

**EECS 16B**

# **Designing Information Devices and Systems II**

## **Lecture 26**

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# Outline

- Linearization of Nonlinear Control Systems
  - Operating points
  - Scalar case and an example
  - Vector case and an example

# Nonlinear Autonomous Systems: Equilibrium Points

$$\frac{d\vec{x}(t)}{dt} = \vec{f}(\vec{x}(t)) \in \mathbb{R}^n$$

$$\vec{x}[i + 1] = \vec{f}(\vec{x}[i]) \in \mathbb{R}^n$$

# Nonlinear Control Systems: Operating Points

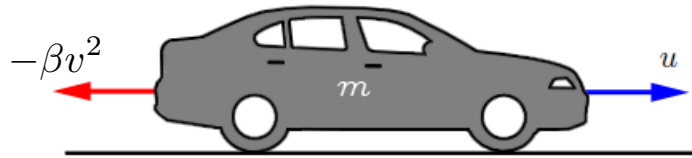
$$\frac{d\vec{x}(t)}{dt} = \vec{f}(\vec{x}(t), \vec{u}(t)) \in \mathbb{R}^n$$

$$\vec{x}[i + 1] = \vec{f}(\vec{x}[i], \vec{u}[i]) \in \mathbb{R}^n$$

# Nonlinear Control Systems: Linearization

**Scalar case:**  $\frac{dx(t)}{dt} = f(x(t), u(t)) \in \mathbb{R}$

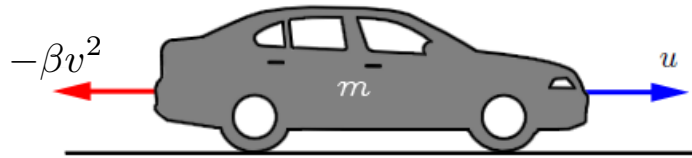
# Nonlinear Control Systems: Example



$$m \frac{v(t)}{dt} = -\beta v(t)^2 + u(t)$$

$$\frac{dx(t)}{dt} = -\frac{\beta}{m} x(t)^2 + \frac{1}{m} u(t) = f(x(t), u(t))$$

# Nonlinear Control Systems: Example



$$\frac{dx(t)}{dt} = -\frac{\beta}{m}x(t)^2 + \frac{1}{m}u(t) = f(x(t), u(t))$$

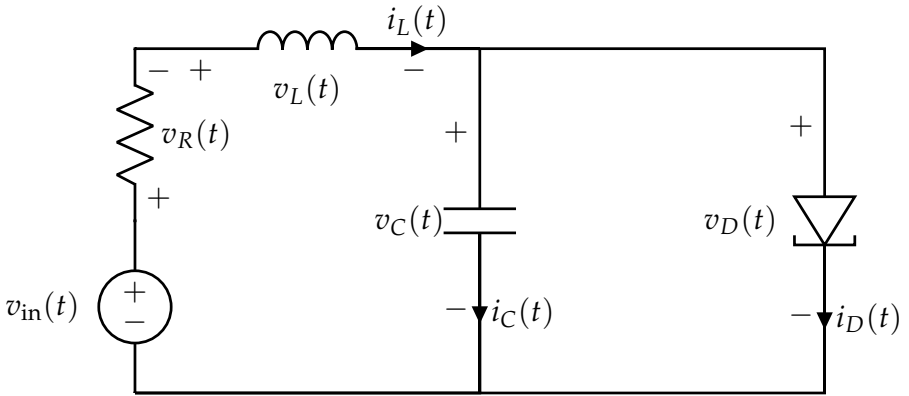
# Nonlinear Control Systems: Linearization

**Vector case:**  $\frac{d\vec{x}(t)}{dt} = \vec{f}(\vec{x}(t), \vec{u}(t))$

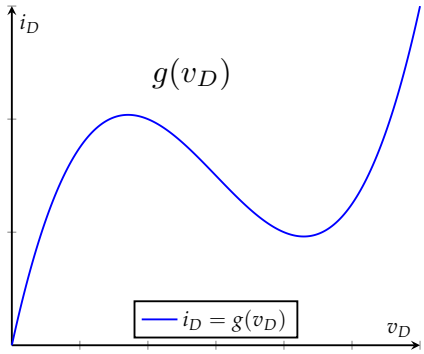
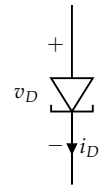


# Nonlinear Control Systems: Linearization

**Example:**  $\frac{d\vec{x}(t)}{dt} = \vec{f}(\vec{x}(t), \vec{u}(t))$

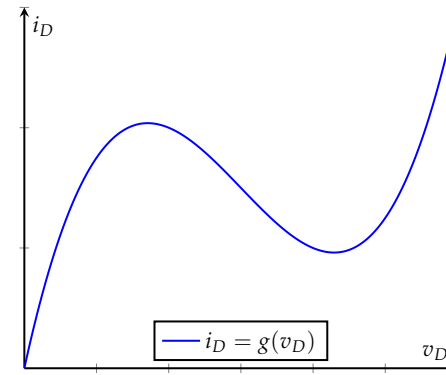
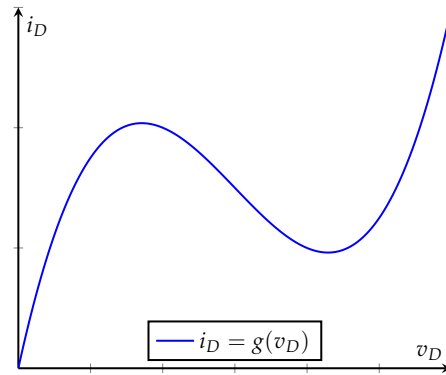
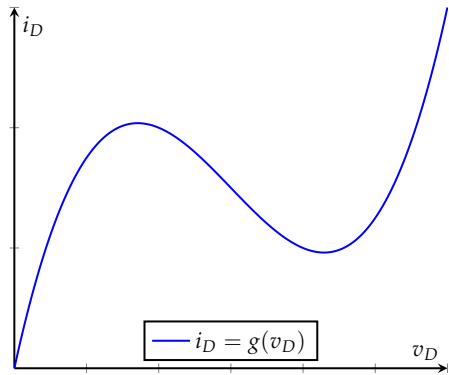


a tunnel diode



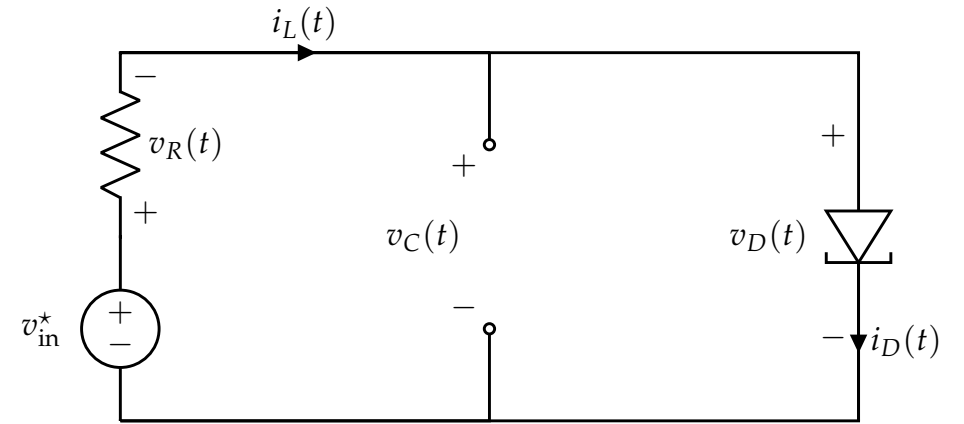
# Nonlinear Control Systems: Linearization

**Example:** operating points.



# Nonlinear Control Systems: Linearization

**Example:** interpretation of operating points.



# Nonlinear Control Systems: Linearization

**Example:** linearized system.

# Nonlinear Control Systems: Linearization

**Example:** *stability* and *controllability* of the linearized system.