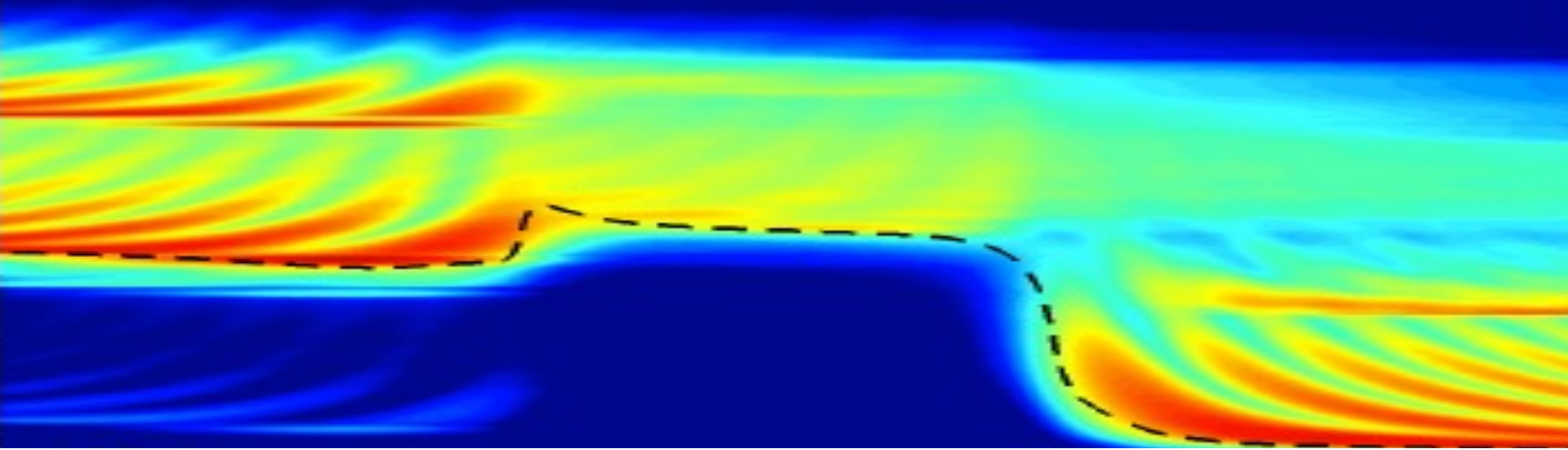

EECS 16B
Designing Information Devices and Systems II
Lecture 12

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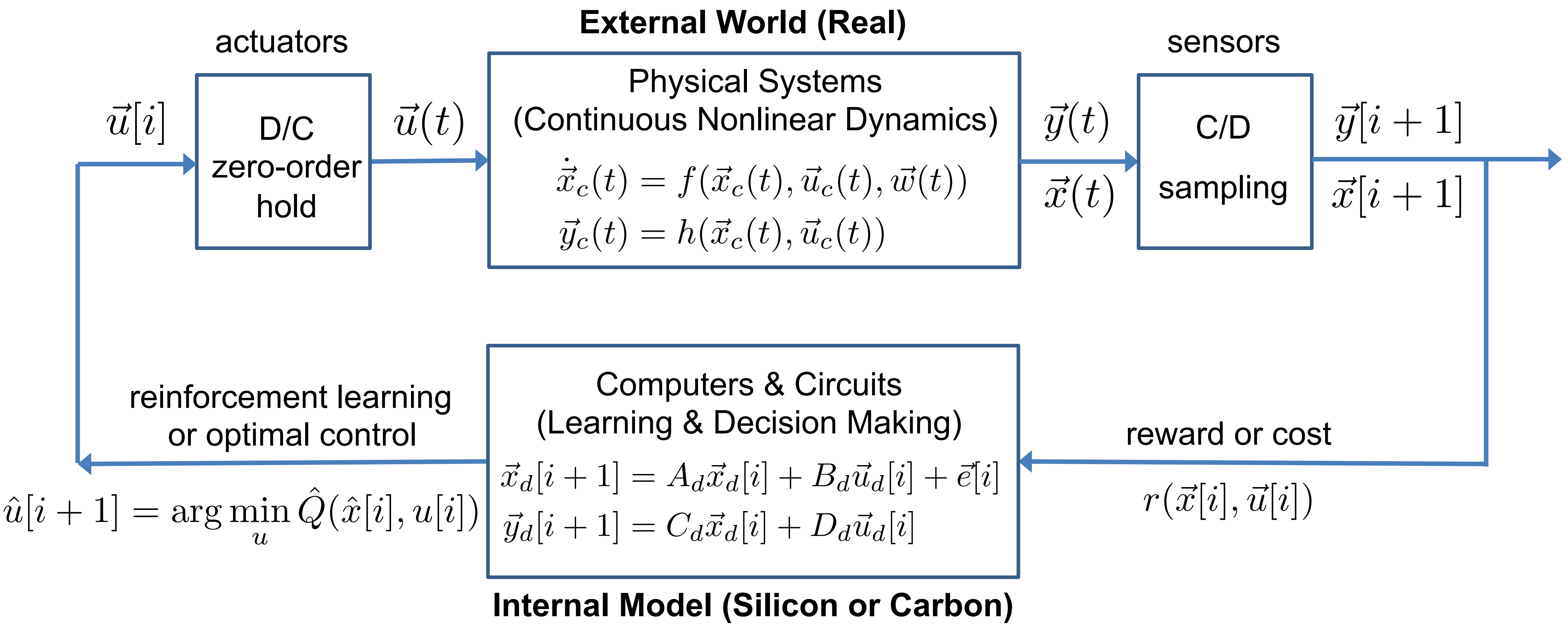
Outline

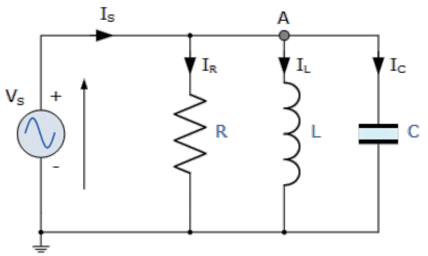
- System Modeling
- Discretization (scalar and vector case)
- System Identification

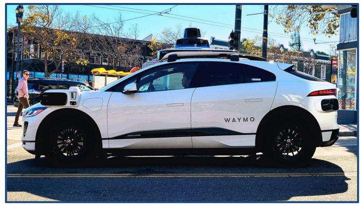
System Modeling & Control

All autonomous intelligent (AI) systems rely on closed-loop learning and control:



System Modeling





mathematical modeling
from first principles

$$\dot{\vec{x}}_c(t) = f(\vec{x}_c(t), \vec{u}_c(t), \vec{w}(t))$$
$$\vec{y}_c(t) = h(\vec{x}_c(t), \vec{u}_c(t))$$

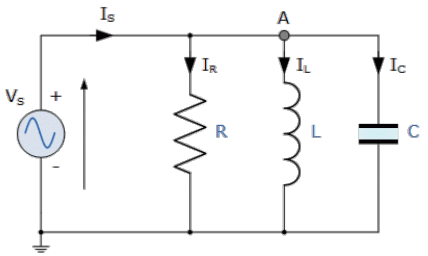
approximation
& linearization

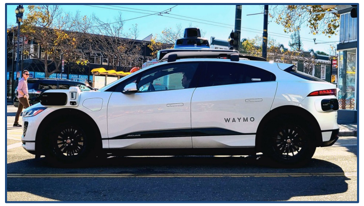
$$\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t) + \vec{n}(t)$$
$$\vec{y}(t) = C\vec{x}(t) + D\vec{u}(t)$$

discretization
& digitization

$$\vec{x}_d[i+1] = A_d\vec{x}_d[i] + B_d\vec{u}_d[i] + \vec{e}[i]$$
$$\vec{y}_d[i+1] = C_d\vec{x}_d[i] + D_d\vec{u}_d[i]$$

System Modeling





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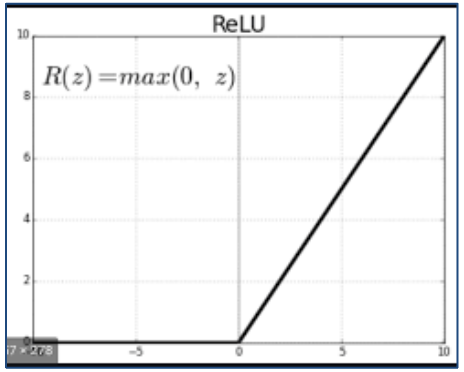
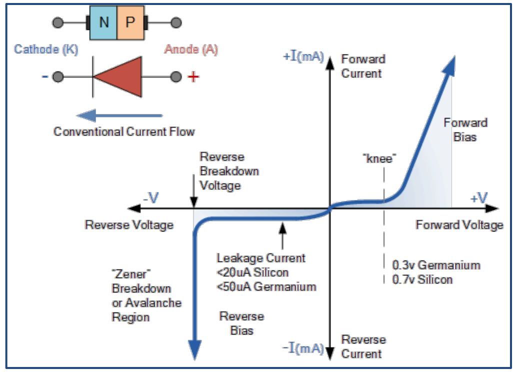
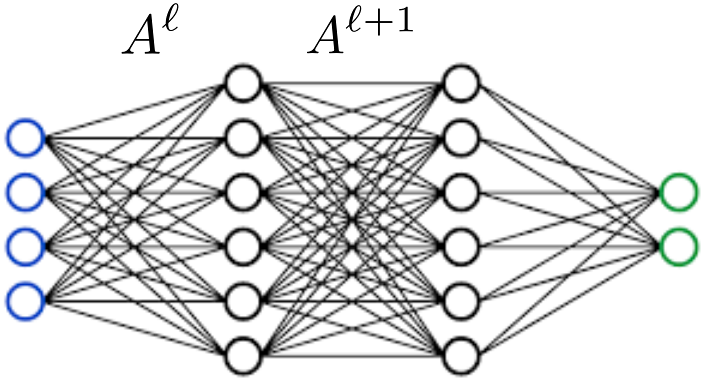


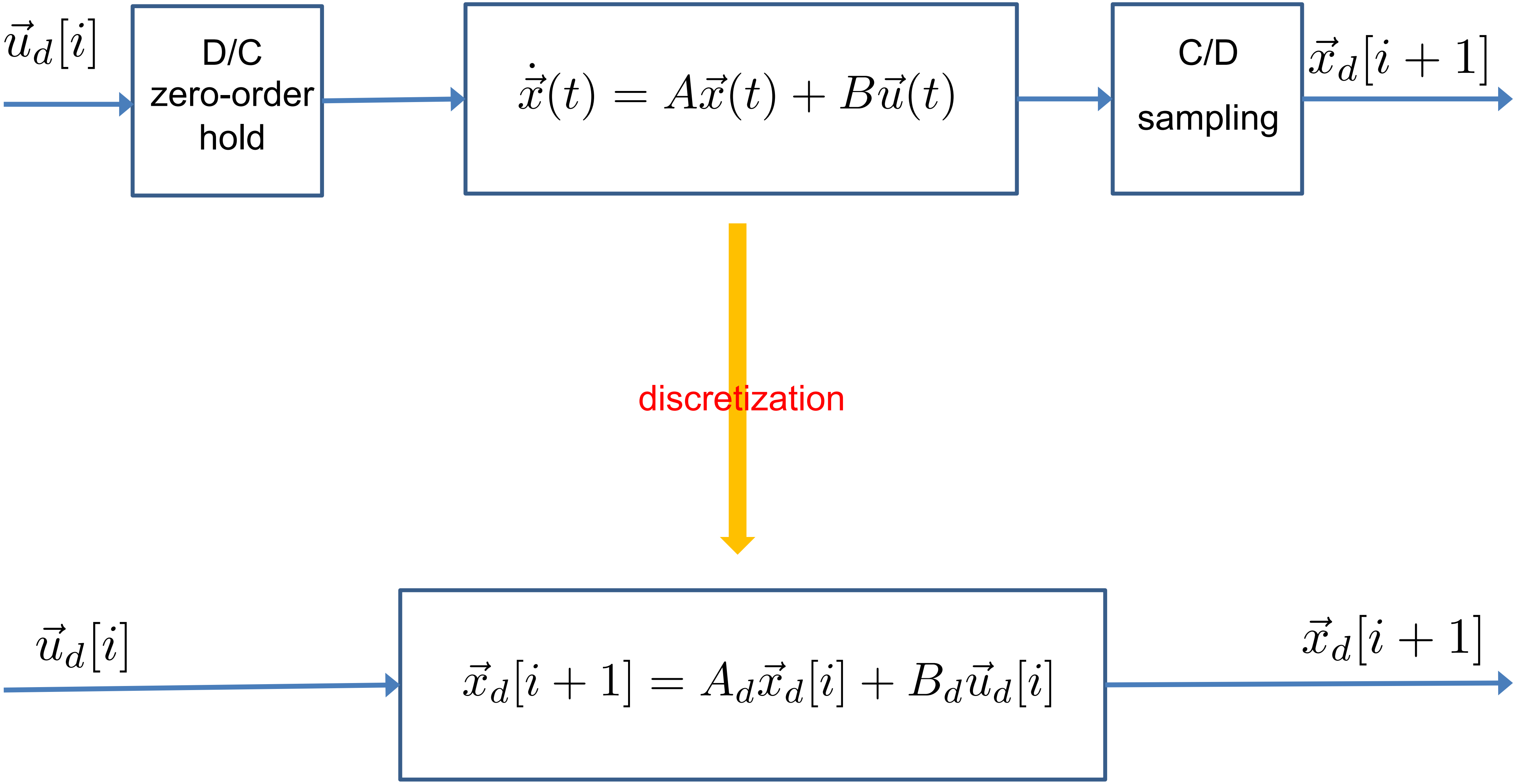


$$\vec{x}_d[i+1] = \sigma_x(A_d\vec{x}_d[i] + B_d\vec{u}_d[i]) + \vec{e}[i]$$
$$\vec{y}_d[i+1] = \sigma_y(C_d\vec{x}_d[i] + D_d\vec{u}_d[i])$$

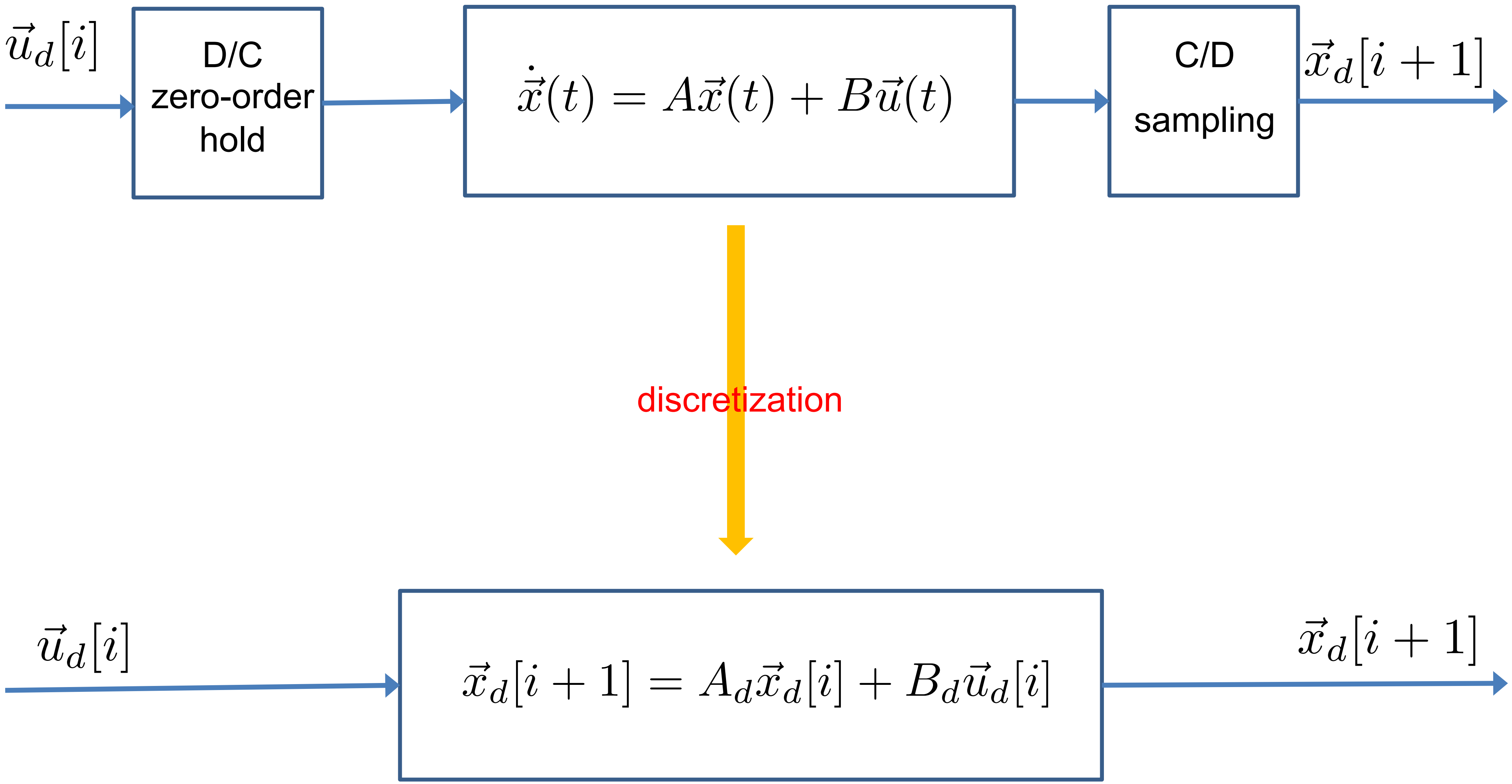
nonlinear activation
and concatenation



System Modeling: Discretization



System Modeling: Discretization



Discretization: Scalar Case

Scalar Case: $x(t) = ax(t) + bu(t)$ $\qquad$ $x_d[i+1] = A_d x[i] + B_d u[i]$

Discretization: Vector Case

Vector Case: $\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$ $\vec{x}_d[i+1] = A_d\vec{x}[i] + B_d\vec{u}[i]$

Diagonalizable: $A = V^{-1}\Lambda V$

Discretization: Vector Case

Vector Case: $\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$ $\vec{x}_d[i+1] = A_d\vec{x}[i] + B_d\vec{u}[i]$

Diagonalizable: $A = V^{-1}\Lambda V$

Discretization: General Case

General Case: $\vec{x}(t) = A\vec{x}(t) + B\vec{u}(t)$ $\quad$ $\vec{x}_d[i+1] = A_d\vec{x}[i] + B_d\vec{u}[i]$

$$\vec{x}(t) = e^{A(t-t_0)}\vec{x}(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}B\vec{u}(\tau)d\tau$$

$$e^{\lambda t} = 1 + \lambda t + \frac{(\lambda t)^2}{2} + \frac{(\lambda t)^3}{6} + \cdots = \sum_{i=0}^{\infty}\frac{(\lambda t)^i}{i!}$$

$$e^{At} = 1 + At + \frac{(At)^2}{2} + \frac{(At)^3}{6} + \cdots = \sum_{i=0}^{\infty}\frac{(At)^i}{i!}$$

$$\vec{x}_d[i+1] = e^{A\Delta}\vec{x}_d[i] + \int_{i\Delta}^{(i+1)\Delta} e^{A(t-\tau)}Bd\tau\vec{u}[i]$$

$$A_d = e^{A\Delta}$$

$$B_d = (e^{A\Delta} - I)A^{-1}B$$

System Identification

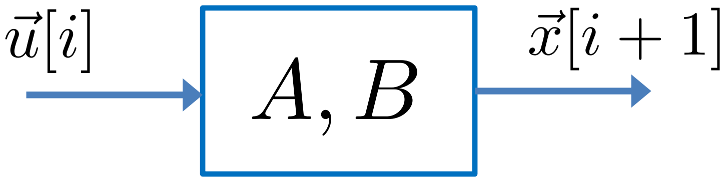
Problem: consider the discrete linear time invariant system:

$$\vec{x}[i+1] = A\vec{x}[i] + B\vec{u}[i] + \vec{e}[i]$$

Given: observed inputs and outputs:

$$\vec{u}[0], \vec{u}[1], \ldots, \vec{u}[l], \ldots$$
$$\vec{x}[0], \vec{x}[1], \ldots, \vec{x}[l], \ldots$$

Objective: learn the system parameters:



Least Squares (Gauss 1809)

$$\vec{s} \in \mathbb{R}^a, \quad D \in \mathbb{R}^{a \times b}, \quad \vec{p} \in \mathbb{R}^b, \quad \vec{e} \in \mathbb{R}^a$$

$$\vec{s} = D \underbrace{\vec{p}}_{\text{unknown}} + \vec{e}, \quad \text{rank}[D] = b \qquad D = [\vec{d}_1, \vec{d}_2, \ldots, \vec{d}_b]$$

$$\vec{p}_\star = \arg\min_{\vec{p}} \|\vec{s} - D\vec{p}\|_2^2$$

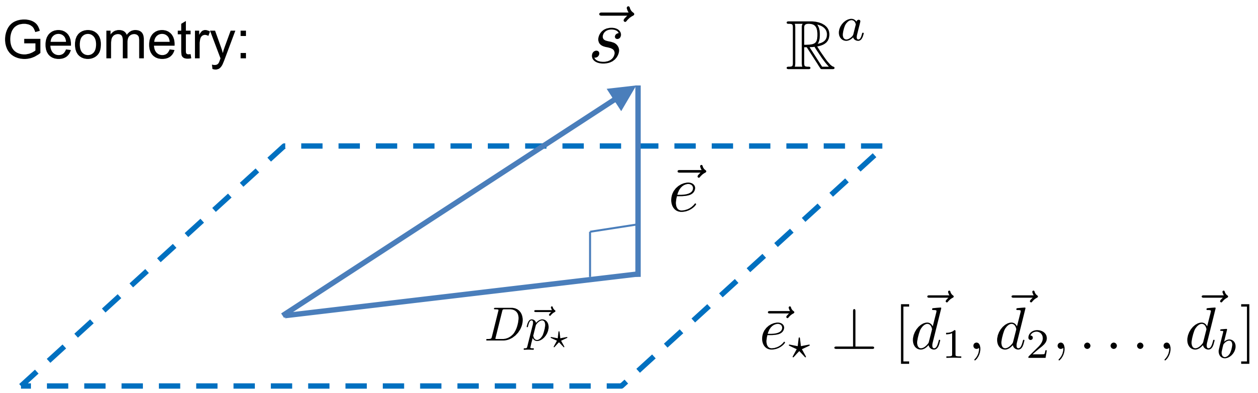
Least Squares (Gauss 1809)

$$\vec{s} \in \mathbb{R}^a, \quad D \in \mathbb{R}^{a \times b}, \quad \vec{p} \in \mathbb{R}^b, \quad \vec{e} \in \mathbb{R}^a$$

$$\vec{s} = D \underbrace{\vec{p}}_{\text{unknown}} + \vec{e}, \quad \text{rank}[D] = b \qquad D = [\vec{d}_1, \vec{d}_2, \ldots, \vec{d}_b]$$

$$\vec{p}_\star = \arg\min_{\vec{p}} \|\vec{s} - D\vec{p}\|_2^2$$

Geometry:



$$D^\top \vec{e} = D^\top (\vec{s} - D\vec{p}_\star) = \vec{0}$$