Lecture 2

* Computing : Transistors & logic
* R-C models
* Solving R-C circuits

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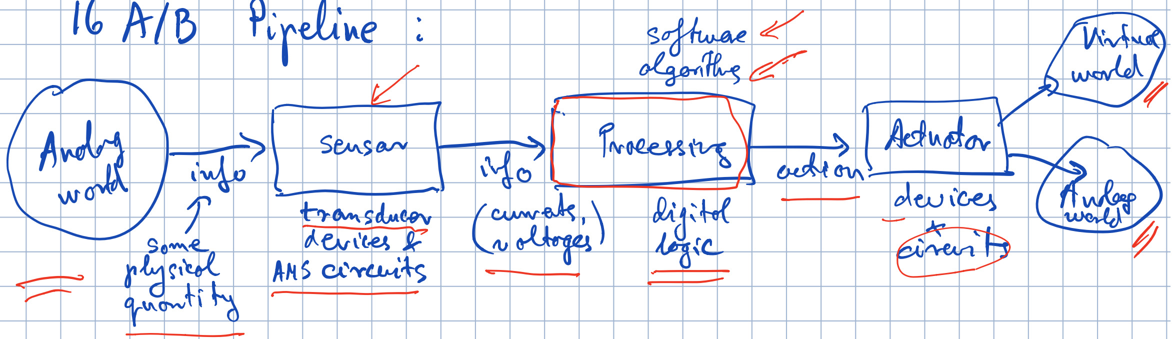
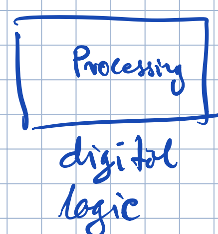
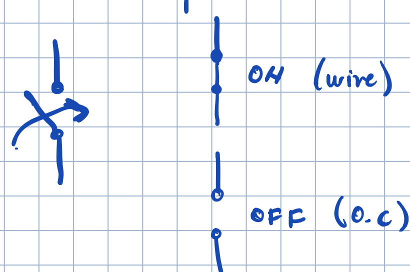
16 A/B Pipeline :



How do we implement computation?

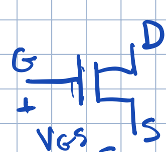


16A : Switch

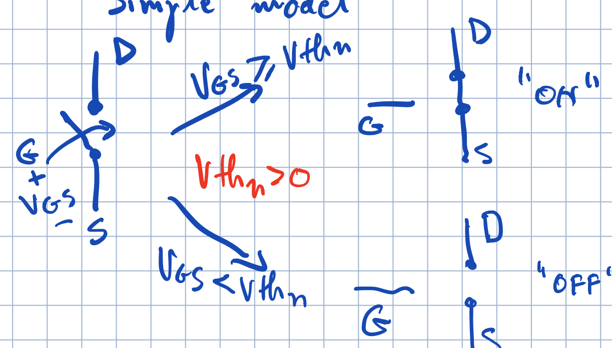


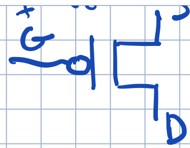
16B : Transistor

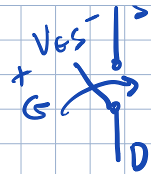
NMOS

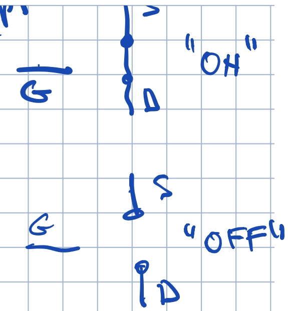


Simple model



PMOS 



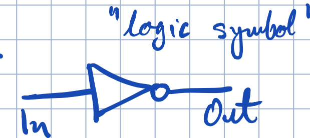
$V_{GS} \le -|V_{thp}|$ 

$V_{thp} < 0$

$V_{GS} > -|V_{thp}|$

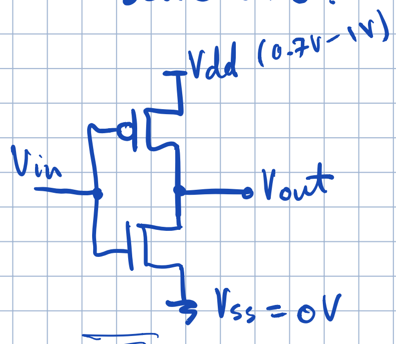
Simplest logic gate : an <u>inverter</u>

"logic symbol"

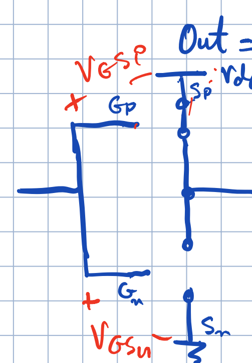


$0 \le V_{thn}, |V_{thp}| \le V_{dd}$

Schematic :



$V_{in} = 0V$
"logic 0"
(False)

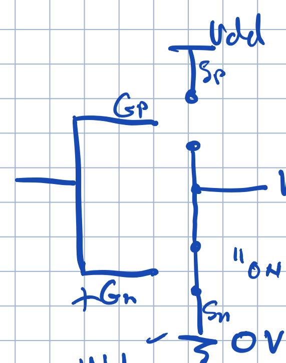


$V_{GSp} = -V_{dd}$
⇓
PMOS on

$V_{out} = V_{dd}$ "logic 1"

$V_{GSn} = 0$
⇓
NMOS OFF

$V_{in} = V_{dd}$
"logic 1"



$V_{GSp} = 0$
⇓ PMOS OFF

$V_{out} = 0V$
"logic 0"

$V_{GSn} = V_{dd}$
⇓
NMOS on

Truth table :

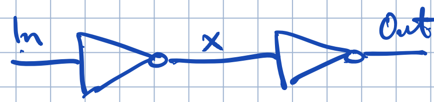
V_{in}	V_{out}
0V	Vdd
Vdd	0V

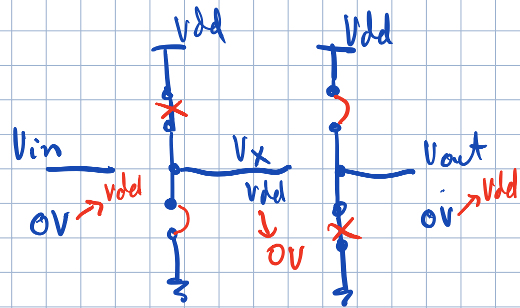
boolean : $Out = \overline{In}$

In	Out
0	1
1	0

Inverter ! 

Let's make a processor: Cascading logic

Simple model: 



State:

① $V_{in} = 0V \Rightarrow V_x = V_{dd} \Rightarrow V_{out} = 0V = V_{in}$

② $V_{in} = V_{dd} \Rightarrow V_x = 0V \Rightarrow V_{out} = V_{dd} = V_{in}$

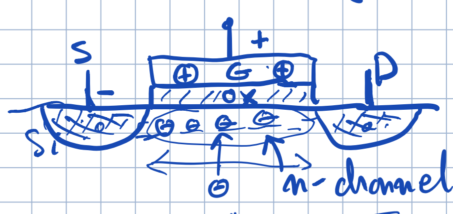
It looks like this process is instantaneous.

Would make a super-fast processor!

Not real!

Need to look at how a device is made.

NMOS (n-channel metal-oxide semiconductor) field effect transistor n-MOSFET



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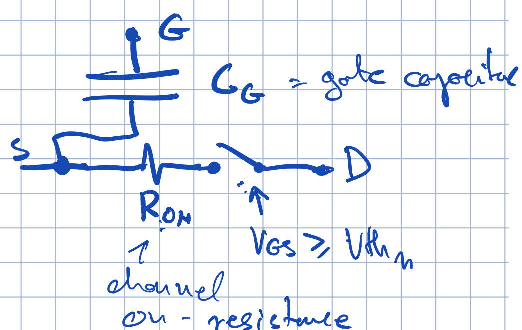
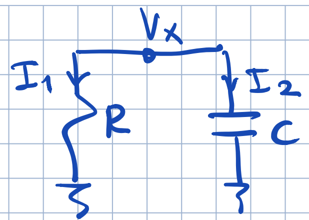


Channel forms when $V_{GS} \geq V_{th_n}$

Better model

To analyze this model need to understand RC circuits.



Elements:

$$I_2 = C\cdot\frac{dV_x}{dt}$$

$$V_x = I_1\cdot R$$

$I_1 + I_2 = 0$: KCL

$$\underbrace{\frac{V_x}{R} + C\frac{dV_x}{dt} = 0}$$

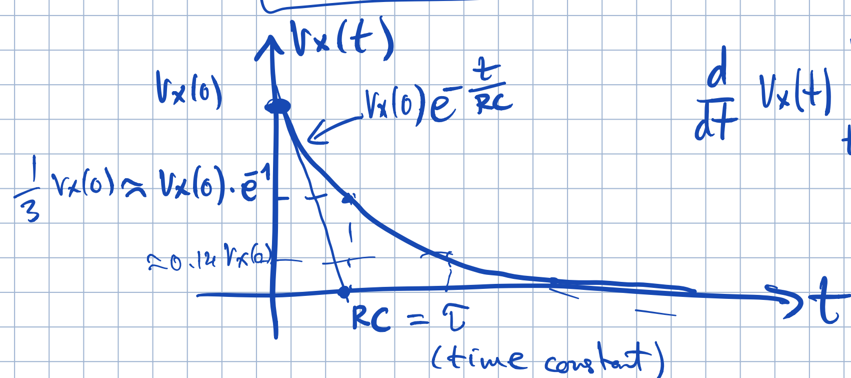
$t \geq 0$ $\Rightarrow$ $\boxed{\frac{dV_x}{dt} = -\frac{V_x}{RC}}$: Differential equation

Guess: $V_x(t) = a\cdot e^{bt}$

$\underbrace{V_x(0)}_{\text{initial state}} = a$, $\frac{dV_x(t)}{dt} = a\cdot b\cdot e^{bt} = b\cdot V_x(t)$

$$b = -\frac{1}{RC}$$

$$\boxed{V_x(t) = V_x(0)\cdot e^{-\frac{t}{RC}}}$$



$$\frac{d}{dt}V_x(t)\Big|_{t=0} = -\frac{V_x(0)}{RC}$$

Check for uniqness :

Suppose $y(t)$ which also solves. //

shorter notation: $x(0) = X_0$ (1)

$$\frac{d}{dt}x(t) = \lambda\cdot x(t) \quad (2) \; *$$

Guessed and checked that $\underline{x_d(t) = X_0\cdot e^{\lambda t}}$, $t \geq 0$ satisfies (1) & (2)

Need to show that $y(t) = x_d(t)$.

Either prove that $\frac{y(t)}{x_d(t)} = 1$ or $y(t) - x_d(t) = 0$

$$\frac{y(t)}{x_d(t)} = \frac{y(t)}{x_0 \cdot e^{\lambda t}} \qquad (2)\ \frac{d}{dt} y(t) = \lambda y(t)$$

$$\frac{d}{dt}\left(\frac{y(t)}{x_d(t)}\right) = \frac{d}{dt}\left(\frac{y(t)}{x_0 e^{\lambda t}}\right) = \frac{1}{x_0}\frac{d}{dt}\left(y(t) \cdot e^{-\lambda t}\right) =$$

$$= \frac{1}{x_0} \cdot \left(\underbrace{\frac{d}{dt} y(t)}_{\overset{(2)}{=}\ \lambda \cdot y(t)} \cdot e^{-\lambda t} - y(t)\, \lambda \cdot e^{-\lambda t}\right) =$$

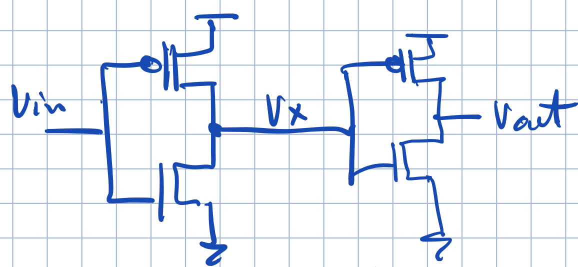
$$= \frac{1}{x_0} \cdot \left(\underbrace{\lambda \cdot y(t) \cdot e^{-\lambda t} - \lambda \cdot y(t) \cdot e^{-\lambda t}}_{0}\right) = 0$$

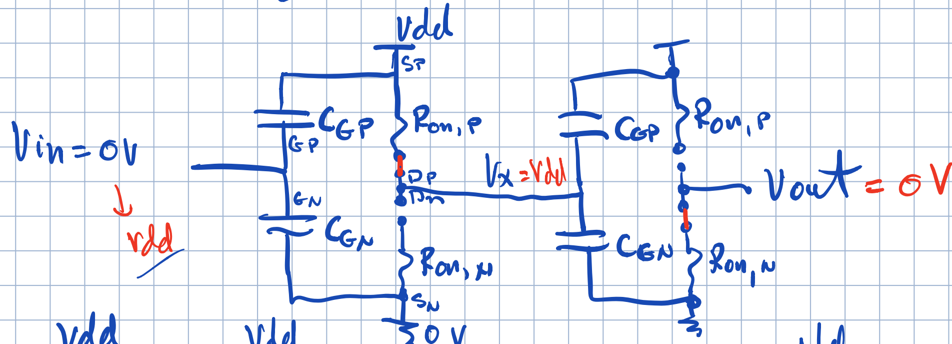
$$\frac{y(t)}{x_d(t)} = \underline{\underline{a}} \quad (\text{constant})$$

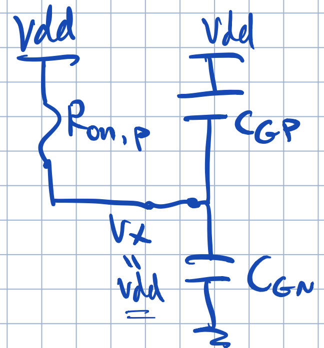
$$y(0) = x_0$$

$$\frac{y(0)}{\underset{x_0}{\overset{\shortparallel}{x_d(0)}}} = \frac{x_0}{x_0} = 1 = a \Rightarrow \frac{y(t)}{x_d(t)} = 1$$

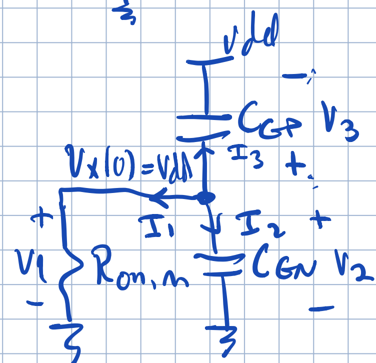
unique







⇒
t=0
switch



KCL: $I_1 + I_2 + I_3 = 0$ $\quad V_1 = V_x = I_1 \cdot R_{on,n}$

$$\frac{V_x}{R_{on,n}} + C_{Gn}\cdot\frac{dV_x}{dt} + C_{GP}\cdot\frac{dV_x}{dt} = 0$$

$$I_2 = C_{GN}\cdot\frac{dV_2}{dt} = C_{GN}\cdot\frac{dV_x}{dt}$$

$$\frac{V_x}{R_{on,n}} + (C_{Gn}+C_{GP})\cdot\frac{dV_x}{dt} = 0$$

$$I_3 = C_{GP}\cdot\frac{dV_3}{dt} = C_{GP}\cdot\frac{d(V_x - Vdd)}{dt} = C_{GP}\cdot\frac{dV_x}{dt}$$

$$\frac{dV_x}{dt} = -\frac{V_x}{\underbrace{R_{on,n}\cdot(C_{Gn}+C_{GP})}_{\tau}}$$

$$V_x(t) = V_x(0) \cdot e^{-\frac{t}{\tau}} \quad , \quad \tau = R_{on,n} \cdot (C_{Gn} + C_{Gp})$$

$$\boxed{V_x(t) = V_{dd} \cdot e^{-\frac{t}{\tau}}}$$