Lecture 9 EECS16B

* Phasor circuit analysis
* Filter examples
* Transfer functions

Phasor analysis steps:

1) Write sources as exponentials:

$v_s(t) = \hat{V}_s e^{j\omega t} + \bar{\hat{V}}_s e^{-j\omega t}$: $\hat{V}_s$ & $\hat{I}_s$ are source phasors

$I_s(t) = \hat{I}_s e^{j\omega t} + \bar{\hat{I}}_s e^{-j\omega t}$

2) Transform the circuit to the phasor domain: s-impedances to impedances ($s = j\omega$)

$Z_R = R$, $Z_C = \frac{1}{j\omega C}$, $Z_L = j\omega L$

3) Cast the branch & element equations in phasor domain:

KVL: $\sum_{i \text{ around the loop}} \hat{V}_i = 0$ KCL: $\sum_{i \text{ into the node}} \hat{I}_i = 0$

Ohm's law: $\hat{V}_i = Z_i \cdot \hat{I}_i$ NVA: $\sum \frac{\hat{V}_j - \hat{V}_k}{Z_{jk}} = 0$

4) Solve for unknown variables:

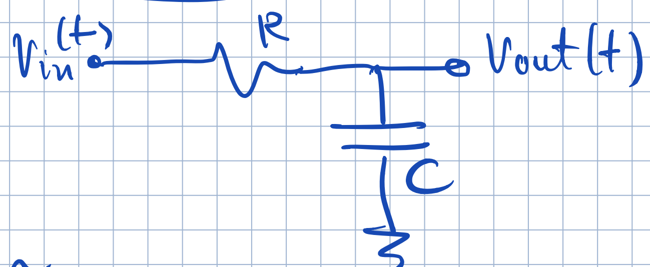
$\hat{V}_R, \hat{I}_R, \hat{V}_C, \hat{I}_C, \hat{V}_L, \hat{I}_L$ (in general $\hat{V}_{el}, \hat{I}_{el}$)

5) Transform solutions (4) from phasor domain to time domain:

$$v_{el}(t) = \hat{V}_{el}\, e^{j\omega t} + \bar{\hat{V}}_{el}\, e^{-j\omega t}$$

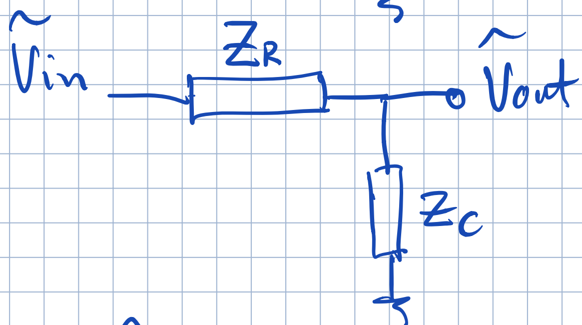
$$i_{el}(t) = \hat{I}_{el}\, e^{j\omega t} + \bar{\hat{I}}_{el}\, e^{-j\omega t}$$

Example 1:



$$V_{in}(t) = V_{in}\cos(\omega t + \phi)$$

$$V_{out}(t) = \,? \qquad \neq \hat{V}_{in}$$



$$Z_R = R \;,\; Z_C = \frac{1}{j\omega C}$$

$$V_{in}(t) = \hat{V}_{in}\, e^{j\omega t} + \bar{\hat{V}}_{in}\, e^{-j\omega t}$$

$$V_{out}(t) = \hat{V}_{out}\, e^{j\omega t} + \bar{\hat{V}}_{out}\, e^{-j\omega t}$$

$$\hat{V}_{out} = \frac{Z_C}{Z_C + Z_R}\hat{V}_{in} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\hat{V}_{in}$$

$$\boxed{\hat{V}_{out} = \frac{1}{1 + j\omega RC}\hat{V}_{in}}$$

$$\hat{V}_{out}(\omega) = \frac{1}{1 + j\omega RC}\hat{V}_{in}(\omega)$$

$$\hat{V}_{in} \longrightarrow \boxed{H(j\omega)} \longrightarrow \hat{V}_{out}$$

also $H(\omega)$

(e.g. a filter)

$$H(j\omega) = \frac{\hat{V}_{out}}{\hat{V}_{in}}$$

$$\boxed{H(j\omega) = \frac{1}{1+j\omega RC}}$$

$$H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1-j\omega RC}{1+(\omega RC)^2}$$

$$\boxed{\text{Magnitude: } |H(j\omega)| = \frac{1}{|1+j\omega RC|} = \frac{1}{\sqrt{1+(\omega RC)^2}}}$$

$$\hat{V}_{out} = H(j\omega)\cdot\hat{V}_{in}$$

$$\hat{V}_{in} = |\hat{V}_{in}|\cdot e^{j\measuredangle \hat{V}_{in}}$$

$$\hat{V}_{out} = |\hat{V}_{out}|\cdot e^{j\measuredangle \hat{V}_{out}}$$

$$|\hat{V}_{out}|\cdot e^{j\measuredangle \hat{V}_{out}} = |H(j\omega)|\cdot e^{j\measuredangle H(j\omega)}\cdot|\hat{V}_{in}|\cdot e^{j\measuredangle \hat{V}_{in}}$$

$$\boxed{|\hat{V}_{out}|\cdot e^{j\measuredangle \hat{V}_{out}} = |H(j\omega)|\cdot|\hat{V}_{in}|\cdot e^{j(\measuredangle H(j\omega)+\measuredangle \hat{V}_{in})}}$$

$$\boxed{\begin{aligned} |\hat{V}_{out}| &= |H(j\omega)|\cdot|\hat{V}_{in}| \\ \measuredangle \hat{V}_{out} &= \measuredangle H(j\omega) + \measuredangle \hat{V}_{in} \end{aligned}}$$

$$\omega\cdot t_d = \measuredangle H(j\omega)$$

$$t_d = \frac{\measuredangle H(j\omega)}{\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \quad \begin{array}{l} \to 1 \text{ as } \omega \to 0 \\ \to 0 \text{ as } \omega \to \infty \\ \text{as } \omega \gg \frac{1}{RC} \end{array}$$

i.e. this is a low-pass filter (first-order)

$$V_{in}(t) = \hat{V}_{in} e^{j\omega t} + \bar{\hat{V}}_{in} e^{-j\omega t} = 2|\hat{V}_{in}| \cos(\omega t + \measuredangle \hat{V}_{in})$$

$$V_{out}(t) = \hat{V}_{out} e^{j\omega t} + \bar{\hat{V}}_{out} e^{-j\omega t} = 2|\hat{V}_{out}| \cos(\omega t + \measuredangle \hat{V}_{out})$$

$$\hat{V}_{out} = H(j\omega)\hat{V}_{in} = \underbrace{|H(j\omega)| \cdot |\hat{V}_{in}|}_{|\hat{V}_{out}|} \cdot e^{j\overbrace{(\measuredangle H(j\omega) + \measuredangle \hat{V}_{in})}^{\measuredangle \hat{V}_{out}}}$$

$$V_{out}(t) = \boxed{|H(j\omega)|} \cdot 2|\hat{V}_{in}| \cdot \cos(\omega t + \measuredangle \hat{V}_{in} \boxed{+ \measuredangle H(j\omega)})$$

Recall: $V_{in}(t) = V_{in}\cos(\omega t) \Rightarrow 2|\hat{V}_{in}| = V_{in}$, $\measuredangle \hat{V}_{in} = 0$

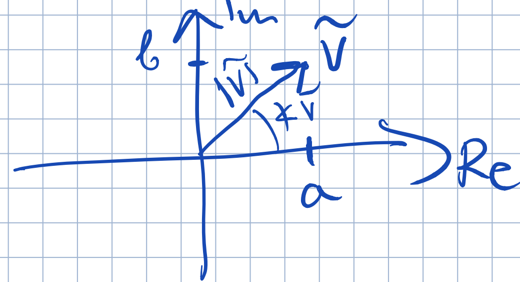
$$= V_{in}\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) = \underbrace{\frac{V_{in}}{2}}_{\hat{V}_{in}} e^{j\omega t} + \underbrace{\frac{V_{in}}{2}}_{\bar{\hat{V}}_{in}} e^{-j\omega t}$$

$$V_{out}(t) = |H(j\omega)| \cdot V_{in} \cdot \cos(\omega t + \measuredangle H(j\omega))$$

RC filter: $|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$, $\measuredangle H(j\omega) = -\tan^{-1}(\omega RC)$

same as derived with diff. equations.

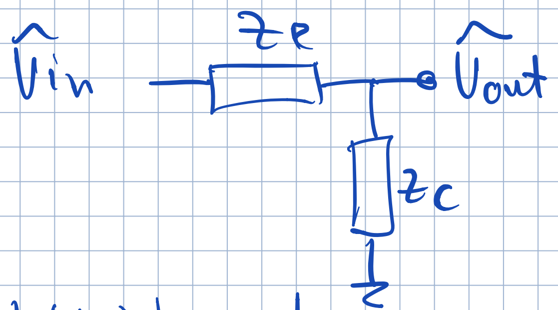
Remember:



$$\hat{V} = a + jb$$

$$|\tilde{V}| = \sqrt{a^2 + b^2}$$

$$\measuredangle \hat{v} = \tan^{-1}\left(\frac{b}{a}\right) = \mathrm{atan2}(b, a)$$



$$H(j\omega) = \frac{\hat{V}_{out}}{\hat{V}_{in}}$$

$$H(j\omega) = \frac{Z_C}{Z_C + Z_R} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

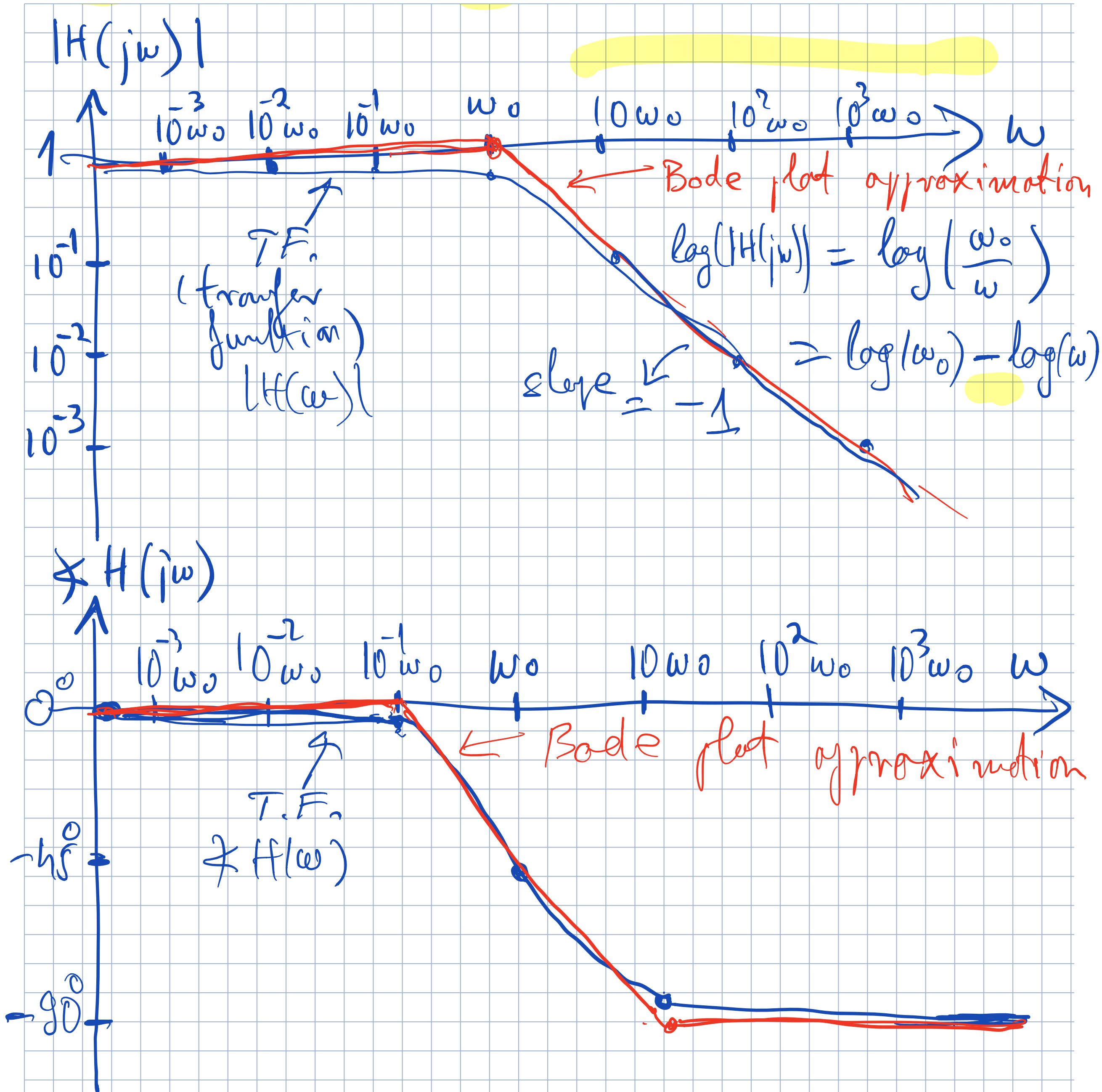
where $\omega_0 = \frac{1}{RC}$

call ω_0 – a "cut-off" frequency

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

ω	$H(\omega)$	$\lvert H(\omega)\rvert$	$\measuredangle H(j\omega)$
$\omega \ll \omega_0$	≈ 1	1	$\sim 0°$
$0.1\omega_0$	$\frac{1}{1+j0.1}$	0.995	$-6°$
ω_0	$\frac{1}{1+j}$	$\frac{1}{\sqrt{2}} = 0.71$	$-45°$
$10\omega_0$	$\frac{1}{1+j10}$	0.1	$-84°$
$\omega \gg \omega_0$	$-j\frac{\omega_0}{\omega}$	$\frac{\omega_0}{\omega}$	$-90°$



|H(jω)|
1
10^{-1}
10^{-2}
10^{-3}
$10^{-3}\omega_0$
$10^{-2}\omega_0$
$10^{-1}\omega_0$
ω_0
$10\omega_0$
$10^{2}\omega_0$
$10^{3}\omega_0$
ω
Bode plot approximation
T.F. (transfer function) |H(ω)|
$\log(|H(j\omega)|) = \log\left(\frac{\omega_0}{\omega}\right)$
$= \log(\omega_0) - \log(\omega)$
slope = −1
∡H(jω)
0°
−45°
−90°
$10^{-3}\omega_0$
$10^{-2}\omega_0$
$10^{-1}\omega_0$
ω_0
$10\omega_0$
$10^{2}\omega_0$
$10^{3}\omega_0$
ω
Bode plot approximation
T.F. ∡H(ω)