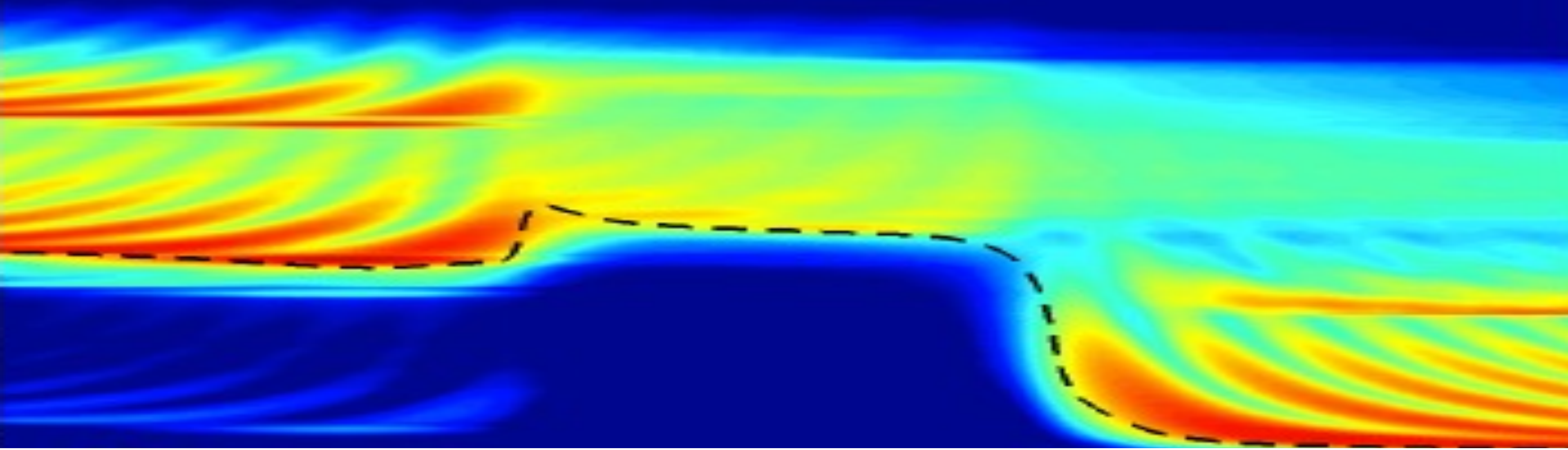

EECS 16B
Designing Information Devices and Systems II
Lecture 14

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Outline

- System BIBS Stability (scalar case)
- System BIBS Stability (vector case)
- Stability of Continuous Time Systems

System Stability (Recap)

Definition: We say a system is *bounded input bounded state (BIBS) stable* if its state stays bounded, $\forall i \; \|\vec{x}[i]\| \leq C$, for any initial condition, any bounded input, and bounded disturbance.

Scalar Case : $x[i+1] = \lambda x[i] + e[i], \quad |\lambda| = 1$

System Stability

Stability for the Scalar Case: $x[i+1] = \lambda x[i] + e[i] \quad (|\lambda| < 1)$

Claim: If $|\lambda| < 1$, then for any $x[0]$ and bounded $|e[i]| < M$ the solution to the system $x[i+1] = \lambda x[i] + e[i]$ remains bounded.

Proof:

System Stability

Stability for the Scalar Case: $x[i+1] = \lambda x[i] + e[i] \quad (|\lambda| < 1)$

Claim: If $|\lambda| < 1$, then for any $x[0]$ and bounded $|e[i]| < M$ the solution to the system $x[i+1] = \lambda x[i] + e[i]$ remains bounded.

Proof:

System Stability

Stability for the Vector Case: $\vec{x}[i+1] = A\vec{x}[i] + \vec{e}[i] \ \in \mathbb{R}^n$

System Stability

Stability for the Vector Case: $\vec{x}[i+1] = A\vec{x}[i] + \vec{e}[i] \ \in \mathbb{R}^n$

Diagonalizable: $\Lambda = V^{-1}AV \qquad \vec{z} = V^{-1}\vec{x}$

System Stability

Stability for the Vector Case: $\vec{x}[i+1] = A\vec{x}[i] + \vec{e}[i] \;\in \mathbb{R}^n$

Triangularizable: $T = V^{-1}AV \quad \vec{z} = V^{-1}\vec{x}$

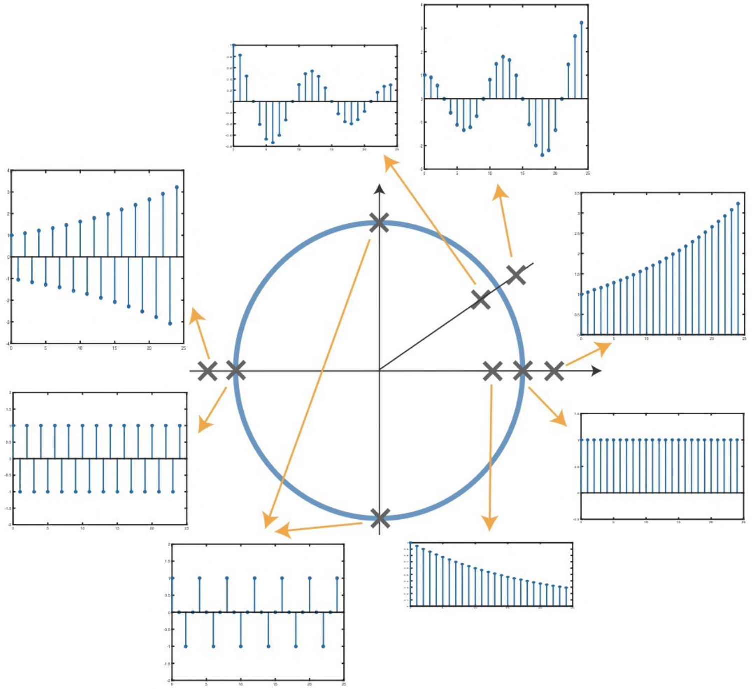
System Stability (Discrete Time)

Stability for the Vector Case: $\vec{x}[i+1] = A\vec{x}[i] + \vec{e}[i] \ \in \mathbb{R}^n$

Summary: The above discrete time system is BIBS stable if all eigenvalues λ_k of A satisfy:

$$|\lambda_k| < 1, \quad k = 1, \ldots, n.$$

That is, all eigenvalues are strictly inside the unit circle in the complex plane.



System Stability (Continuous Time)

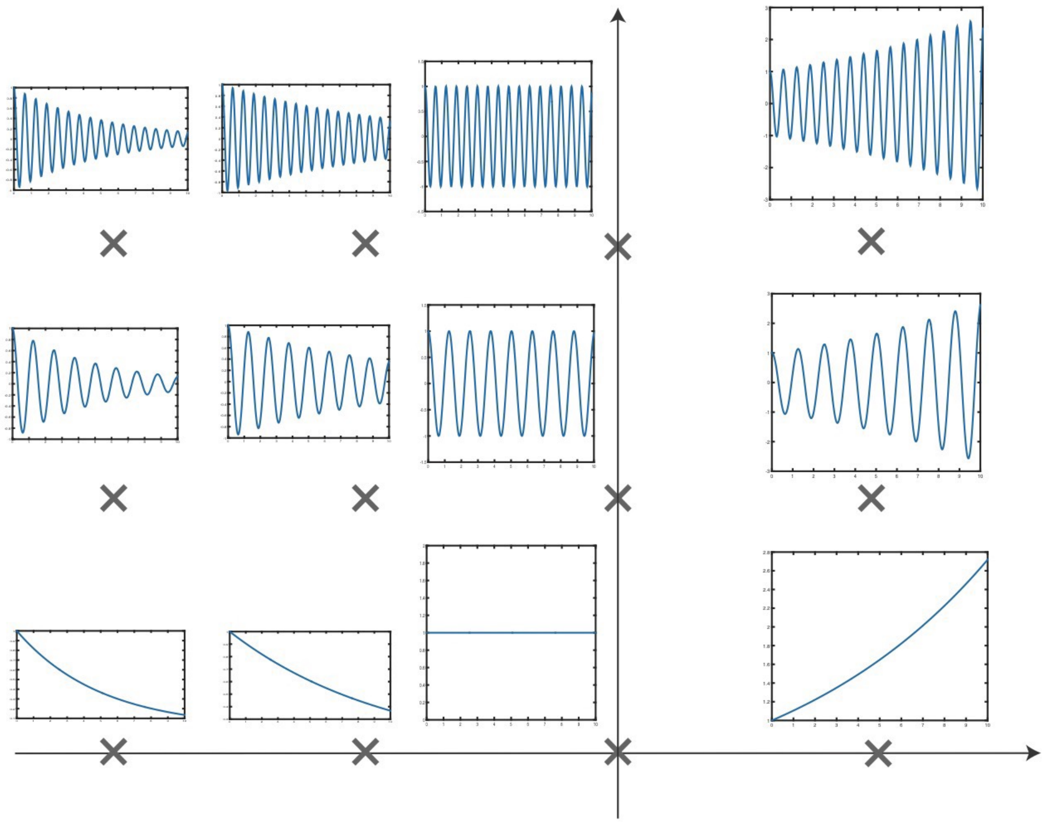
Stability for the Scalar Case: $\frac{d}{dt}x(t) = \lambda x(t) + w(t)$ $\frac{x[i+1] - x[i]}{\Delta} = \lambda x[i] + w[i]$

$$x(t) = e^{\lambda t}x(0) + \int_0^t e^{\lambda(t-\tau)}w(\tau)d\tau$$

$$|\int_0^t e^{\lambda(t-\tau)}w(\tau)d\tau| \leq \int_0^t e^{\lambda(t-\tau)}d\tau M = \frac{e^{\lambda t} - 1}{\lambda}M$$

System Stability (Continuous Time)

Stability for the Scalar Case: $\frac{d}{dt}x(t) = \lambda x(t) + w(t)$



System Stability (Continuous Time)

Stability for the Vector Case: $\dot{\vec{x}}(t) = A\vec{x}(t) + \vec{w}(t) \ \in \mathbb{R}^n$

Diagonalize or triangularize: $T = V^{-1}AV \quad \vec{z} = V^{-1}\vec{x}$

System Stability (Summary)

Continuous time: $\dot{\vec{x}}(t) = A\vec{x}(t) + \vec{w}(t) \in \mathbb{R}^n$

Discrete time: $\vec{x}[i+1] = A\vec{x}[i] + \vec{e}[i] \in \mathbb{R}^n$