Miterm Exam#2

Name:

SID#: ______________________________
(after the exam begins, add your SID# in the top right corner of each page)

Discussion Section and TA: ____________________

Discussion Section and TA: ____________________

Lab Section and TA: ____________________

Name of left neighbor: ____________________

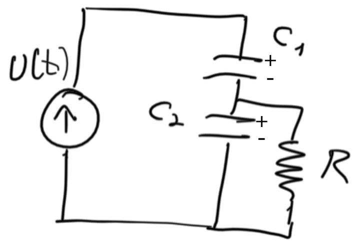
Name of right neighbor: ____________________

You have 1:20 hours for 5 problems.

- Scan through the exam before you start. Try not to spend too much time on any one part. Some problems are longer than others, take that into consideration.
- Show your work. An answer without explanation is not acceptable and does not guarantee any credit.
- Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.
- Do not remove pages, as this disrupts the scanning. If needed, cross out any parts that you don't want us to grade.

Problem	Score
1	
2	
3	
4	
5	
Total	

Problem 1: State Space Modeling of Circuits *(30 points)*



a) Write a state-space model for the system. Use $\vec{x}(t) = \begin{bmatrix} v_{c1}(t) \\ v_{c2}(t) \end{bmatrix}$ as state variables. Note, that $U(t)$ is a current source.

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \vec{x}(t) + \begin{bmatrix} \quad \\ \quad \end{bmatrix} u(t)$$

b) Is the system stable? Explain the physical meaning of stability or instability for this circuit.

Stable? Y / N Explain:

c) Is the system controllable?

Controllable? Y / N

Problem 2: Observability and Observers *(30 points)*

Consider the system:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

with the output:

$$y(t) = [c_1 \quad c_2]\vec{x}(t)$$

a) What are the constraints on c_1,c_2 such that the system is observable?

Constraints:

b) Now, let $y = [0 \quad 1]\vec{x}(t)$ and consider the observer:

$$\frac{d\vec{x}(t)}{dt} = \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} ([0 \quad 1]\hat{x}(t) - y(t))$$

Which of the following eigenvalues of $(A + LC)$ will guarantee the *fastest* convergence of $\hat{x}(t)$ to $\vec{x}(t)$?

i: $(0.5, 0.5)$ ii: $(0.9, 0.9)$ iii: $(-0.3, -0.3)$ iv :(-0.9+0.9j,-0.9-0.9j)

Eigenvalues with fastest convergence:

c) For the system in part (b), find $l_1, \ l_2$ such that the observer will converge with eigenvalues of $(-0.5, \ -0.5)$

l_1:	l_2:

Problem 3: Controllability and Controllers *(30 points)*

You are given the system:

$$\vec{x}(t+1) = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

a) Is the system stable?

Stable? Y / N

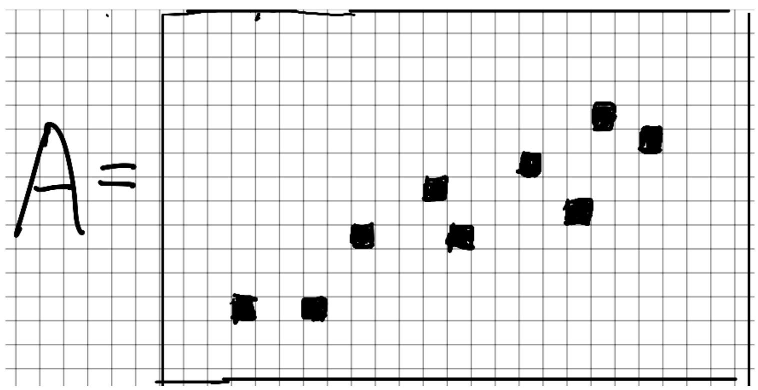
b) Design a state-feedback controller that will take the system from any state to $\vec{x}(t) = 0$ in 2 steps.

k_1=	k_2=

c) Design a controller that will take the system from $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ in the minimum number of steps. In your answer provide the values for $\vec{U} = U(t)$ till the target is hit. For example, if you need 5 steps, report $\vec{U} = [U(0), U(1), U(2), U(3), U(4)]$.

steps: $\quad\quad\quad\quad$ $\vec{U} = [\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad]$

Consider the following matrix with non-zero entries in the black squares and zeros elsewhere:

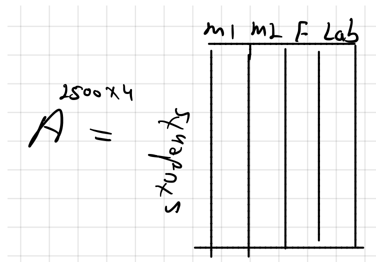


What is the number of non-zero singular values of A? explain.

# of singular values:	Explain:

Problem 5: Data Science *(30 points)*

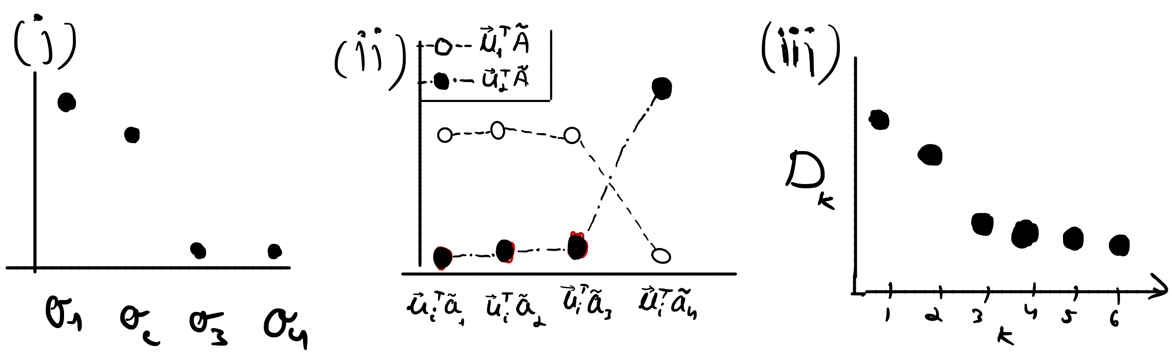
Consider a matrix $A \in \mathrm{R}^{2500\times 4}$ which represents the EE16B Sp' 2020 miterm 1, miterm 2, final and lab grades for all 2500 students taking the class.



To perform PCA, you subtract the mean of each column and store the results in $\tilde{A}$. Your analysis includes:

1. Computing the SVD: $\tilde{A} = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \sigma_3 \vec{u}_3 \vec{v}_3^T + \sigma_4 \vec{u}_4 \vec{v}_4^T$ and plot the singular values.
2. Computing and graph the projection of: $\vec{u}_1^T \tilde{A}$ and $\vec{u}_2^T \tilde{A}$
3. Performing k-means on the rows of $\tilde{A}$ with $k = 1, 2, 3, 4, 5, 6$ and plotting the cost function $D_k = \sum_{i=1}^{k} \sum_{\vec{x} \in S_i} ||\vec{x} - \mu_i||$, as function of k (μ_i are cluster centers)

The analysis data are plotted below:



Based on the analysis, answer the following questions. Briefly explain your answer.

Example) The data can be accurately represented by two principle components.

Example Solution: True! There are two significant singular values, and the rest are very small.

True False

a) Students' exams scores are highly correlated with each other.

True False

b) The middle plot (ii) shows that students who did well on the exam did not do well in the labs and vice versa.

True False

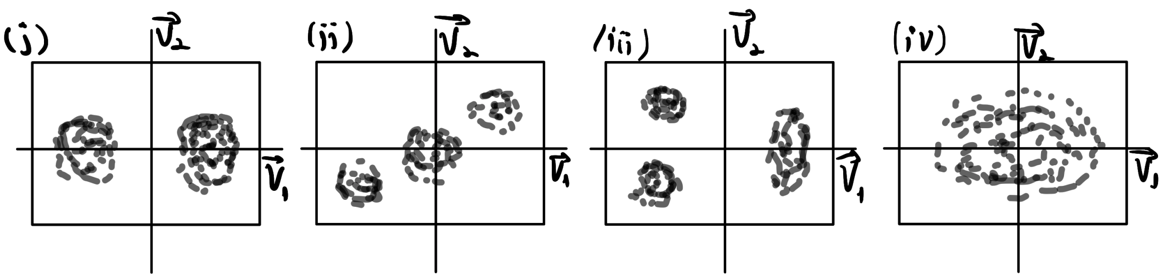
c) The data shows that one of the clusters must be solely associated with exam scores independently of lab scores.

True False

d) One of the principal components attributes is solely associated with lab scores and not with exam scores.

True False

e) Circle all the scatter plots that could describe the data projected on the largest two principal components.



(i), (ii), (iii), (iv)