

EECS 16B

Designing Information Devices and Systems II

Lecture 8

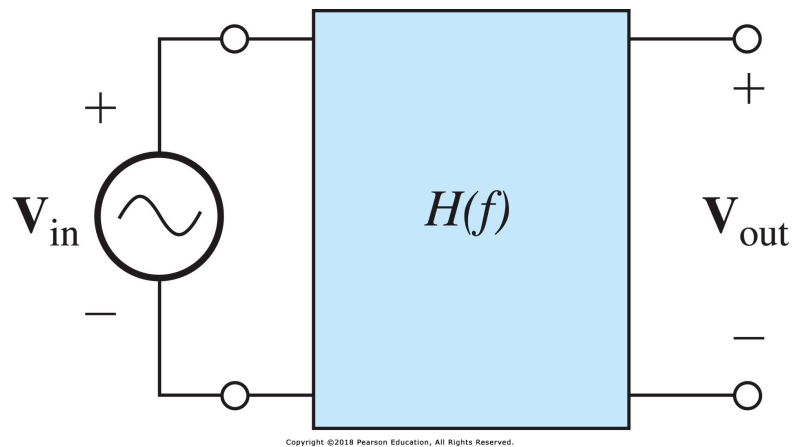
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Transient Response

- Outline
 - High Pass Filters
 - Series and Parallel Resonance
 - Amplifiers and Devices
- Reading- Hambley text sections 6.4, 6.5, 6.6, 6.7, slides

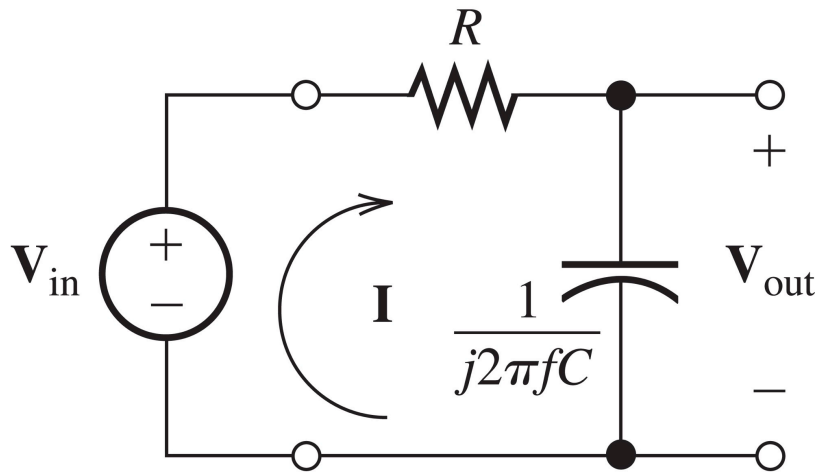
Recap: Concept of Transfer Function



$$H(f) = \frac{V_{out}}{V_{in}}$$

$H(f)$ is a complex number

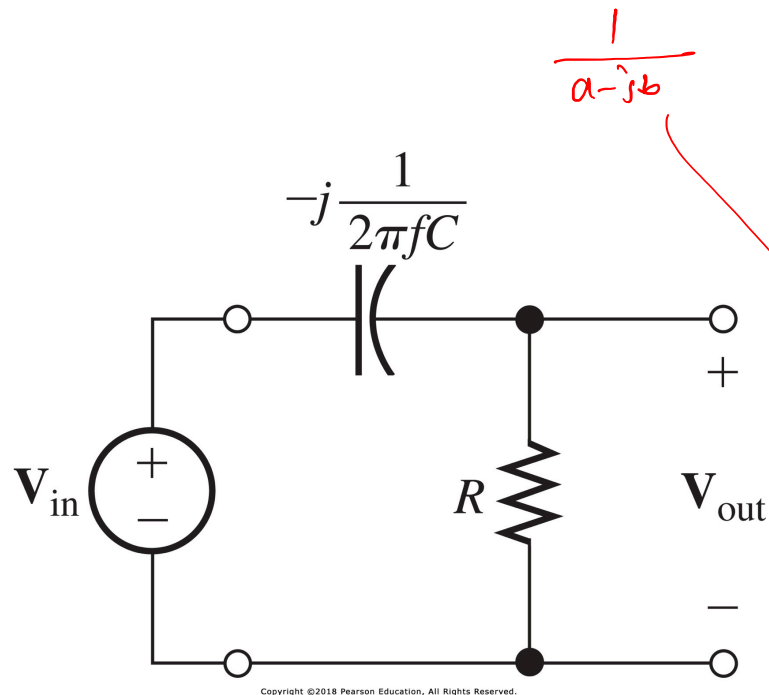
Recap: First order low pass filter



$$H(f) = \frac{1}{\sqrt{\left(1 + \frac{f^2}{f_B^2}\right)}} \angle -\tan^{-1}\left(\frac{f}{f_B}\right)$$

$$f_B = \frac{1}{2\pi RC}$$

First order High Pass Filter



$$\frac{1}{a-jb}$$

$$H(f) = \frac{V_{out}}{V_{in}}$$

$$= \frac{R}{R + \frac{1}{j\omega C}} \frac{V_{in}}{V_{in}}$$

$$= \frac{1}{1 - j \frac{1}{\omega RC}}$$

$$= \frac{1}{1 - j \frac{\omega_B}{\omega}}$$

$$= \frac{1}{\sqrt{1 + \frac{\omega_B^2}{\omega^2}}}$$

$$\angle \tan^{-1} \frac{\omega_B}{\omega}$$

$$H(f) = \frac{1}{\sqrt{1 + \left(\frac{f_B}{f}\right)^2}} \angle \tan^{-1} \frac{f_B}{f}$$

First order High Pass Filter

$$H(f) = \frac{1}{\sqrt{1 + \frac{f_B^2}{f^2}}} \angle \tan^{-1} \left(\frac{f_B}{f} \right)$$

$$|H(f)|_{dB} = 20 \log_{10} \left(\frac{1}{\sqrt{1 + \left(\frac{f_B}{f}\right)^2}} \right)$$

$$= 20 \log_{10} \left[1 + \left(\frac{f_B}{f}\right)^2 \right]^{-1/2}$$

$$= -10 \log_{10} \left[1 + \left(\frac{f_B}{f}\right)^2 \right]$$

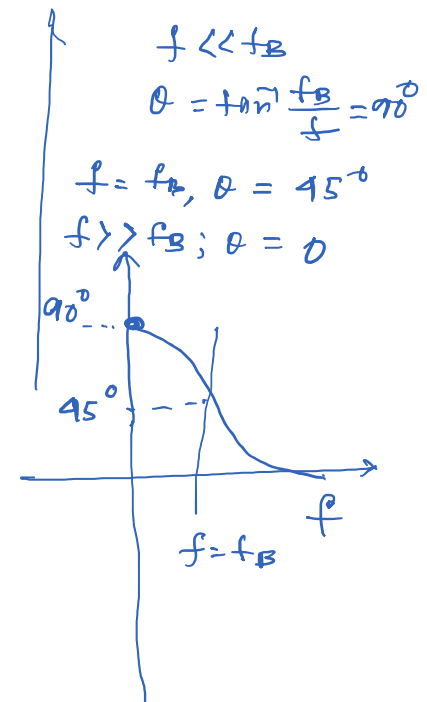
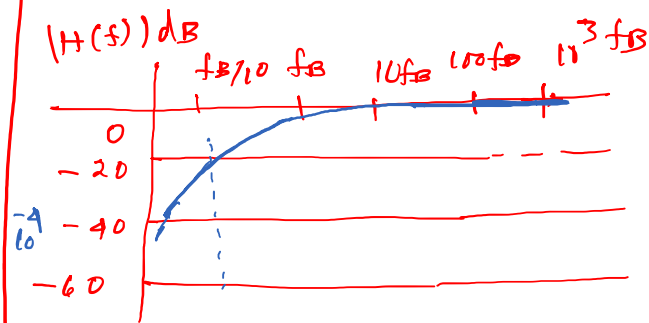
$$f \ll f_B \rightarrow |H(f)|_{dB} \approx -10 \log_{10} \left(\frac{f_B}{f} \right)^2$$

$$= -20 \log_{10} \frac{f_B}{f}$$

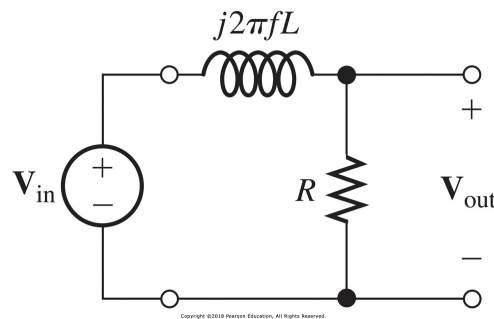
$$|H(f)|_{dB} = 20 \log_{10} f - 20 \log_{10} f_B$$

$$f = f_B, |H(f)|_{dB} = -10 \log_{10} 2 = -3$$

$$f \gg f_B, |H(f)|_{dB} \approx -10 \log_{10} 1 = 0$$



Low Pass and High Pass Filters with Inductors



$$\omega_B = \frac{1}{RC}$$

$$\omega_L = \frac{R}{L}$$

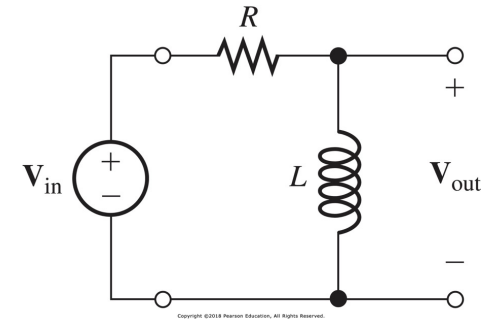
$$V_{out} = \frac{R}{R + j\omega L} V_{in}$$

$$\therefore \frac{V_{out}}{V_{in}} = H(f) = \frac{1}{1 + j\omega \frac{L}{R}}$$

$$= \frac{1}{1 + j \frac{\omega}{\omega_L}}$$

$$= \frac{1}{1 + j f/f_c}$$

similar form to a RC LP filter



$$V_{out} = \frac{j\omega L}{R + j\omega L}$$

$$= \frac{1}{1 + \frac{R}{j\omega L}}$$

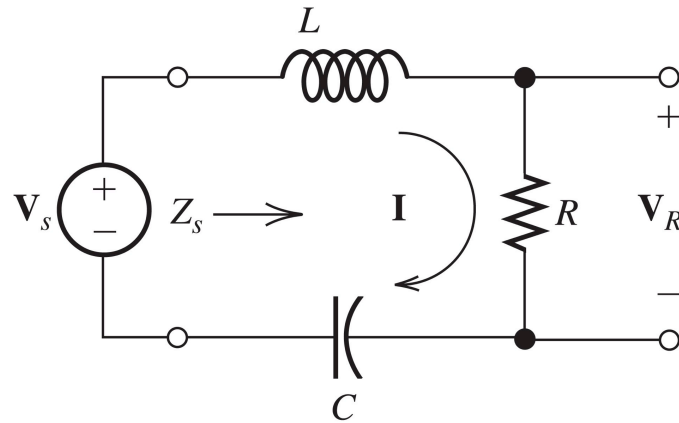
$$= \frac{1}{1 - j \frac{\omega_L}{\omega}}$$

$$= \frac{1}{1 - j(f_c/f)}$$

same form as RC HP filter

Resonant Circuits

Series Resonance



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Recap: R-L-C circuits: Response in time

$$\frac{d^2 v_c}{dt^2} + \frac{1}{L/R} \frac{dv_c}{dt} + \frac{1}{LC} v_c = \frac{v_s}{Le}$$

$$2\alpha = \frac{1}{L/R} \Rightarrow \alpha = \frac{R}{2L}$$

$$\frac{d^2 v_c}{dt^2} + 2\alpha \frac{dv_c}{dt} + \omega_0^2 v_c = \frac{v_s}{Le}$$

$$\omega_0^2 = \frac{1}{LC}$$

Homogeneous solution

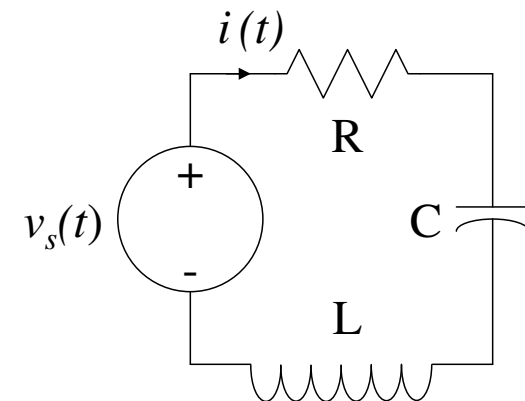
$$\frac{d^2 v_c}{dt^2} + 2\alpha \frac{dv_c}{dt} + \omega_0^2 v_c = 0$$

From previous discussions we have seen that an exponential solution works

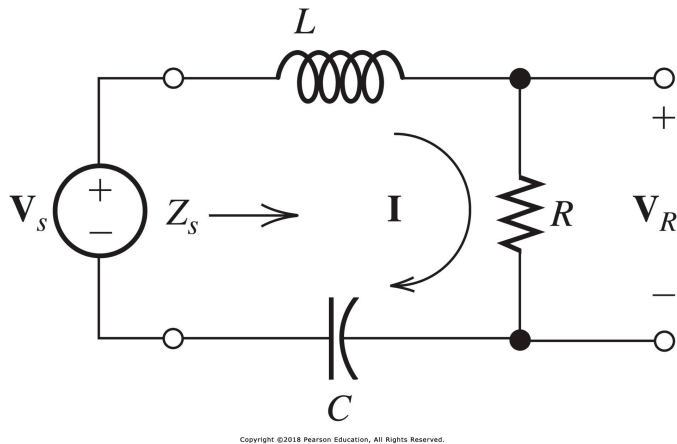
Lets try: $v_c(t) = Ae^{st}$

$$As^2 e^{st} + 2\alpha As e^{st} + \omega_0^2 A e^{st} = 0$$

$$\Rightarrow \boxed{s^2 + 2\alpha s + \omega_0^2 = 0}$$



Series Resonance



$$Z = R + j\omega L - \frac{j}{\omega C}$$

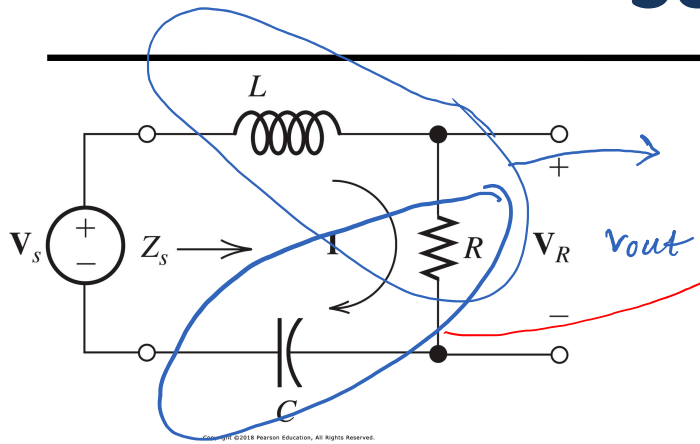
What happens at $\omega = \omega_0$?

$$z = R + j\omega L \left[1 - \frac{1}{\omega^2 LC} \right]$$

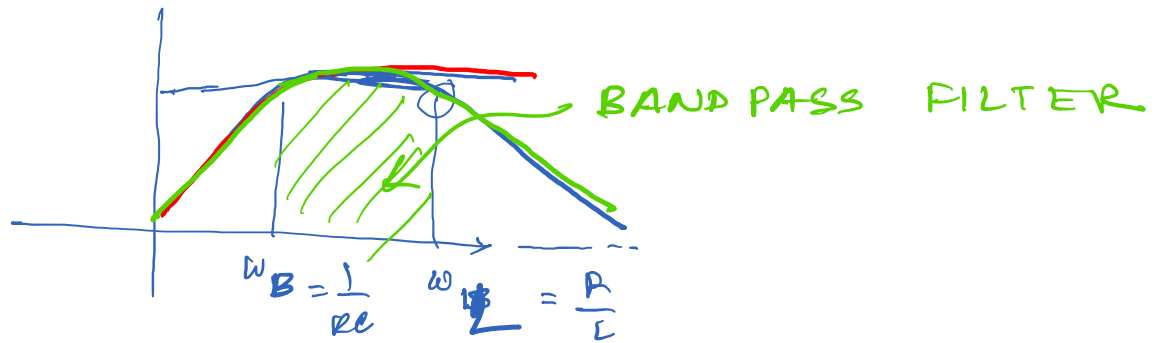
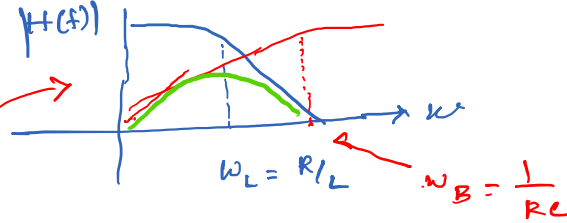
$$= R + j\omega L \left[1 - \frac{\omega_0^2}{\omega^2} \right] = R \Big|_{\omega = \omega_0}$$

At resonance, all reactive impedances
sum up to zero

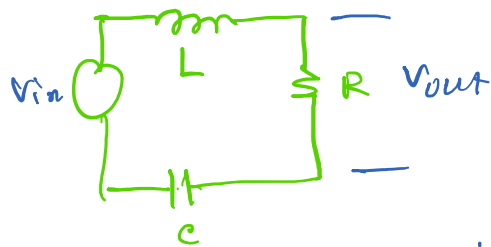
Series Resonance



Qualitatively



Series Resonance



$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$V_{out} = \frac{R}{Z} v_{in}$$

$$H(f) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L - \frac{j}{\omega C}}$$

$$= \frac{R}{R + j\omega L \left(1 - \frac{\omega_0^2}{\omega^2}\right)}$$

$$V_{out} = IR$$

$$I = \frac{V_{in}}{R + j\omega L - \frac{j}{\omega C}}$$

$$Z = \frac{R}{I}$$

$$V_{out} = \frac{R}{Z} v_{in}$$

$$H(f) = \frac{1}{1 + \frac{j\omega L}{R} \left(1 - \frac{\omega_0^2}{\omega^2}\right)}$$

$$= \frac{1}{1 + j\frac{L}{R} \left(\omega - \frac{\omega_0^2}{\omega}\right)}$$

$$= \frac{1}{1 + j\frac{\omega_0 L}{R} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$= \frac{1}{1 + \underbrace{j\frac{\omega_0 L}{R}}_{Q_s} \left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

Series Resonance

$$H(f) = \frac{1}{1 + jQ_s \left(\frac{f}{f_0} - \frac{f_0}{f} \right)}$$

$$\angle \tan^{-1} Q_s \left(\frac{f}{f_0} - \frac{f_0}{f} \right)$$

$$= \frac{1}{\sqrt{1 + Q_s^2 \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2}}$$

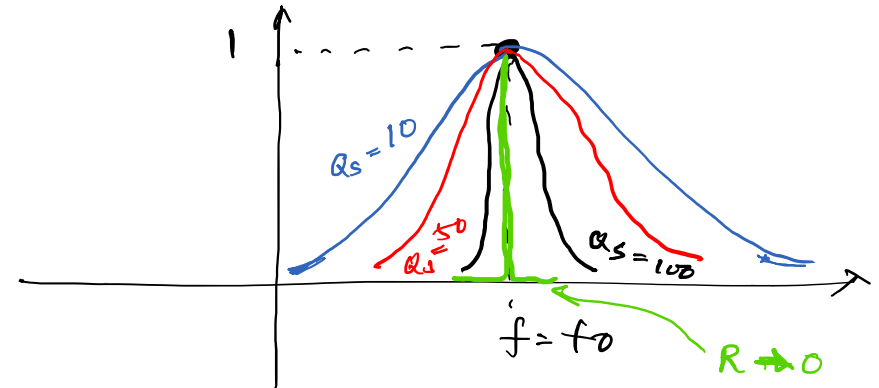
At $f = f_0$, $|H(f)| = \frac{1}{\sqrt{1 + Q_s^2 \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2}}$

$$= 1$$

$f \gg f_0$, $|H(f)| \approx \frac{1}{\sqrt{Q_s^2 \left(\frac{f}{f_0} \right)^2}}$

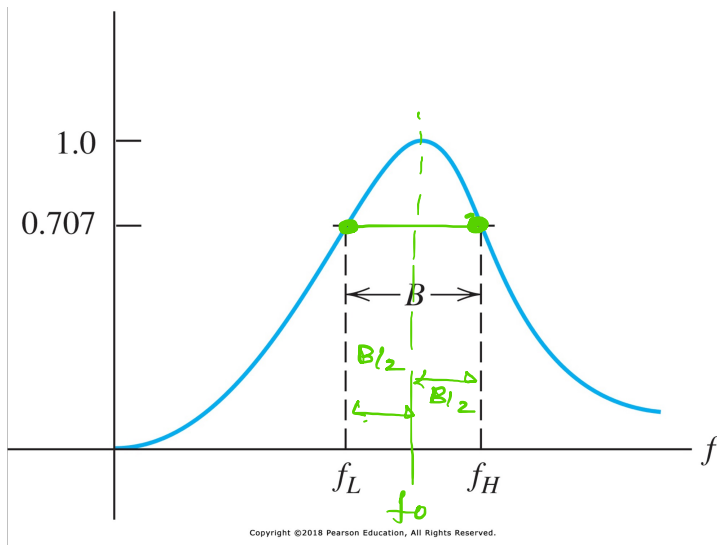
$$= \frac{1}{Q_s} \frac{f_0}{f} \approx 0$$

$f \ll f_0$, $|H(f)| \approx \frac{1}{Q_s} \frac{f}{f_0} \approx 0$



$$Q_s = \frac{\omega_0 L}{R}$$

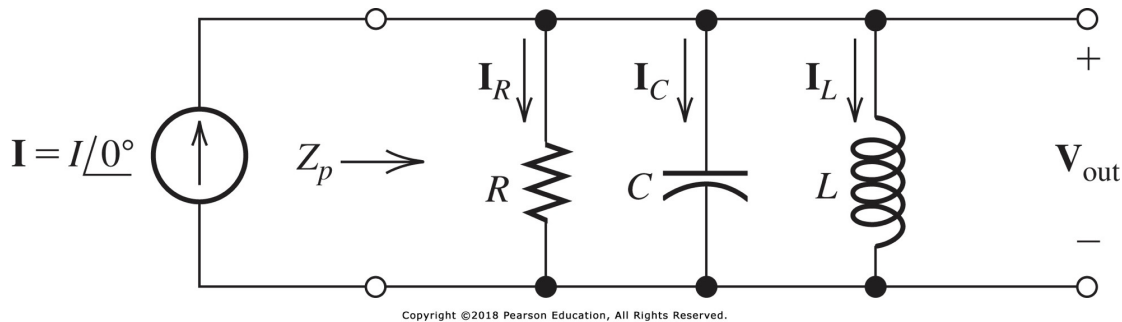
Series Resonance Bandpass Filter



Half-power frequencies are defined as the frequencies where the magnitude of the transfer function has fallen by a factor of $\frac{1}{\sqrt{2}} = 0.707$

It can be shown that
$$B = f_H - f_L = \frac{f_0}{Q_s}$$

Parallel Resonance



$$H(s) = \frac{V_{out}}{I}$$

$$V_{out} = I Z_p$$

$$\begin{aligned} \frac{V_{out}}{I} = Z_p &= \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} \\ &= \frac{1}{\frac{1}{R} + j\omega C \left[1 - \frac{\omega_0^2}{\omega^2}\right]} \end{aligned}$$

$$= \frac{R}{1 + j\omega_0 RC \left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right]}$$

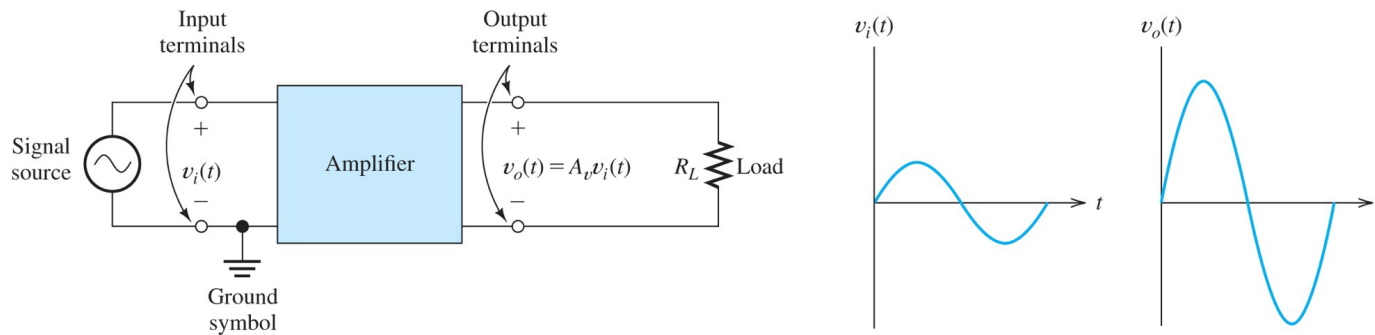
$$Q_s = \frac{\omega_0 L}{R}$$

$$Q_p = \omega_0 RC$$

$$= \frac{R}{1 + jQ_p \left(f/f_0 - f_0/f \right)}$$

same form as series resonant ckt

Active Devices



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- Active devices are made of semiconductors
 - Semi-conductors are materials whose resistance is in between a metal and insulator
- Half
- More interestingly, one is able to change the resistance of the semiconductor materials by using external control such as voltage or current