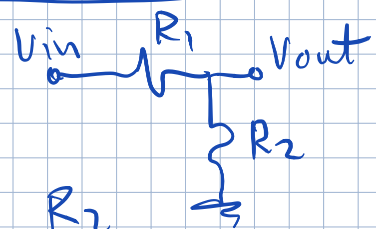
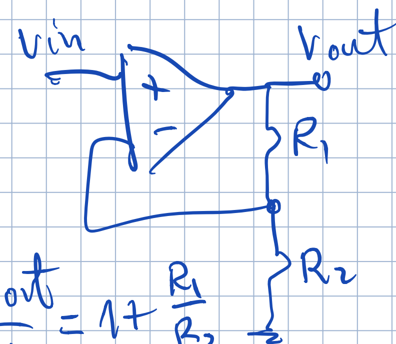
Lecture 10 — EEC S 16B

* Cascading ckt blocks transfer function
* Filter design example

In 16A:

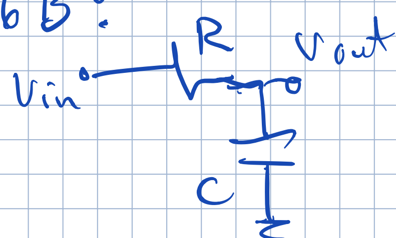


$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1+R_2}$$



$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$$

In 16B:



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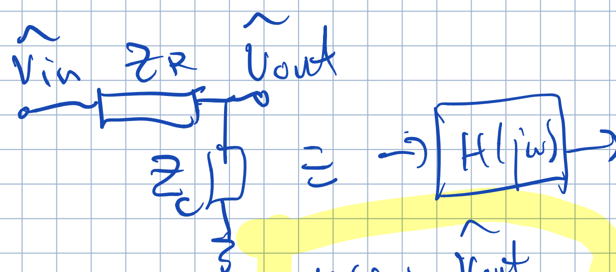


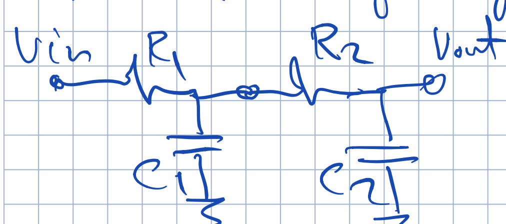
$$\boxed{H(j\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}}}$$

$$= \frac{1}{1 + j\frac{\omega}{\omega_0}}, \quad \omega_0 = \frac{1}{RC}$$

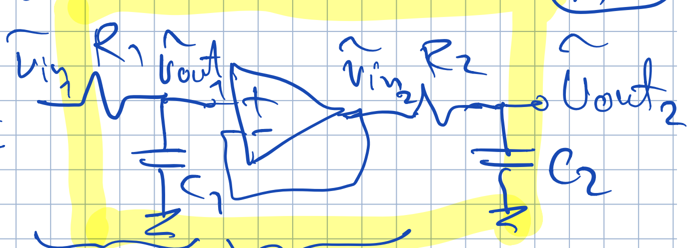
How do we cascade these circuits?

Circuit blocks should not "load" each other in order to preserve the transfer function.

Ex 1



$\neq$



$$H(j\omega) = H_1(j\omega)\, H_{buf}(j\omega)\, H_2(j\omega)$$

$$H_1(j\omega) = \frac{\hat{V}_{out1}}{\hat{V}_{in1}} \; ; \; H_{buf}(j\omega) = \frac{\hat{V}_{in2}}{\hat{V}_{out1}} \; ; \; H_2(j\omega) = \frac{\hat{V}_{out2}}{\hat{V}_{in2}}$$

$$H(j\omega) = \frac{\hat{V}_{out2}}{\hat{V}_{in1}} = \frac{\hat{V}_{out2}}{\hat{V}_{in2}} \cdot \frac{\hat{V}_{in2}}{\hat{V}_{out1}} \cdot \frac{\hat{V}_{out1}}{\hat{V}_{in1}} =$$

$$\boxed{H(j\omega) = H_2(j\omega) \cdot H_{buf}(j\omega) \cdot H_1(j\omega)}$$

For Ex 1:

$$H_1(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{01}}} \; , \; H_{buf}(j\omega) = 1$$

$$H_2(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{02}}} \qquad \omega_{01} = \frac{1}{R_1 C_1}$$

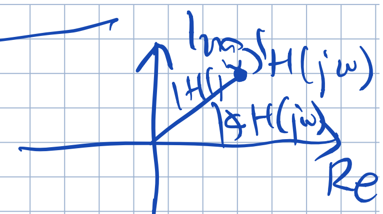
$$\omega_{02} = \frac{1}{R_2 C_2}$$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{01}}} \cdot 1 \cdot \frac{1}{1 + j\frac{\omega}{\omega_{02}}}$$

$$= \frac{1}{\left(1 + j\frac{\omega}{\omega_{01}}\right)\left(1 + j\frac{\omega}{\omega_{02}}\right)}$$

$$\hat{V}_{in1} \longrightarrow \boxed{H_1(j\omega)} \xrightarrow{\hat{V}_{out1}} \boxed{H_{buf}(j\omega)} \xrightarrow{\hat{V}_{in2}} \boxed{H_2(j\omega)} \longrightarrow \hat{V}_{out2}$$

As operators $H(j\omega)$



In general:

$$\underbrace{\tilde{V}_{in} \rightarrow \boxed{H_1(j\omega)} \rightarrow \dots \rightarrow \boxed{H_n(j\omega)}}_{H(j\omega)} \rightarrow \hat{V}_{out}$$

$$H_k(j\omega) = |H_k(j\omega)| e^{j\measuredangle H_k(j\omega)}$$

$$\frac{\hat{V}_{out}}{\tilde{V}_{in}} = H(j\omega) = H_1(j\omega) \cdots H_n(j\omega)$$

$$= \underbrace{|H_1(j\omega)| \cdots |H_n(j\omega)|}_{|H(j\omega)|} \cdot e^{j\overbrace{}^{}\underbrace{(\measuredangle H_1(j\omega) + \dots + \measuredangle H_n(j\omega))}_{\measuredangle H(j\omega)}}$$

Time-domain:

$$v_{out}(t) = |H(j\omega)| \, 2|\tilde{V}_{in}| \cos(\omega t + \measuredangle \hat{V}_{in} + \measuredangle H(j\omega))$$

for $v_{in}(t) = V_{in} \cos(\omega t + \phi)$

$$v_{out}(t) = |H(j\omega)| \cdot V_{in} \cos(\omega t + \phi + \measuredangle H(j\omega))$$

Recap:

$$v_{in}(t) = v_{in} \cos(\omega t + \phi) =$$

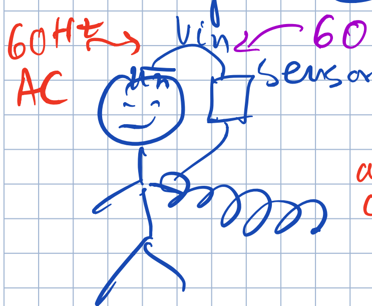
$$= V_{in} \left(\frac{e^{j(\omega t + \phi)}}{2} + \frac{e^{-j(\omega t + \phi)}}{2} \right) =$$

$=$

$$= \underbrace{\frac{V_{in} e^{j\phi}}{2}}_{\hat{V}_{in}} \cdot e^{j\omega t} + \underbrace{\frac{V_{in} e^{-j\phi}}{2}}_{\bar{V}_{in}} \cdot e^{-j\omega t}$$

$$= \hat{V}_{in} e^{j\omega t} + \bar{V}_{in} e^{-j\omega t}$$

Design example:



signal :	600 Hz	1mV
alternating current AC :	60Hz	10mV
fluorescent light :	60 kHz	20mV

Want to attenuate AC and fluorescent by 100x.

$$V_{in}(t) = V_{AC}\cos(\omega_{AC} t + \phi_{AC}) + \boxed{V_S \cos(\omega_S t + \phi_S)} + V_F \cos(\omega_F t + \phi_F)$$

ω_{AC} : $2\pi \cdot 60\,\text{Hz}$

ω_S : $2\pi \cdot 600\,\text{Hz}$

ω_F : $2\pi \cdot 60\,\text{kHz}$

$$\hat{V}_{in} \rightarrow \boxed{H(j\omega)} \rightarrow \hat{V}_{out}$$

want to build to attenuate V_{AC} & V_F

$$V_{out}(t) = |H(j\omega_{AC})|\, V_{AC} \cos(\omega_{AC} t + \phi_{AC} + \measuredangle H(j\omega_{AC}))$$

$$+ |H(j\omega_S)| \cdot V_S \cos(\omega_S t + \phi_S + \measuredangle H(j\omega_S))$$

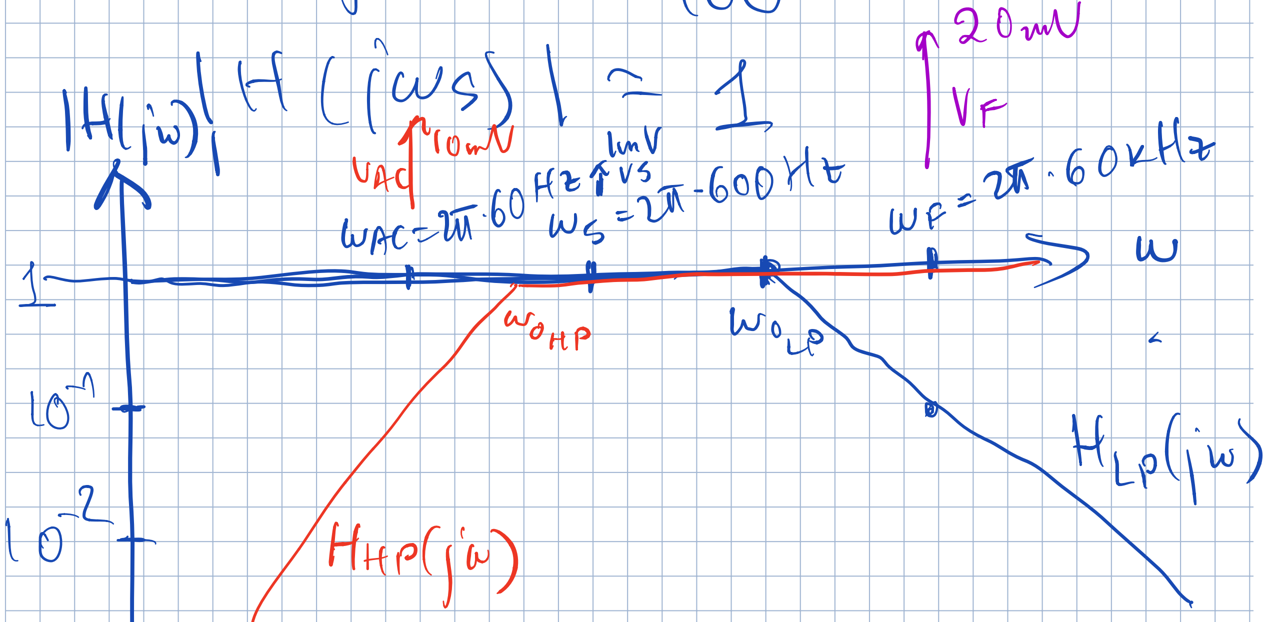
$$+ |H(j\omega_F)| \cdot V_F \cos(\omega_F t + \phi_F + \measuredangle H(j\omega_F))$$

Goal was to attenuate AC & Fluos by 100x :

$$\Rightarrow |H(j\omega_{AC})| = \frac{1}{100}$$

$$|H(j\omega_F)| = \frac{1}{100}$$

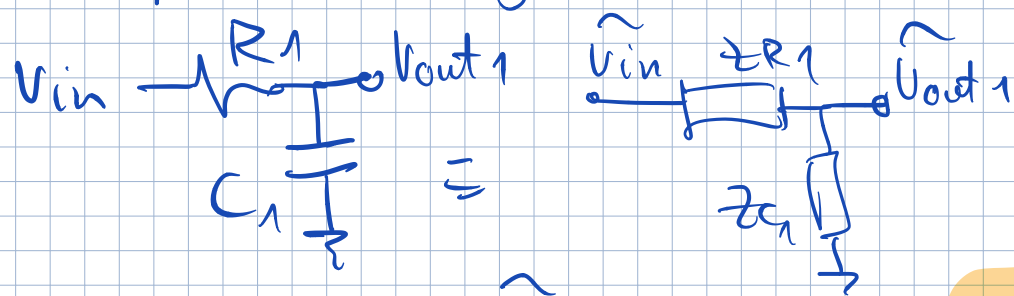
$$|H(j\omega_S)| \approx 1$$



$$\hat{v}_{in} \rightarrow \boxed{H_{LP}(j\omega)} \rightarrow \boxed{H_{Eq}(j\omega)} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \hat{v}_{out}$$

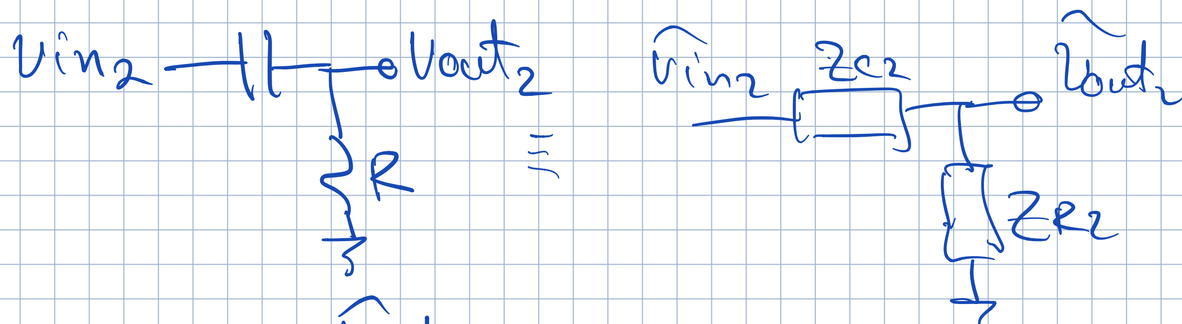
a low-pass filter

$H_{LP}(j\omega)$ already know how to build:



$$= H_{LP}(j\omega) = \frac{\hat{V}_{out1}}{\hat{V}_{in}} = \frac{1}{1 + j\frac{\omega}{\omega_{0LP}}} \quad ; \boxed{\omega_{0LP} = \frac{1}{R_1 C_1}}$$

$H_{HP}(j\omega)$ – a "high-pass" filter:



$$H_{HP}(j\omega) = \frac{\hat{V}_{out_2}}{\hat{V}_{in_2}} = \frac{Z_{R2}}{Z_{R2} + Z_{C2}} = \frac{R_2}{R_2 + \frac{1}{j\omega C_2}}$$

$$= \frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2} = \boxed{\frac{j\frac{\omega}{\omega_{0HP}}}{1 + j\frac{\omega}{\omega_{0HP}}}} \quad ; \boxed{\omega_{0HP} = \frac{1}{R_2 C_2}}$$

$$= \boxed{\frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}}$$

$\omega \to 0$ $\quad |H_{HP}(j\omega)| \to 0$

$\omega \to \infty$ $\quad |H_{HP}(j\omega)| \to 1$

$$H(j\omega) = H_{LP}(j\omega) \cdot \cancel{H_{log}(j\omega)}^{\,1} \cdot H_{HP}(j\omega)$$

$$= \frac{1}{1 + j\frac{\omega}{\omega_{0LP}}} \cdot \frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}$$

$$\omega_{0HP} = \sqrt{\omega_{AC} \cdot \omega_S}$$

$$\left[\log \omega_{0HP} = \frac{1}{2}\log \omega_{AC} + \frac{1}{2}\log \omega_S\right]$$

$$\omega_{0LP} = \sqrt{\omega_S \cdot \omega_F}$$

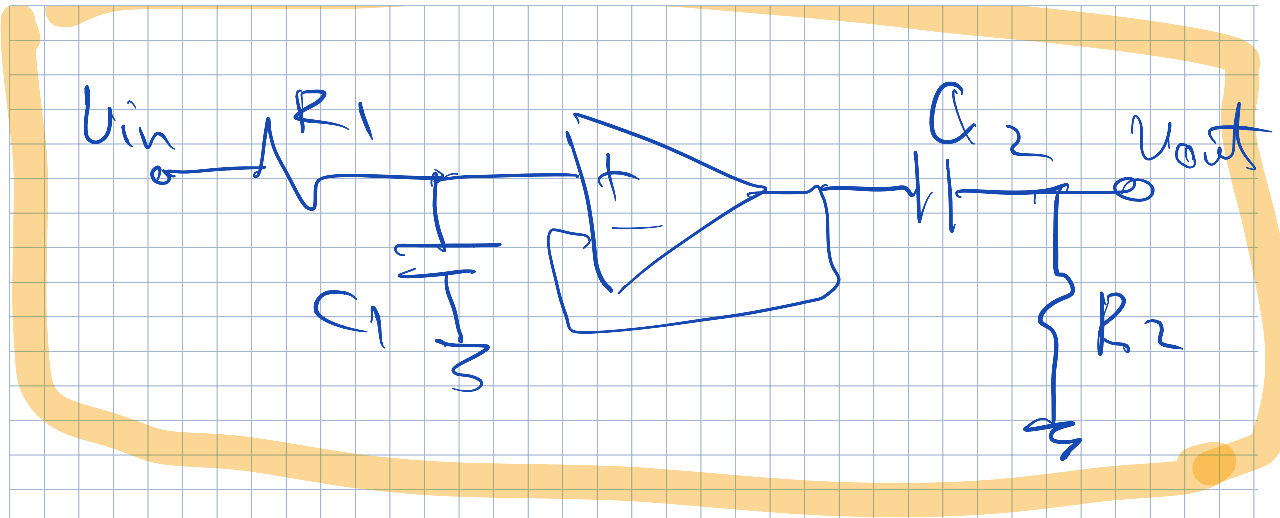
$$\omega_{0HP} = \frac{1}{R_2 C_2} \; ; \quad \omega_{0LP} = \frac{1}{R_1 C_1}$$

$$\omega_{0HP} = \sqrt{2\pi \cdot 60\,Hz \cdot 2\pi \cdot 600\,Hz} = 2\pi \cdot 190\,Hz$$

$$\omega_{0LP} = \sqrt{2\pi \cdot 600\,Hz \cdot 2\pi \cdot 60\,kHz} = 2\pi \cdot 6000\,Hz$$

For $C_1 = 1\,nF \Rightarrow R_1 = \frac{1}{\omega_{0LP} C_1} = 26\,k\Omega$

For $C_2 = 1\,\mu F \Rightarrow R_2 = \frac{1}{\omega_{0HP} C_2} = 840\,\Omega$



Vin
R1
+
−
C1
C2
Vout
R2