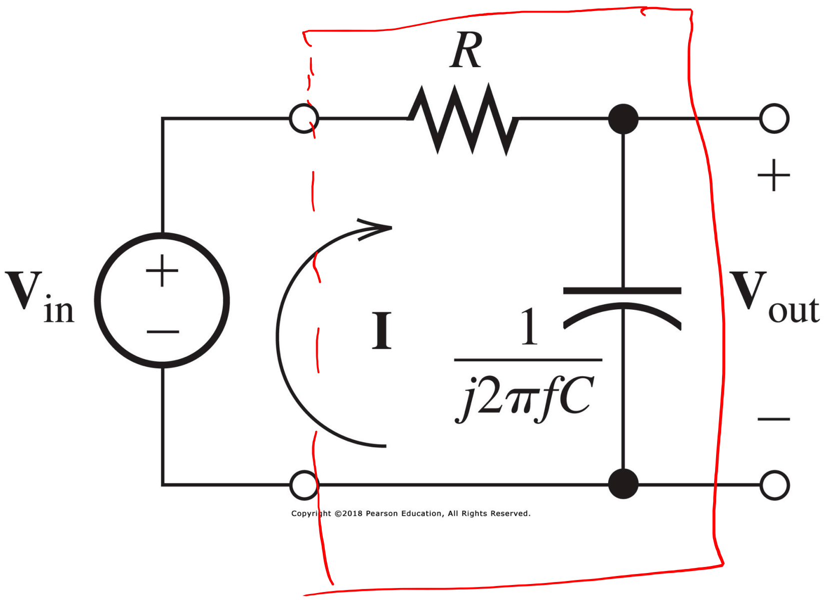


$$H(f) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}}$$

Transfer function

$H(f)$ is a complex number

A simple RC Circuit



$$H(f) = \frac{V_{out}}{V_{in}}$$

$$V_{out} = I Z_c = \frac{V_{in}}{R + Z_c} Z_c = \frac{V_{in}}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

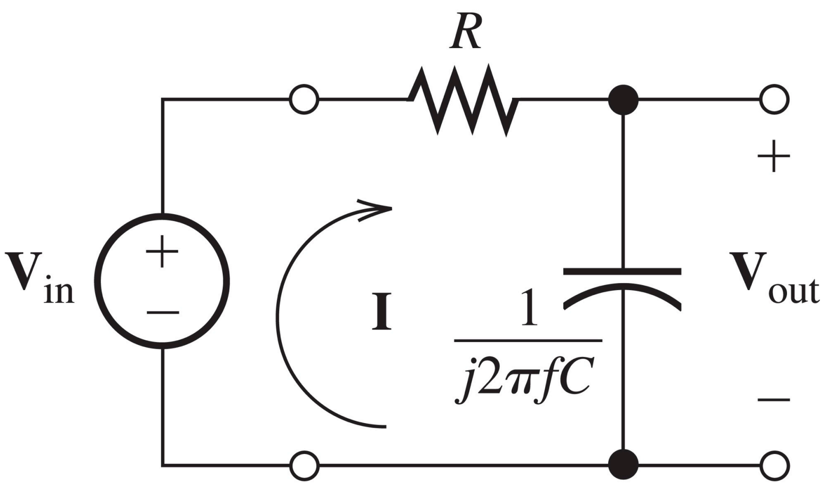
$$\frac{V_{out}}{V_{in}} = H(f) = \frac{1}{j\omega RC + 1}$$

$$= \frac{1}{\sqrt{1 + (\omega RC)^2} \angle \tan^{-1}(\omega RC)}$$

$$= \frac{\angle -\tan^{-1}\left(\frac{\omega}{\omega_B}\right)}{\sqrt{1 + \left(\frac{\omega}{\omega_B}\right)^2}}$$

$$\boxed{\frac{1}{RC} = \omega_B}$$

A simple RC Circuit

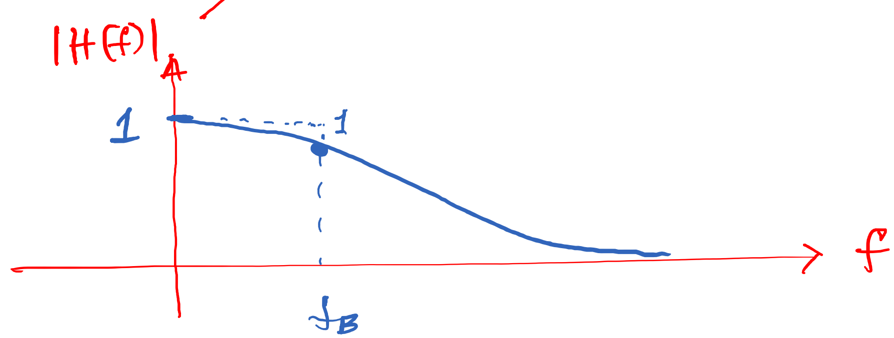


$$H(f) = \frac{1}{\sqrt{1+(\omega/\omega_B)^2}} \angle -\tan^{-1}\frac{\omega}{\omega_B}$$

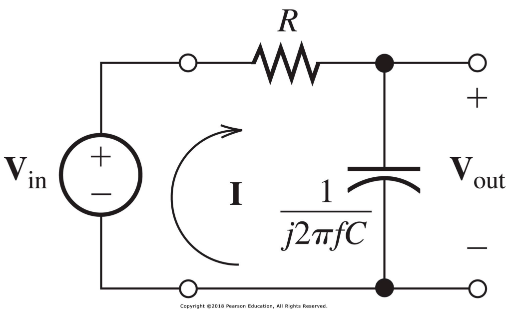
$$\omega = 2\pi f$$

$$\omega_B = \frac{1}{RC} = 2\pi f_B$$

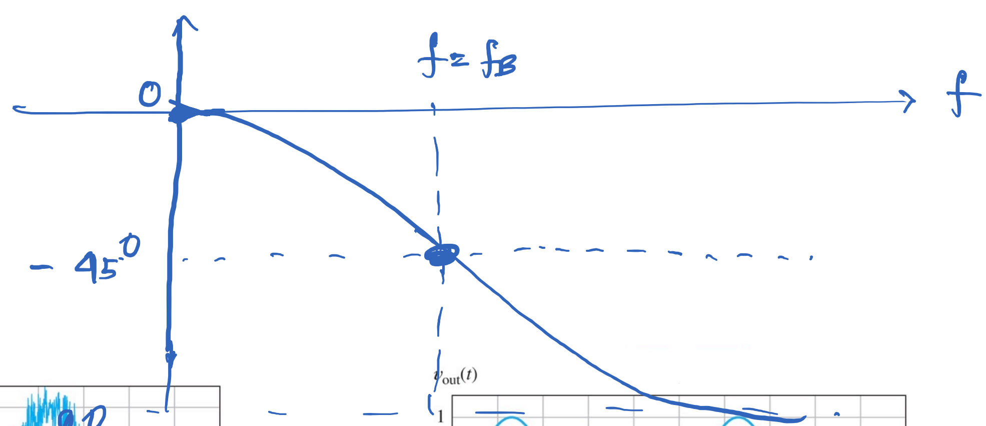
$$H(f) = \boxed{\frac{1}{\sqrt{1+(f/f_B)^2}}} \angle -\tan^{-1}(f/f_B)$$

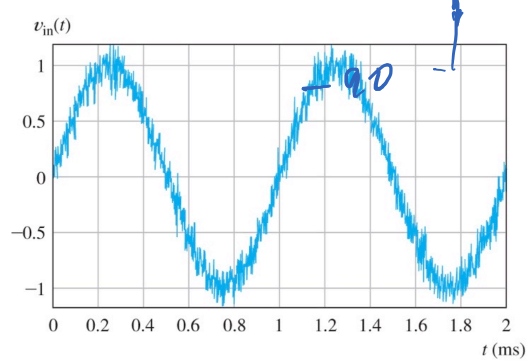


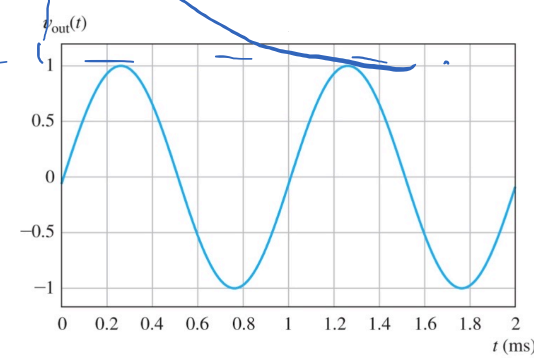
First Order low pass filter



phase : $-\tan^{-1}(f/f_B)$







Decibels

Decibel:

$$|H(f)|_{dB} = 20\log_{10}|H(f)|$$

$$20\log_{10}10^{2}$$

$$= 40\underbrace{\log_{10}10}_{1}$$

$$= 40\ dB$$

$$20\log_{10}\frac{1}{\sqrt{2}} = 20\log_{10}2^{-\frac{1}{2}}$$

$$= -10\underbrace{\log_{10}2}_{0.3}$$

$$= -3\ dB$$

Table 6.2 Transfer-Function Magnitudes and Their Decibel Equivalents

$\lvert H(f)\rvert$	$\lvert H(f)\rvert_{dB}$
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	−3
1/2	−6
0.1	−20
0.01	−40

Cascaded Networks

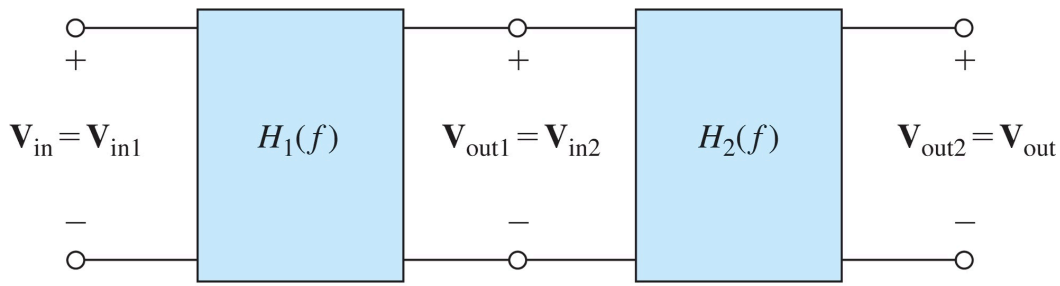
$$H(f) = \frac{V_{out}}{V_{in}}$$

$$= \frac{V_{out_2}}{V_{in_2}} \times \frac{V_{in_2}}{V_{in}}$$

$$= \frac{V_{out_2}}{V_{in_2}} \times \frac{V_{out_1}}{V_{in_1}}$$

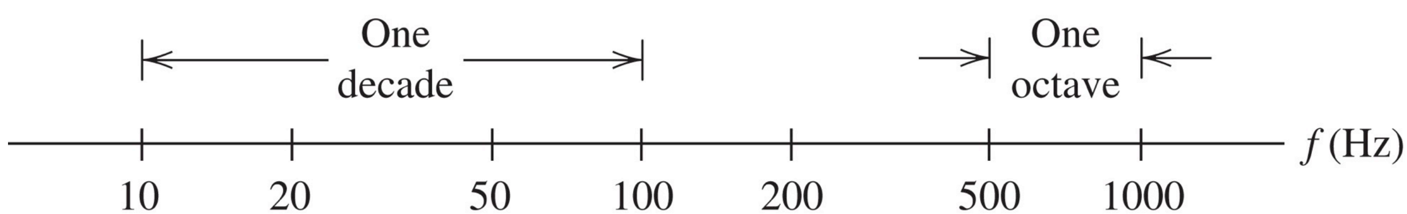
$$= H(f)|_1 \times |H(f)|_2$$

$$|H(f)|_{dB} = 20\log_{10}|H(f)|_1 + 20\log_{10}|H(f)|_2$$



Logarithmic Frequency Scales

- A **decade** is a range of frequencies for which the ratio of the highest frequency to the lowest is **10**
- An **Octave** is a range of frequencies for which the ratio of the highest frequency to the lowest is **2**



Logarithmic Frequency Scales

10 Hz $\longleftrightarrow$ 1000 Hz $\longrightarrow$ Octaves:

$$n = \log_2\left(\frac{f_2}{f_1}\right) = \frac{\log_{10} f_2/f_1}{\log_{10} 2} \doteq \frac{2}{0.3}$$

$$\simeq 6$$

10 — 100 — 1000

$$n = \log_{10}\left(\frac{f_2}{f_1}\right)$$

$$= \log_{10}\frac{10^3}{10^1}$$

$$= \log_{10} 10^2$$

$$= 2$$

10
20
40
80
160
320
640

Bode Plots

A **Bode** plot is a plot of the decibel magnitude of a newtwork function versus log-scale frequency

$f = 0, \quad |H(f)|_{dB} = 0$

$f = f_B, \quad |H(f)|_{dB} = -10 \log_{10}(1+1) = -3\,dB$

$f \gg f_B, \quad |H(f)|_{dB} \cong -10 \log_{10} (f/f_B)^2$

$= -20 \log_{10} (f/f_B)$

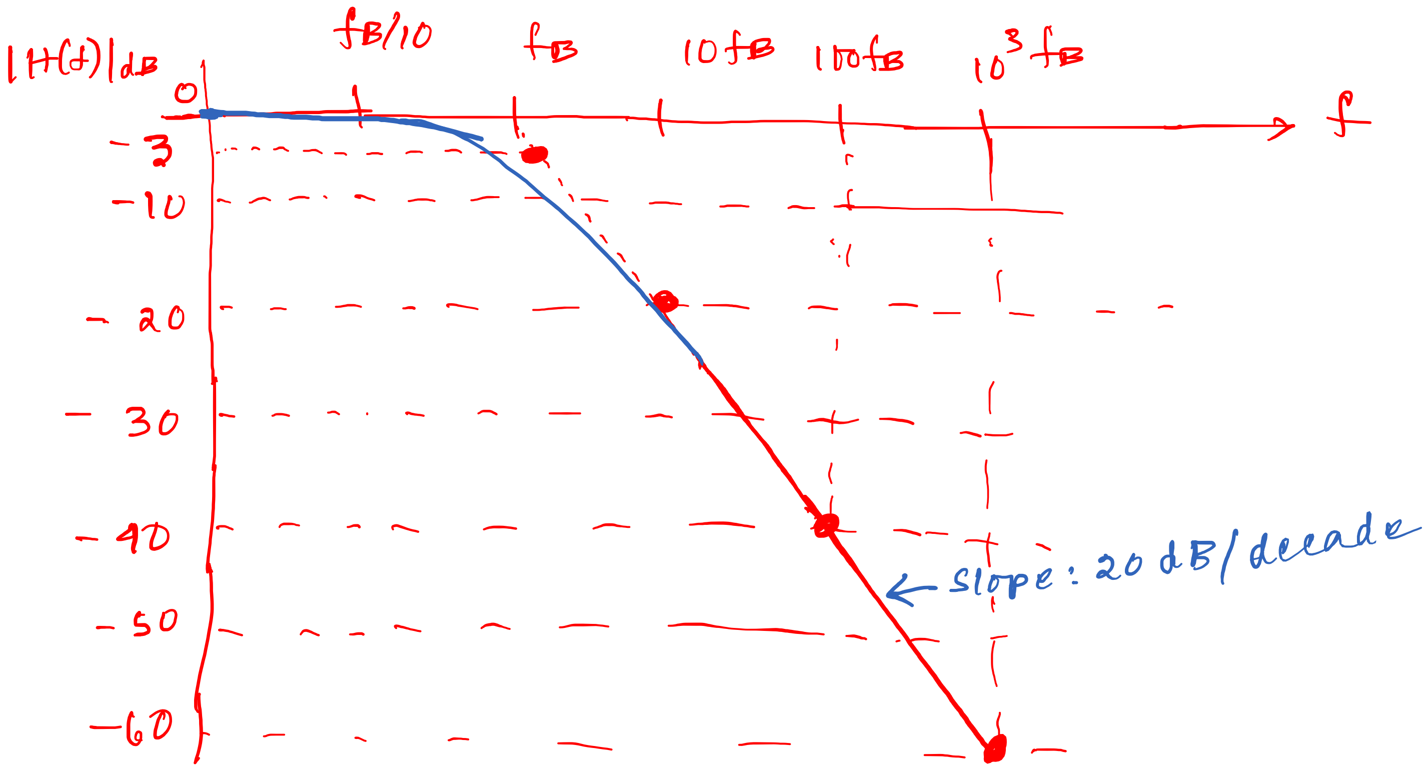
$\boxed{|H(f)|_{dB} = -20 \log_{10} f + 20 \log_{10} f_B}$ $\leftarrow y = mx + c$

$|H(f)|_{dB} = 20 \log_{10} |H(f)|$

$= 20 \log_{10} \frac{1}{\sqrt{1+(f/f_B)^2}}$ $-1/2$

$= 20 \log_{10} [1+(f/f_B)^2]$

$= -10 \log_{10} [1+(f/f_B)^2]$


|H(f)|dB
0
- 3
- 10
- 20
- 30
- 40
- 50
-60
fB/10
fB
10fB
100fB
10^3 fB
f
slope: 20 dB/decade

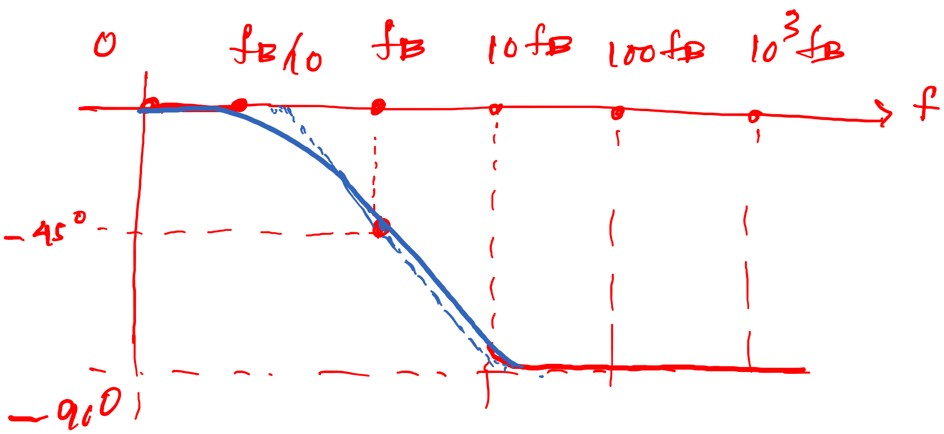
Bode Plots

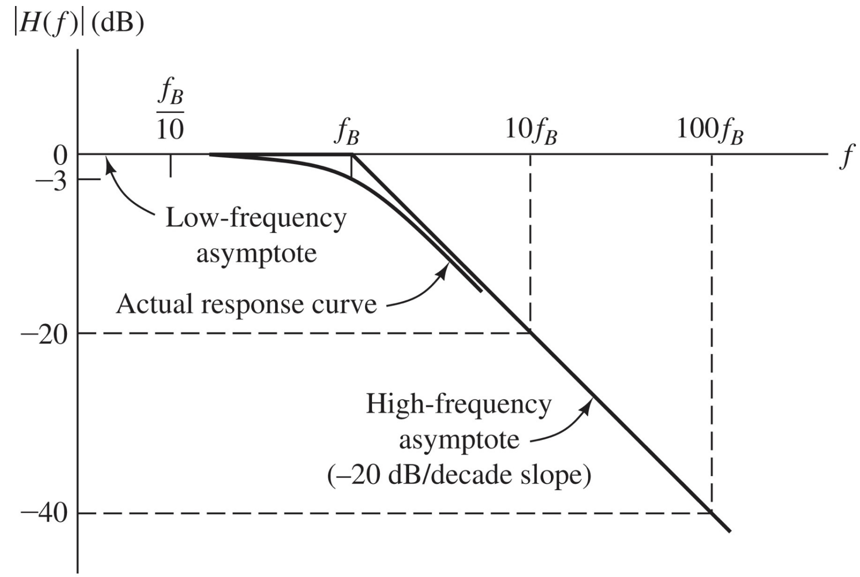
phase: $\angle -\tan^{-1} f/f_B$

$f = 0 \rightarrow$ phase $= 0$

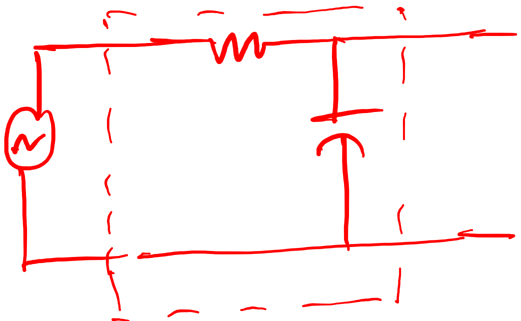
$f = f_B \rightarrow$ phase $= -45^\circ$

$f \gg f_B \rightarrow$ phase $= -90^\circ$

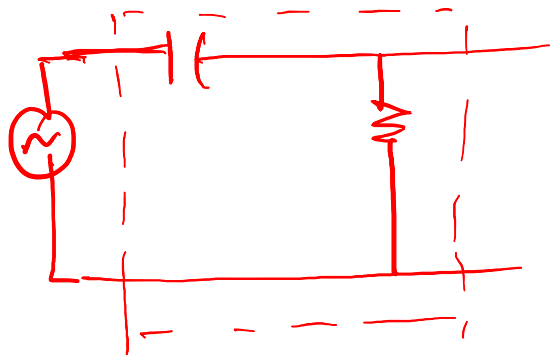




High Pass Filter



LP



HP