Midterm

PRINT your student ID: _____

PRINT AND SIGN your name: _____ , _____ _____
(last)   (first)   (sign)

PRINT your discussion sections and (u)GSIs (the ones you attend): _____

Instructions

Remember, you are supposed to write your answers for every part of every question on a fresh sheet of paper. Because you are taking this exam remotely, keep the following in mind:

(a) Whenever you see a bubble to fill in, write down the answers you would have bubbled on your sheet instead.

(b) If the problem asks you to mark something on an existing plot, just write where you would have marked on your sheet instead.

(c) If the problem asks you to sketch a plot on a template, then just sketch the plot on your sheet by drawing and labeling the axes yourself. You don't have to draw all the gridlines on a log-scale plot - — just the axes are enough to draw along with labeled tick-marks on the axes sufficient to understand your plot sketch.

1. **Honor Code (0 pts.)**

   Please copy the following statement in the space provided below and sign your name.

   As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will follow the rules and do this exam on my own.

   Note that if you do not copy the honor code and sign your name, you will get a 0 on the exam.

2. **What are you planning to do after the midterm? (2 pts.)**

3. **What's your favorite thing to do in Berkeley? (2 pts.)**

4. Capacitor and Inductor Basics (20 pts.)

(a) (4 pts.) **What is the capacitance of the following parallel plate capacitor?** The permittivity of the material is known to be $10^{-10} \frac{F}{m}$, the dimensions of the larger face of the plate is $2\,\mu m$ by $3\,\mu m$, and the thickness of the insulator is $1\,\mu m$.
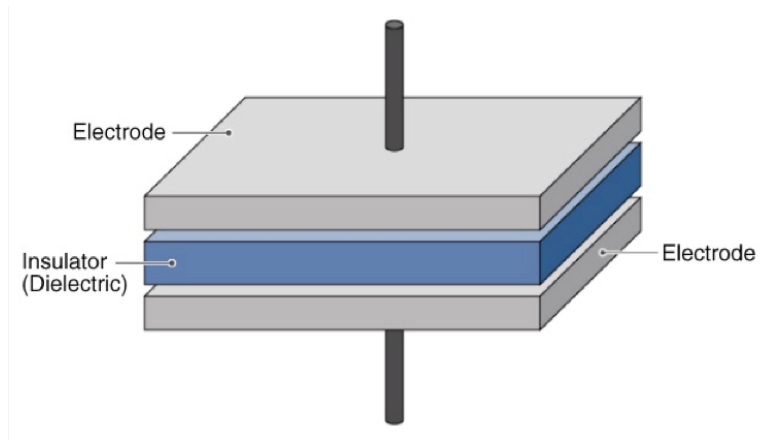
(b) (8 pts.) A capacitance and its voltage with respect to time is shown below. **Qualitatively sketch the**

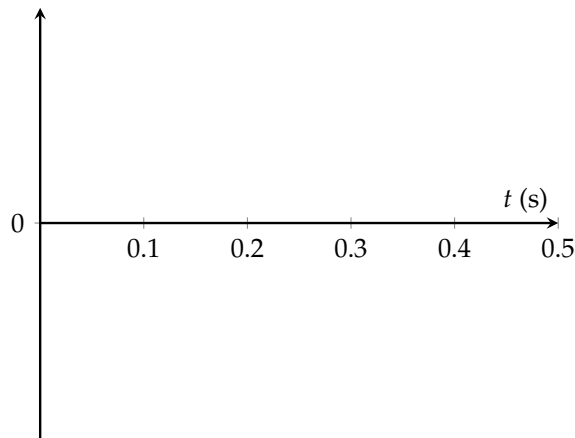 i. **Current**

  ii. **Power**

as functions of time. When you are qualitatively sketching, you may define your own axes and scaling. The sketches you make do not have to be to scale. **The important part is to note the general shape of the curve – you do not have to denote exact values on your coordinate axes.** Feel free to use the blank axes below.
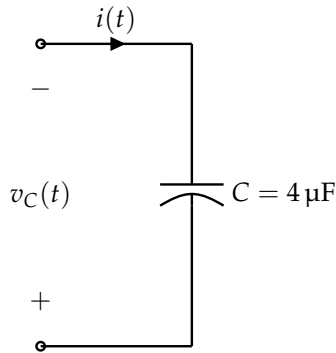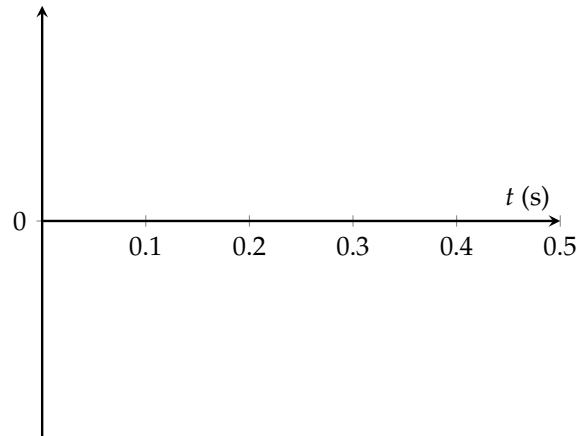

(a) Capacitor Circuit

(b) Plot of $v_C(t)$

$$t \text{ (s)}$$

| 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
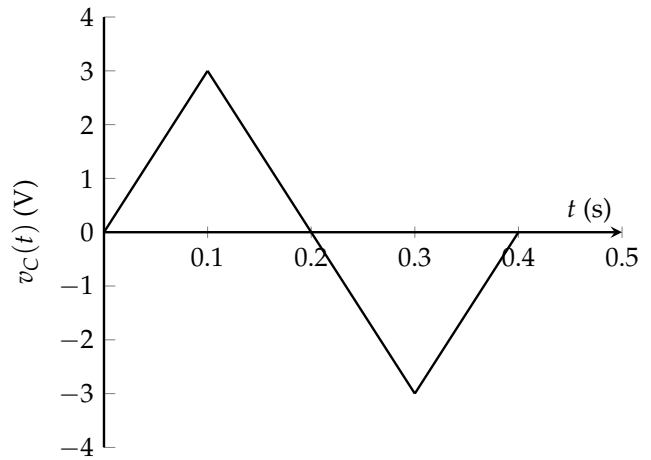
(c) (8 pts.) At $t = 0$, an inductor $L = 1\,\mu\text{H}$ which was carrying a current of $1\,\text{mA}$ is connected across a resistor, $R = 1\,\text{k}\Omega$. **Find the expression for current through the inductor as a function of time, and find the total energy that will be dissipated through the resistor.**
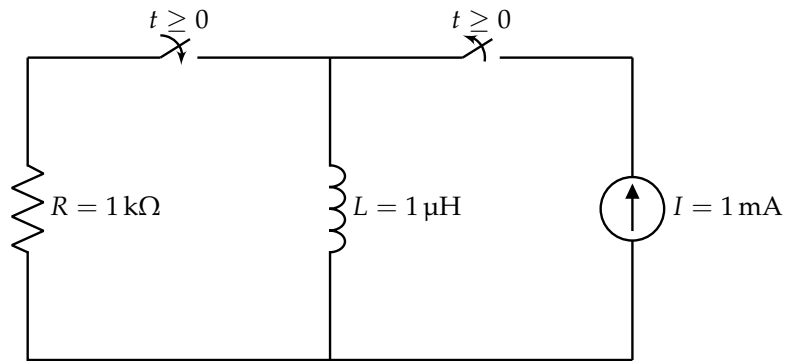
(HINT: Remember, $E = \int P\,\mathrm{d}t$.)

Figure 2: Sample Inductor Circuit

5. **The Inevitable RLC Circuit (20 pts.)**

   Suppose you are given the following RLC Circuit as shown in fig. 3. V_{DC} is some unknown DC voltage.
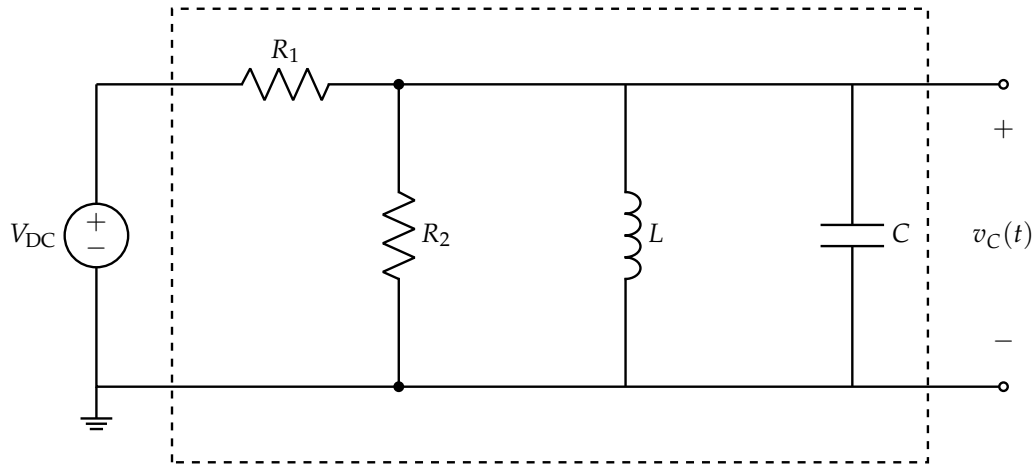

Figure 3

   (a) (8 pts.) **Show that the differential equation that describes the voltage across the capacitor $v_C(t)$ as a function of time can be written as**

   $$\frac{\mathrm{d}^2 v_C(t)}{\mathrm{d}t^2} + M\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} + Nv_C(t) = 0 \tag{1}$$

   where M and N are constants that may include $R_1, R_2, L,$ and C.

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

(b) (8 pts.) Suppose that, if you were to plug in appropriate values of R_1, R_2, L, and C, you arrive at the following differential equation:

$$\frac{d^2 v_C(t)}{dt^2} + 10^9 \frac{dv_C(t)}{dt} + 10^{18} v_C(t) = 0 \tag{2}$$

Solve this differential equation to derive an expression for $v_C(t)$**.** Note that your solution should consider both homogeneous and particular solutions. The solution can also contain constants to be determined from initial conditions (no initial conditions have been specified for this problem).

What is the steady state solution for $v_C(t)$**?**

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

(c) (4 pts.) **Qualitatively sketch $v_C(t)$ as a function of time t.** When you are qualitatively sketching, you may define your own axes and scaling. The important part is to note the general shape of the curve – you do not have to denote exact values on your coordinate axes. In your qualitative plot, indicate whether the function is decaying or increasing with time, and make sure to note other important aspects of the function where applicable (e.g., oscillations, discontinuities, etc.).

6. **Let's Transfer to a Different Approach (15 pts.)**

   For this problem we will be using the same RLC circuit as the previous problem. It has been copied below for your convenience in fig. 4.
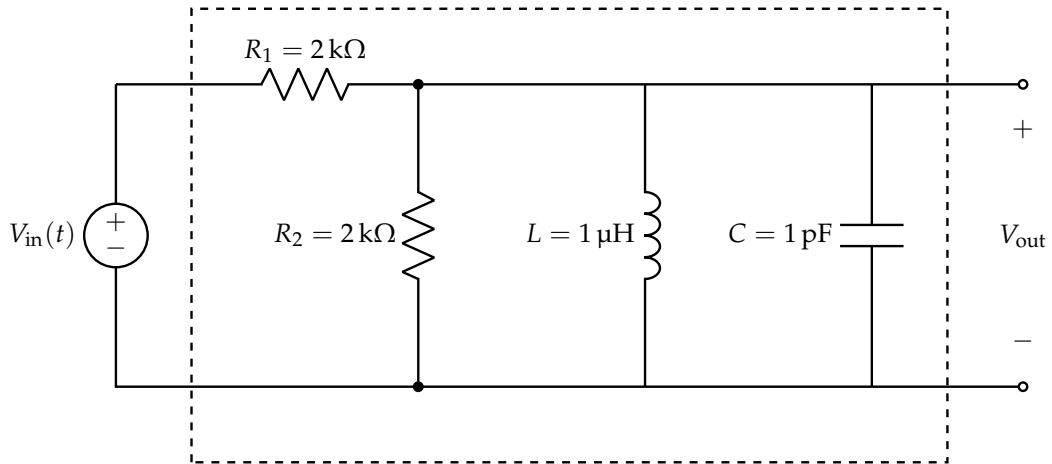

Figure 4

   (a) (8 pts.) For the circuit provided in fig. 4, **find the transfer function** $H(f)$ **considering the two port network indicated by the box in terms of** R_1, R_2, L, C, f**.** Put your answer in the form

   $$H(f) = \frac{1}{k_1 + jk_2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)} \tag{3}$$

   where k_1 and k_2 are some real constants in terms of R_1, R_2, L, C and $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$.

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

(b) (5 pts.) **Find $|H(f)|$ as $f \to \infty$ and as $f \to 0$. Then, find the value(s) of f for which $|H(f)|$ is maximized. Qualitatively sketch the magnitude of the transfer function as a function of frequency (f).** You may put your answers in terms of k_1 and k_2 if you wish. When you are qualitatively sketching, you may define your own axes and scaling. The important part is to note the general shape of the curve – you do not have to denote exact values on your coordinate axes. **In your qualitative plot, make sure to indicate the behavior of $|H(f)|$ as $f \to 0$ and $f \to \infty$, and label the point in the graph where $|H(f)|$ is maximized.**

(c) (2 pts.) **Using the symbolic transfer function eq. (3), what is the dB magnitude of the transfer function at a frequency of** $f = f_0$**?** Write your answer in terms of k_1 and k_2, and you do not need to simplify your answer fully.

7. Power is a Complex Topic (15 pts.)

Once again, we are considering the following RLC Circuit with the provided values for R_1, R_2, L, C.
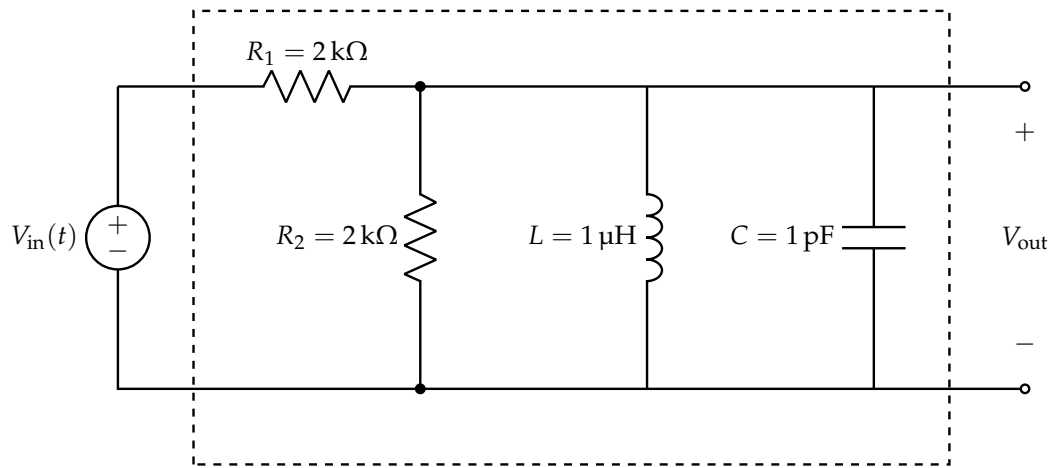

Figure 5

(a) (10 pts.) Now, let's say that the circuit is being driven by a voltage source where

$$V_{in}(t) = 2\cos\left(10^9 t + 30°\right) \qquad (4)$$

   i. **Calculate the real power P.**

   (HINT: First, find the resonant frequency.)

ii. **Calculate the reactive power Q.**

iii. **Calculate the power factor and comment on the relative magnitude of the real and reactive powers.**

(b) (5 pts.) Consider the following circuit in fig. 6. It is known that when $V_{\text{GS}} = 1\,\text{V}$, $I_{\text{DS}} = 1\,\text{mA}$ whereas when $V_{\text{GS}} = 0\,\text{V}$, $I_{\text{DS}} = 10^{-9}\,\text{A}$.
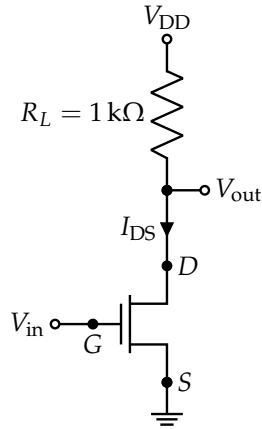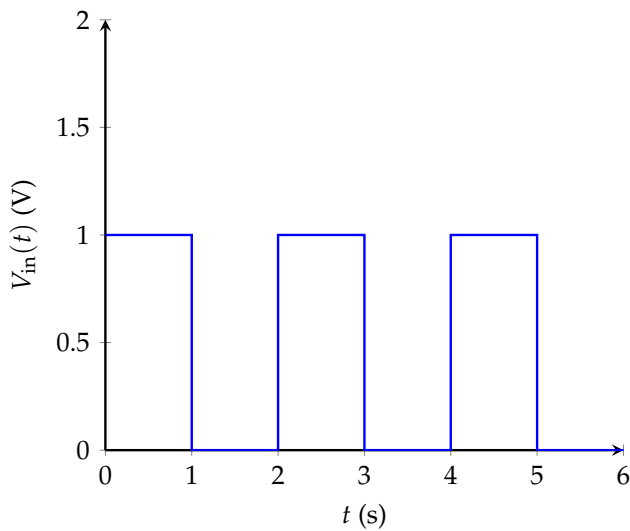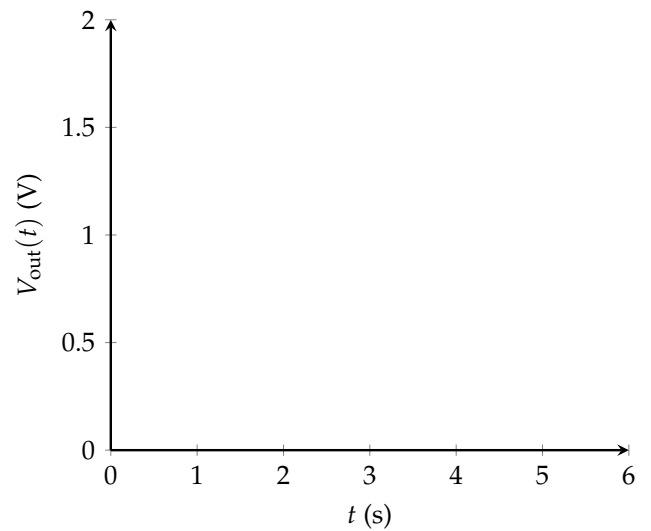
Figure 6: NMOS Transistor Circuit

Given that $V_{\text{DD}} = 1\,\text{V}$ and the graph for $V_{\text{in}}(t)$ is shown in fig. 7a, **sketch V_{out} as a function of time t on the plot provided in fig. 7b.**

(a) Plot of $V_{\text{in}}(t)$

(b) Plot of $V_{\text{out}}(t)$

8. **Least Squares, Uncertainty, and Safety (30 pts.)**

Suppose we have some two dimensional, discrete time system of the form

$$\vec{x}[i+1] = \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \vec{x}[i] + \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\vec{b}} u[i] \tag{5}$$

where $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2$ are all unknown, real numbers. For ease of notation, let us denote $\vec{x}[i] := \begin{bmatrix} x_1[i] \\ x_2[i] \end{bmatrix}$.

(a) (5 pts.) Let's say we wanted to estimate these parameters using system ID. Upon applying some control inputs, we observe the following data:

i	$x_1[i]$	$x_2[i]$	$u[i]$
0	1	0	2
1	3.41	2.41	1
2	9.95	8.95	3
3	30.97	29.97	7
4	96.86	95.86	3
5	291.71	290.71	3

Set up a least squares problem to solve for the unknowns, i.e., $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2$. Then, symbolically describe how to solve the least squares problem. Note: you do not need to actually perform any matrix computations.

(b) (8 pts.) Regardless of your answer to the previous part, suppose that, when solving the least squares problem, we obtained the following values of A and $\vec{b}$:

$$A = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} \tag{6}$$

$$\vec{b} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \tag{7}$$

This gives us the following discrete time system:

$$\vec{x}[i+1] = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} \vec{x}[i] + \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} u[i] \tag{8}$$

Diagonalize the system to obtain

$$\vec{x}_\lambda[i+1] = \Lambda \vec{x}_\lambda[i] + \widetilde{\vec{b}} u[i] \tag{9}$$

where Λ is diagonal and $\vec{x}_\lambda[i]$ is the representation of $\vec{x}[i]$ in the eigenbasis of A.

(HINT: You may want to use the fact that the eigenvectors of A are $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.)

(c) (2 pts.) **Is the system in eq. (8) stable? Explain your answer.**

(d) (15 pts.) Regardless of your answers to the previous parts, suppose that we could write

$$\vec{x}_\lambda[i] = \begin{bmatrix} \lambda_1^i x_{\lambda_1}[0] \\ \lambda_2^i x_{\lambda_2}[0] \end{bmatrix} + \sum_{j=0}^{i-1} \begin{bmatrix} \lambda_1^{i-1-j} u[j] \\ 0 \end{bmatrix} \tag{10}$$

Let's say that, based on our error in our estimate for least squares, we knew that $2 \le \lambda_1 \le 4$ and $0 \le \lambda_2 \le 2$. Now, suppose that your initial state is $\vec{x}_\lambda[0] = \vec{0}$, and that you want to uniformly bound your control inputs $u[j]$ for $j = 0, \ldots, i-1$, so that $|x_{\lambda_1}[i]| + |x_{\lambda_2}[i]| \le k$. **In other words, find a $\varepsilon > 0$ such that, if $|u[j]| \le \varepsilon$ for $j = 0, \ldots, i-1$, then $|x_{\lambda_1}[i]| + |x_{\lambda_2}[i]| \le k$.** Your value of ε may depend on i and k.

(HINT: You may use the triangle inequality, i.e., $|\sum_{i=1}^{n} a_i| \le \sum_{i=1}^{n} |a_i|$. You may also use the geometric series formula, i.e., $\sum_{i=0}^{n-1} r^i = \frac{1-r^n}{1-r}$.)

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.

If needed, you can also use this space to work on problems. But if you want the work on this page to be graded, make sure you tell us on the problem's main page.]

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]