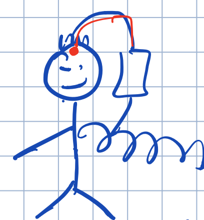
Lecture 11 EECS 16B

* Filter design

Design example:



AC: $\overset{10mV}{V_{AC}} \cos(\overbrace{2\pi\cdot 60Hz}^{\omega_{AC}} + \phi_{AC})$

Sig: $\overset{1mV}{V_S} \cos(2\pi\, 600Hz + \phi_S)$

fluoresc. flicker: $V_F \cos(2\pi\, 60kHz + \phi_F)$

$V_F = 20mV$

$\omega_{AC} = 2\pi \cdot 60Hz$

$= 377 \frac{rad}{s}$

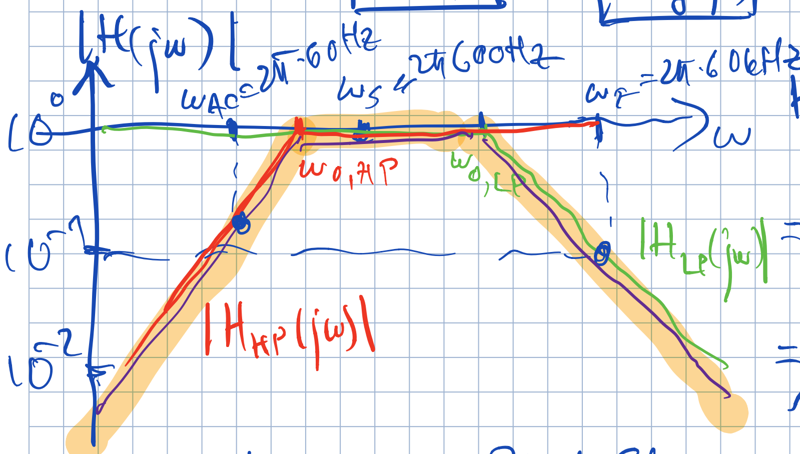
$\omega_S = 2\pi \cdot 600Hz$

$\omega_F = 2\pi \cdot 60kHz$

Want to attenuate V_{AC} & V_F by 100x:

Designed a filter:

$\tilde{v}_{in} \rightarrow \boxed{H_{LP}(j\omega)} \rightarrow \boxed{H_{buf}(j\omega)} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \hat{v}_{out}$



$H(j\omega) = \frac{\hat{V}_{out}}{\tilde{V}_{in}} = H_{LP}(j\omega) \cdot \cancel{H_{buf}(j\omega)} \cdot H_{HP}(j\omega)$

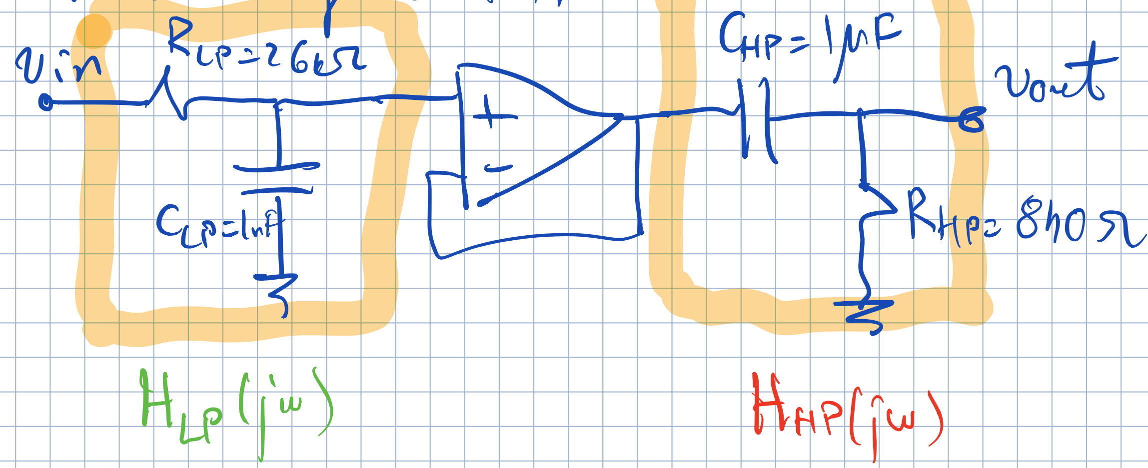
$|H_{LP}(j\omega)|$

$= \frac{1}{1 + j\frac{\omega}{\omega_{0,LP}}} \cdot \frac{1}{1 - j\frac{\omega_{0,HP}}{\omega}}$

determine R's & C's

$\omega_{0,LP} = \frac{1}{R_{LP} \cdot C_{LP}}, \quad \omega_{0,HP} = \frac{1}{R_{HP} \cdot C_{HP}} \Leftarrow \begin{cases} \omega_{0,LP} = \sqrt{\omega_S \cdot \omega_F} \\ \omega_{0,HP} = \sqrt{\omega_{AC} \cdot \omega_S} \end{cases}$

Circuit representation:



$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{0,LP}}} \cdot \frac{1}{1 - j\frac{\omega_{0,HP}}{\omega}}$$

ω	$\lvert H_{LP}(j\omega)\rvert$	$\lvert H_{HP}(j\omega)\rvert$	$\lvert H(j\omega)\rvert$	$V_{in} \cdot \lvert H(j\omega)\rvert$
$2\pi \cdot 60Hz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi \cdot 60Hz}{2\pi \cdot 6kHz}\right)^2}} \approx 1$	$\frac{1}{\sqrt{1+\left(\frac{2\pi \cdot 190}{2\pi 60}\right)^2}} \approx 0.3$	$1 \cdot 0.3 = 0.3$	$10mV \cdot 0.3 = 3mV$
$2\pi \cdot 600Hz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi 600Hz}{2\pi 6kHz}\right)^2}} = 1$	$\frac{1}{\sqrt{1+\left(\frac{190}{600}\right)^2}} = 0.95$	$1 \cdot 0.95 = 0.95$	$1mV \cdot 0.95 = 0.95 mV$
$2\pi \cdot 60kHz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi 60k}{2\pi 6k}\right)^2}} \approx 0.1$	$\frac{1}{\sqrt{1+\left(\frac{190}{60k}\right)^2}} \approx 1$	$0.1 \cdot 1 = 0.1$	$20mV \cdot 0.1 = 2mV$

$$V_{out}(t) = |H(j\omega_{AC})| \cdot V_{AC} \cdot \cos(\omega_{AC} t + \phi_{AC} + \measuredangle H(j\omega_{AC}))$$
$$+ |H(j\omega_S)| \cdot V_S \cdot \cos(\omega_S t + \phi_S + \measuredangle H(j\omega_S))$$
$$+ |H(j\omega_F)| \, V_F \cdot \cos(\omega_F t + \phi_F + \measuredangle H(j\omega_F))$$

Want $|H(j\omega_{AC})|$ & $|H(j\omega_F)| = \frac{1}{100}$

& $|H(j\omega_S)| \approx 1$

Current design is not enough!

Only get $|H(j\omega_{AC})| = 0.3$

$|H(j\omega_F)| = 0.1$

$|H(j\omega_S)| = 0.95$

$$\rightarrow \boxed{H_{LP}(j\omega)} \rightarrow \boxed{Buff} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \boxed{Buf} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \dots \rightarrow$$

$$\boxed{H^*(j\omega) = H_{LP}^{n}(j\omega)\, H_{HP}^{m}(j\omega)}$$

Want: $|H^*(j\omega_{AC})| = \frac{1}{100} = 0.3^m \Rightarrow m = 4$

$|H^*(j\omega_F)| = \frac{1}{100} = 0.1^n \Rightarrow n = 2$

$|H^*(j\omega_S)| = 0.95^4 = 0.8$ o.k. - not great

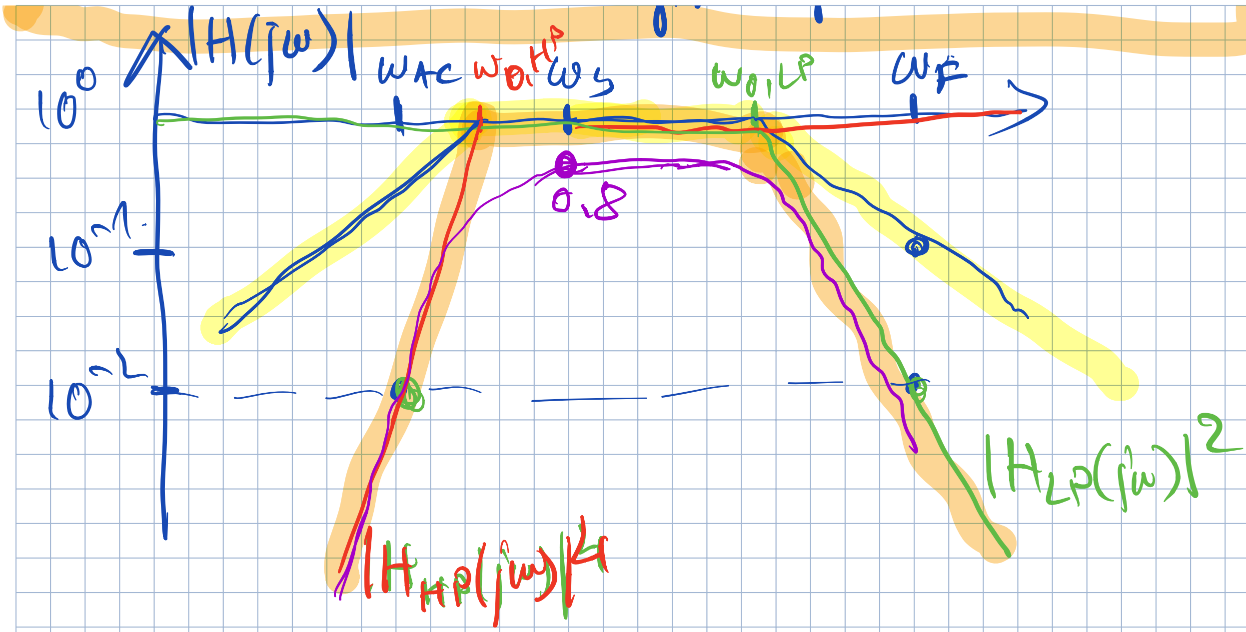
In general:

origin zeros

$$H(j\omega) = K \frac{(j\omega)^{N_{zo}} \cdot \left(1 + j\frac{\omega}{\omega_{z1}}\right) \cdots \left(1 + j\frac{\omega}{\omega_{zn}}\right)}{(j\omega)^{N_{po}} \cdot \left(1 + j\frac{\omega}{\omega_{p1}}\right) \cdots \left(1 + j\frac{\omega}{\omega_{pm}}\right)}$$

origin poles

ω_{zn} - zero $\qquad \omega_{pm}$ - pole



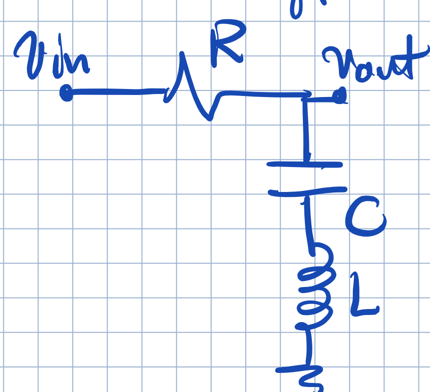
What if our desired signal is at 100Hz?

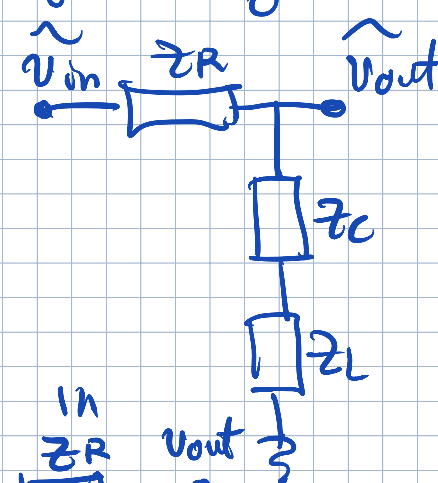
Our previous design won't work.

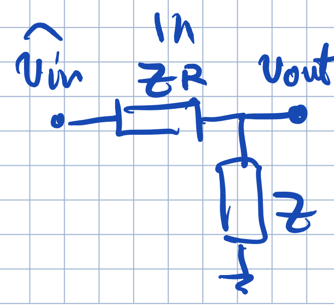
Too close to ω_{AC}.

Need a different filter!

Consider:







where $Z = Z_L + Z_C$

$$\boxed{\tilde{V}_{out} = \frac{Z}{Z+Z_R} \cdot \tilde{V}_{in}}$$

$$Z = j\omega L + \frac{1}{j\omega C}$$

$$= j\left(\omega L - \frac{1}{\omega C}\right)$$

fantastic!

Want $Z(\omega_0) = 0$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\omega_0^2 = \frac{1}{LC}$$

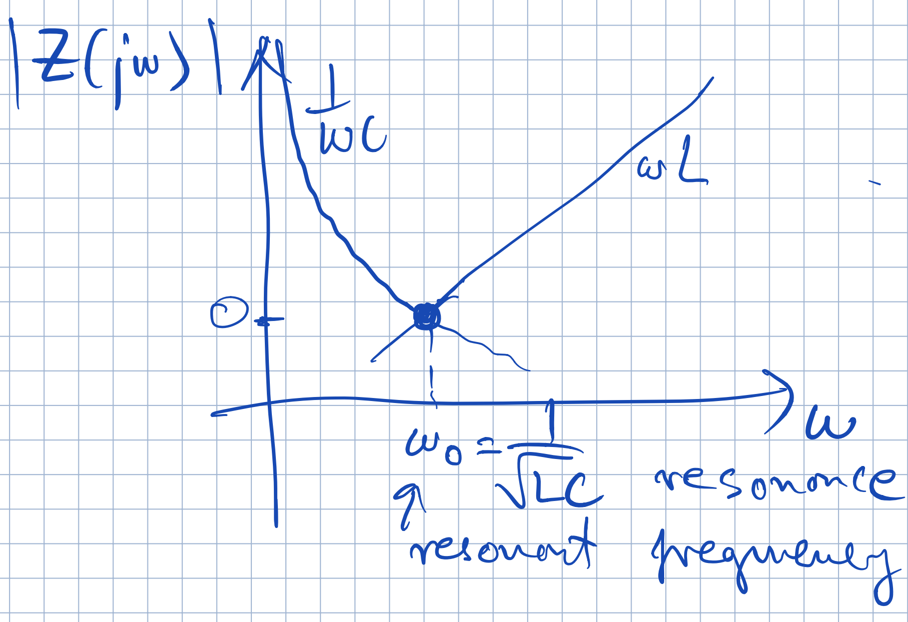
choose $\omega_0 = \omega_{AC} = 2\pi\, 60\,Hz$

Pick $C = 100\,\mu F \Rightarrow L \approx 70\,mH$

$$Z(\omega_0) = Z(\omega_{AC}) = 0$$

$$\tilde{V}_{out}(j\omega) = \frac{Z}{Z+Z_R} \cdot \tilde{V}_{in}(j\omega) \Rightarrow V_{out}(j\omega_{AC}) = \frac{0}{0+Z_R} \hat{V}_{in}(j\omega_{AC})$$

$$= 0$$



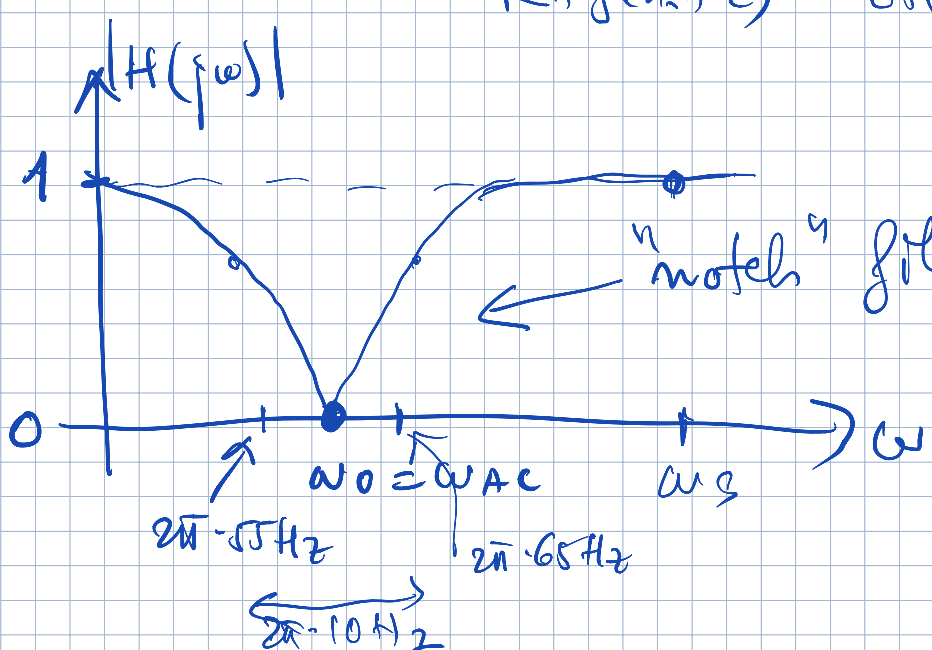
$$H(j\omega) = \frac{\hat{V}_{out}}{V_{in}} = \frac{Z}{Z + Z_R} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$H(j\omega_0) = 0 \qquad \omega_0 = 2\pi \cdot 60\,Hz$$

$$H(j\,2\pi \cdot 65\,Hz) = \frac{j \cdot 3.5\,\Omega}{R + j(3.5\,\Omega)}$$

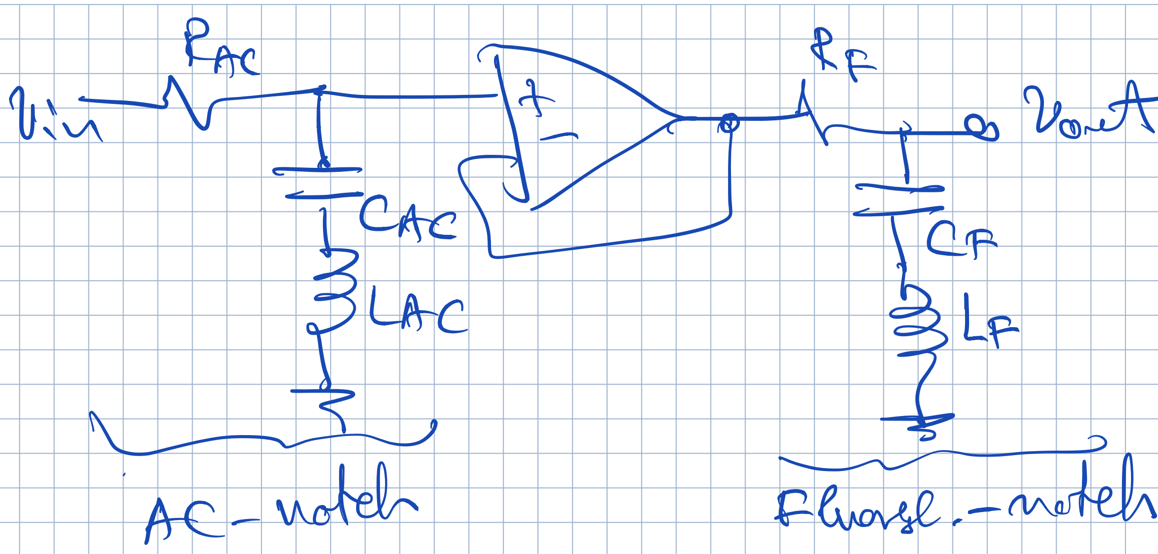
$$H(j\,2\pi\,55\,Hz) = \frac{j \cdot 4.7\,\Omega}{R + j(4.7\,\Omega)}$$

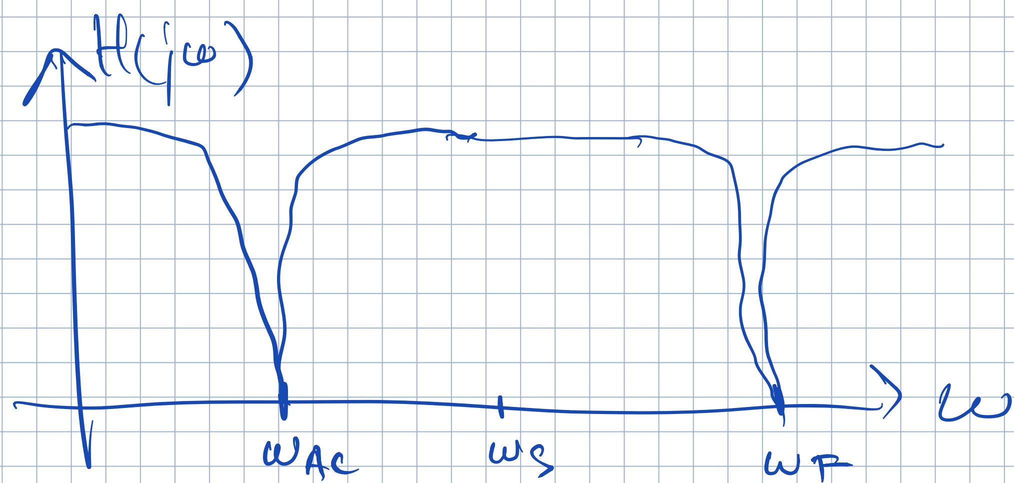
Pick $R \simeq 3\,\Omega$ less than 50% attenuation



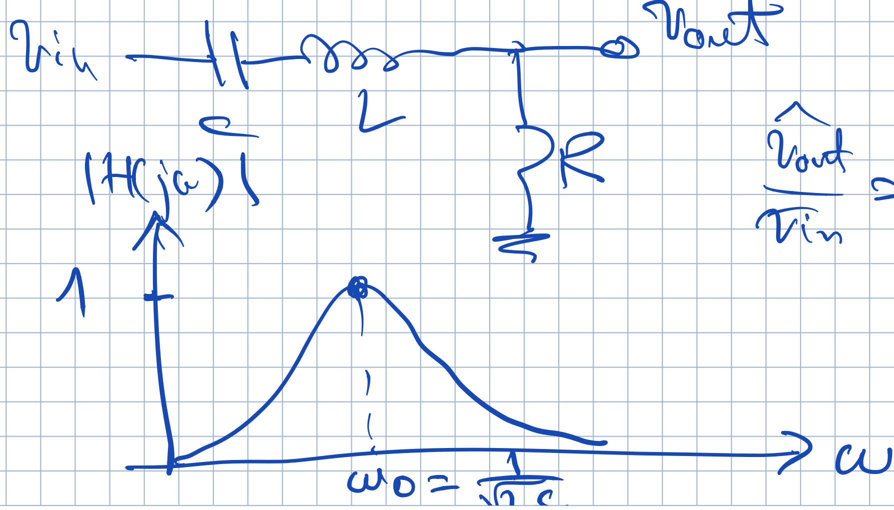
Can also set $\omega_{0,2} = \omega_F \Rightarrow L_F$ & C_F

such that $\frac{1}{\sqrt{L_F \cdot C_F}} = \omega_F$

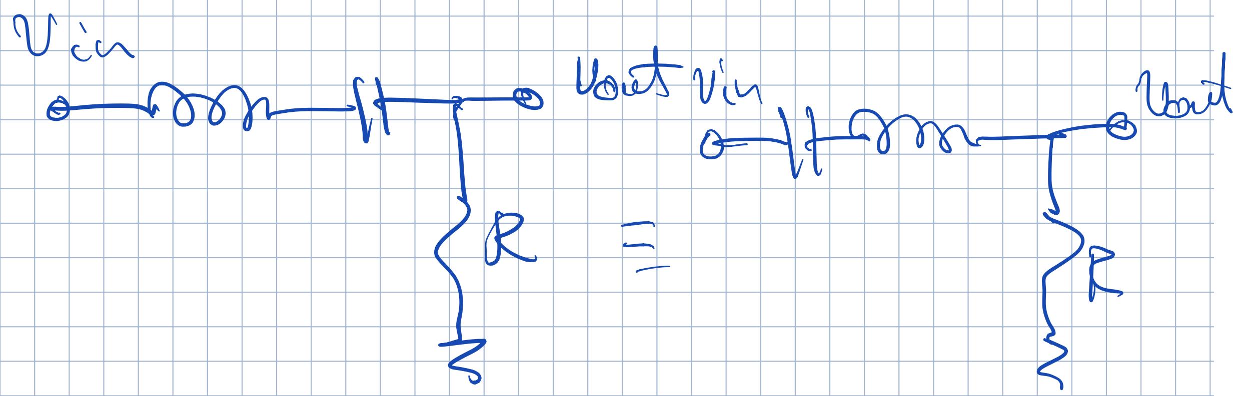




Can also design a sharp band-pass:



$$\frac{\hat{v}_{out}}{v_{in}} = \frac{R}{R + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}}$$



Vin
Vout
R
=
Vin
Vout
R