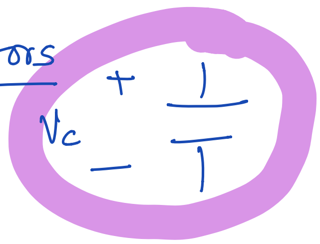
EECS 16B

Review of 16A material

Topics:

- ★ Eigenvalues / Eigenspaces ← 16
- · Voltage dividers ← 16
- ★ Capacitors ← ~~~~ 40
- ★ Op-Amps / NFB. ← 40
- ★ Least squares. ← 25-30.

Capacitors

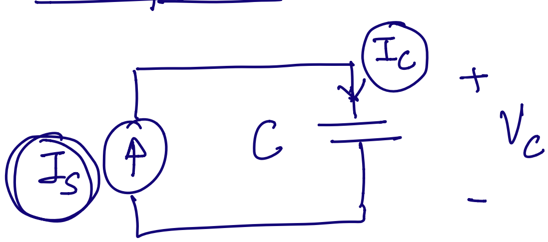


$$\boxed{Q = C V_c}$$

Charge, Capacitance, Voltage

$$I = \frac{dQ}{dt} \qquad I = C \cdot \frac{dV}{dt}$$

Example 1: $V_c(t)$, $V_c(T)$ Find $V_c(t)$ in terms of I_s, C.



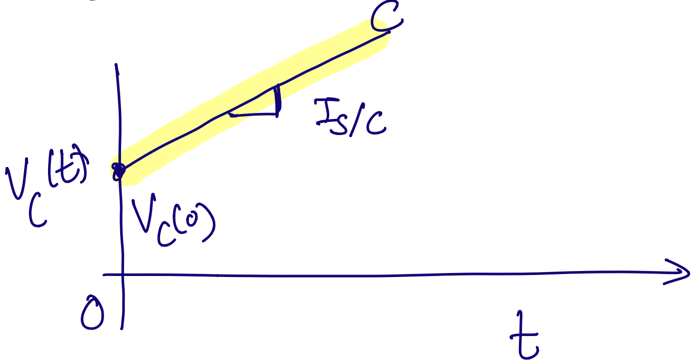
$$I_c(t) = C \cdot \frac{dV_c(t)}{dt}$$

$$I_c(t) = I_s$$

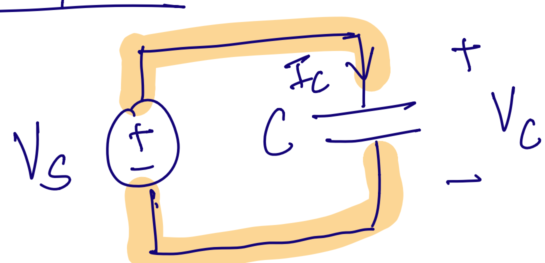
$$\int_0^T I_c(t)\,dt = \int_0^T I_s\,dt = \int_0^T C \cdot \frac{dV_c(t)}{dt}$$

$$I_s(T-0) = C \cdot [\underbrace{V_c(T)}_{} - V_c(0)].$$

$$V_c(T) = \frac{I_s \cdot T}{C} + V_c(0)$$



Example 2: "Capacitors do not pass DC"

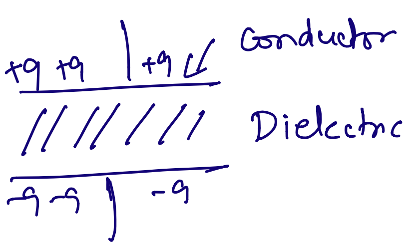


$$I_c = C \cdot \frac{dV_c}{dt}$$

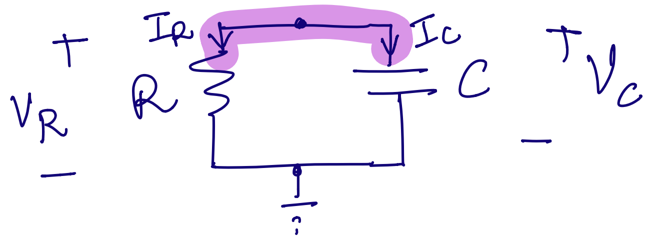
$$V_c = \text{constant}$$

$$I_c = C \cdot 0 = 0$$

$$\frac{dV_c}{dt} = 0$$



Example 3: $V_c(0) = V_0$



KCL: $I_R + I_C = 0$. (1)

$V_R = I_R \cdot R$ Ohms Law.

$$I_c = C \cdot \frac{dV_c}{dt}$$

$$V_c = V_R$$

$$f(x) = e^{\alpha x}$$
$$\frac{df}{dx} = \alpha \cdot e^{\alpha x}$$

$$\frac{V_R}{R} + \frac{c\,dV_c}{dt} = 0.$$

$$\frac{V_c}{R} + \frac{C \cdot dV_c}{dt} = 0.$$

$$\frac{dV_c(t)}{dt} = -\frac{V_c(t)}{RC}$$ Differential equation.

$$\boxed{V_c(t) = a \cdot e^{bt}}$$ GUESS

$$\frac{dV_c(t)}{dt} = a \cdot b \cdot e^{bt}$$

$$\boxed{\text{LHS} \quad a \cdot b \cdot e^{bt}} = \boxed{-\frac{a \cdot e^{bt}}{RC} \quad \text{RHS}}$$

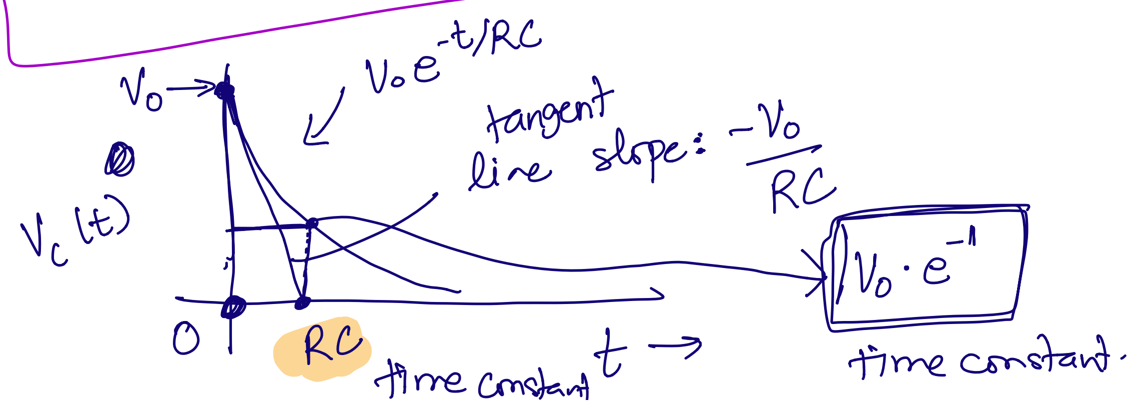
$$\boxed{b = -\frac{1}{RC}} \checkmark$$

$$V_c(0) = a \cdot e^{b \cdot 0} = a \cdot 1 = a$$

$$V_c(0) = V_0 .$$

$$\Rightarrow \boxed{a = V_0}$$

$$\boxed{V_c(t) = V_0 \cdot e^{-t/RC}}$$



$mx + c = y.$

$-\frac{V_0}{RC} \cdot x + c = y$ if $x = 0$, $mx + c = V_0$.

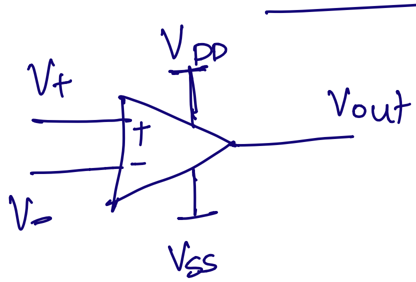
$\Rightarrow c = V_0$

Eqⁿ of tangent: $y = \frac{-V_0}{RC} \cdot x + V_0$

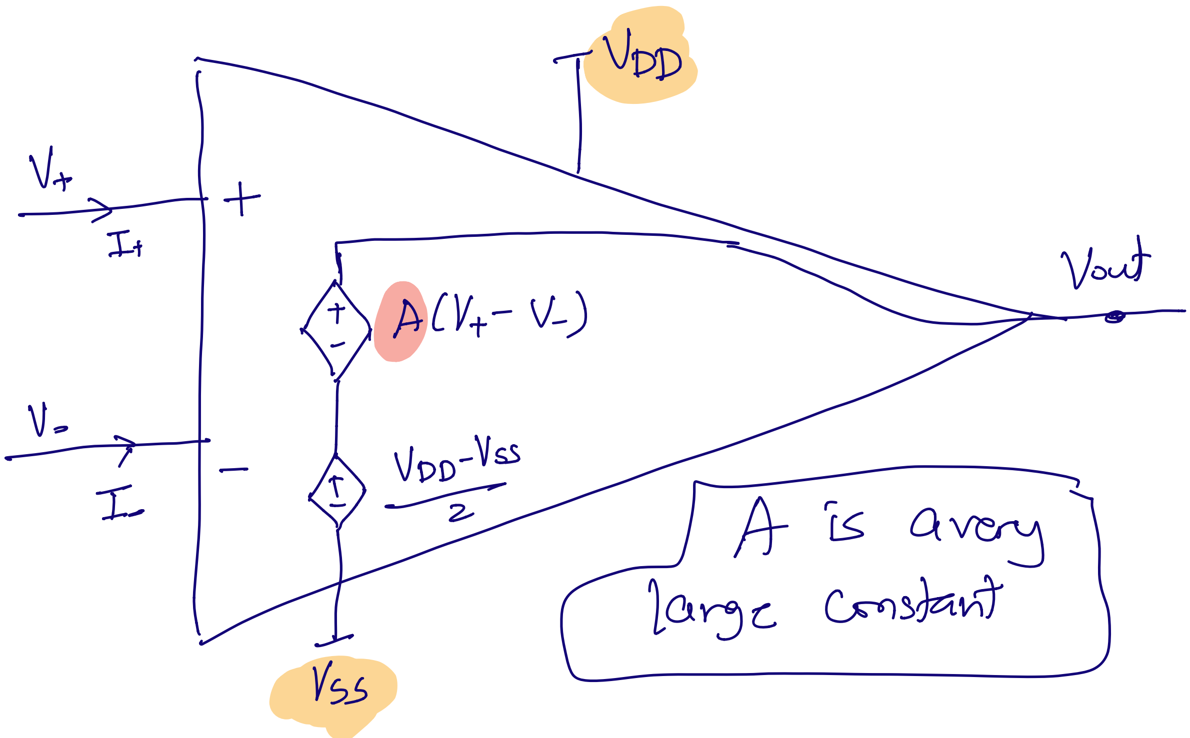
& $y = 0$, $\Rightarrow x = RC$

Op-Amps Operational Amplifier.

Negative Feedback.



Innards of an Op-Amp.



Golden Rule #1: $I_+ = I_- = 0$

$$V_{out} = V_{SS} + \frac{V_{DD}-V_{SS}}{2} + A(V_+ - V_-)$$

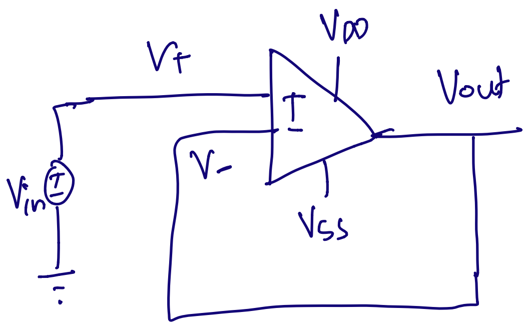
subject to:

Max. value of $V_{out} = V_{DD}$

Min. value of $V_{out} = V_{SS}$.

$$V_{out} = \frac{V_{DD}+V_{SS}}{2} + A(V_+ - V_-)$$

Connect an op-amp in negative feedback.



"Buffer circuit"

$V_{out} = V_{in}$

$$V_{out} = \frac{V_{DD}+V_{SS}}{2} + A(V_+ - V_-)$$

$$V_{out} = V_-$$

$$V_{in} = V_+$$

$$V_{out} = \frac{V_{DD}+V_{SS}}{2} + A(V_{in} - V_{out})$$

$$V_{out}(1+A) = \frac{V_{DD}+V_{SS}}{2} + A \cdot V_{in}$$

$$V_{out} = \left(\frac{1}{1+A}\right) \cdot \frac{V_{DD}+V_{SS}}{2} + \frac{A}{1+A} \cdot V_{in}$$

$\hookrightarrow A \to \infty.$

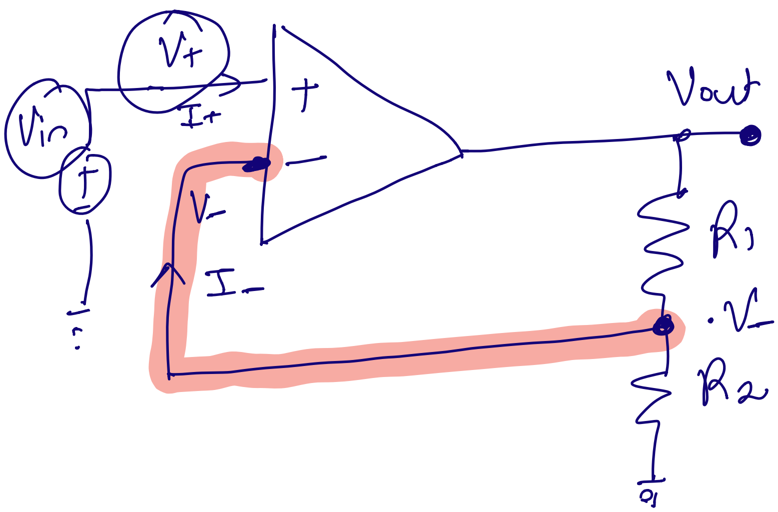
$V_{out} = 0 + 1 \cdot V_{in}.$

$$\boxed{V_{out} = V_{in}}$$

$V_+ = V_-$

Golden Rule #2: For op-amp in negative feedback: $V_+ = V_-$

Non-inverting amplifier



NFB:

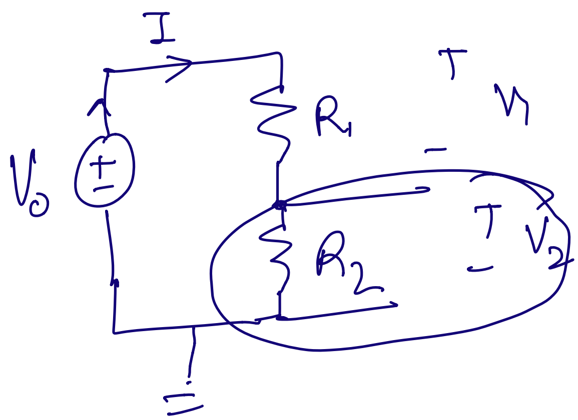
$V_+ = V_-$

$I_+ = I_- = 0$

$$\boxed{V_- = \frac{R_2 \cdot V_{out}}{R_1 + R_2}}$$

How is V_- connected to V_{out}?

Voltage dividers



$V_1 = IR_1$

$\boxed{V_2 = IR_2.}$

$V_1 + V_2 = V_0$

$I(R_1 + R_2) = V_0$

$$I = \frac{V_0}{R_1 + R_2}$$

$$\boxed{V_2 = \frac{V_0 \cdot R_2}{R_1 + R_2}}$$

$V_+ = V_-$

$V_+ = V_{in}$

$$V_- = \frac{R_2 \cdot V_{out}}{R_1 + R_2}$$

||

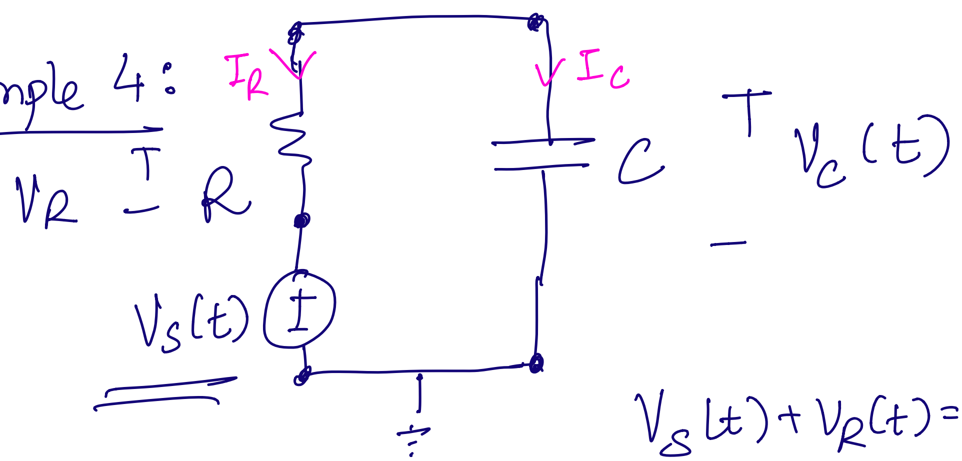
$$V_{in} = \frac{R_2 \quad V_{out}}{R_1 + R_2}$$

$$\underline{\underline{V_{out}}} = \frac{R_1 + R_2}{R_2} \cdot V_{in} = \underbrace{\left(1 + \frac{R_1}{R_2}\right)}_{\text{"Gain"}} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$$

Back to RC circuits.

Example 4:



$$V_s(t) + V_R(t) = V_C(t)$$

KVL

KCL $I_R + I_C = 0$.

$$I_c(t) = C \cdot \frac{dV_c(t)}{dt}$$

$$V_R(t) = I_R(t) \cdot R$$

$$\frac{V_R(t)}{R} + C \cdot \frac{dV_c(t)}{dt} = 0$$

$$\Rightarrow \frac{V_C(t) - V_S(t)}{R} + \frac{C \cdot dV_c(t)}{dt} = 0$$

$$\Rightarrow \frac{C \cdot dV_c(t)}{dt} = -\frac{V_C(t)}{R} + \frac{V_S(t)}{R}$$

$$= \frac{dV_c(t)}{dt} = -\frac{V_c(t)}{RC} + \frac{V_S(t)}{RC}$$

$$\frac{d\,x(t)}{dt} = \lambda \cdot x(t) + u(t)$$

Eigenvalues / Eigenspaces.

$$A \in \mathbb{R}^{n \times n}.$$

$\vec{x}$ = eigenvector

λ = eigenvalue.

$$A\vec{x} = \lambda \vec{x}$$

Direction unchanged by the operation of the matrix.

$\lambda \in \mathbb{R}$

$$A\vec{x} = \lambda \vec{x}$$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$A\vec{x} - \lambda\vec{x} = 0$$

$$\underbrace{(A - \lambda I)}_{\in \mathbb{R}^{n \times n}} \vec{x} = \underline{0}$$

$$\vec{x} \in \text{Nullspace } (A - \lambda I)$$

Find λ such that determinant

$$\det(A - \lambda I) = 0.$$

$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $\quad A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$

$\det(A - \lambda I)$

$= (1-\lambda)(3-\lambda) - 8$

$= \lambda^2 - 4\lambda - 5$

$= (\lambda+1)(\lambda-5)$

$\lambda_1 = -1$, $\lambda_2 = 5$ are our eigenvalues

Null $(A - 5I)$

$(A - 5I) = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix}$

Null:

GE:

$$\left[\begin{array}{cc|c} -4 & 2 & 0 \\ 4 & -2 & 0 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad x_1 - \tfrac{1}{2}x_2 = 0.$$

basic free

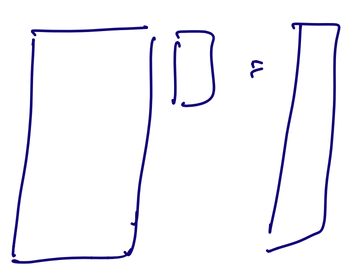
$$\vec{x} = \begin{bmatrix} 1/2\, x_2 \\ x_2 \end{bmatrix}$$

eigenspace: $\text{span}\left\{ \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \right\}$

Least - Squares:

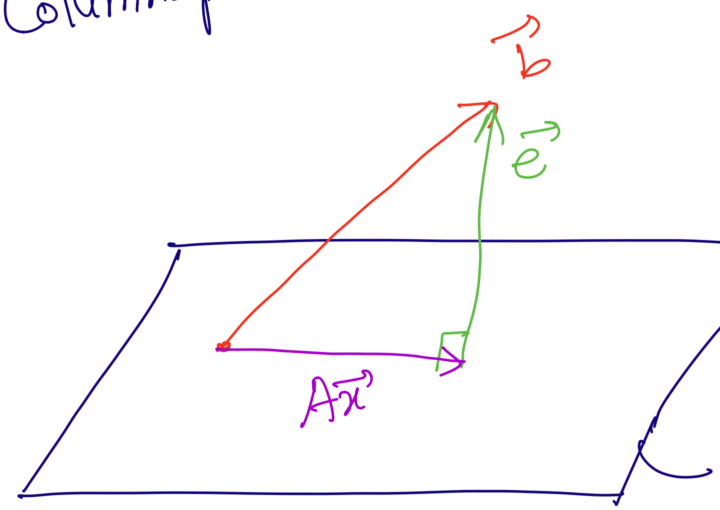
$A\vec{x} \approx \vec{b}$

More eq^n than unknowns



$A\vec{x}$ $\quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

Columnspace (A)



$A\vec{x} + \vec{e} = \vec{b}$

$\vec{e}$ is orthogonal to the plane $Col(A)$

$A\vec{x}$

$\vec{e} \perp$ Columns of A

$$\underset{\vec{x}}{\text{minimize}} \; \| A\vec{x} - \vec{b} \|^2 = \underset{\vec{x}}{\min} \| \vec{e} \|^2$$

$$A = \begin{bmatrix} | & | & & | \\ \vec{a}_1, & \vec{a}_2 & \cdots & \vec{a}_n \\ | & | & & | \end{bmatrix} \Rightarrow$$

$\vec{a}_1^T \cdot \vec{e} = 0$

$$\left.\begin{aligned} &\langle \vec{a_1}, \vec{e} \rangle = 0 \\ &\quad\vdots \\ &\langle \vec{a_n}, \vec{e} \rangle = 0 \end{aligned}\right\}$$

$$\underline{\underline{A^T \cdot \vec{e} = 0}}$$

$$A^T(\vec{b} - A\vec{x}) = 0$$

$$A^T\vec{b} = A^TA\vec{x}$$

$$\Rightarrow \vec{x} = \underbrace{(A^TA)^{-1}}_{} A^T\vec{b}$$

<u>if invertible</u>

A^TA

Square