

SID# \_\_\_\_\_

**EE 16B Midterm 1  
Spring 2018**

Name: \_\_\_\_\_

SID #: \_\_\_\_\_

**(after the exam begins add your SID# in the top right corner of each page)**

Discussion Section and TA: \_\_\_\_\_

Discussion Section and TA: \_\_\_\_\_

Lab Section and TA: \_\_\_\_\_

Name of left neighbor: \_\_\_\_\_

Name of right neighbor: \_\_\_\_\_

**Instructions:**

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

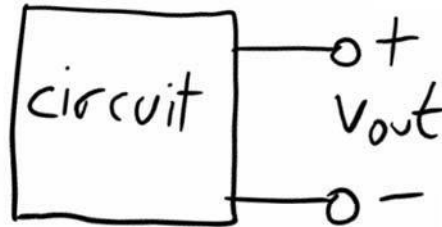
Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

Do not remove pages, as this disrupts the scanning. If needed, cross out any parts that you don't want us to grade.

<b>PROBLEM</b>	<b>MAX</b>
<b>1</b>	<b>15</b>
<b>2</b>	<b>25</b>
<b>3</b>	<b>15</b>
<b>4</b>	<b>20</b>
<b>5</b>	<b>25</b>

**Problem 1 Warm up** (15 points)

a) Consider the following circuit.



a) For  $t \geq 0$ , the following equation applies to  $v_{out}(t)$ . In addition,  $v_{out}(0) = V_0$  and  $\frac{dv_{out}}{dt} = 0$  at  $t=0$ .

$$\frac{d^2 v_{out}}{dt^2} + A \frac{dv_{out}}{dt} + B v_{out} = 0$$

If  $A < 2\sqrt{B}$ , provide an expression for  $v_{out}(t) \geq 0$ . (5 points)

**Solution:**

$$v_{out}(t) = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t}$$

$$\lambda_1 = \frac{-A}{2} + \sqrt{\left(\frac{A}{2}\right)^2 - B}$$

$$k_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1} V_0$$

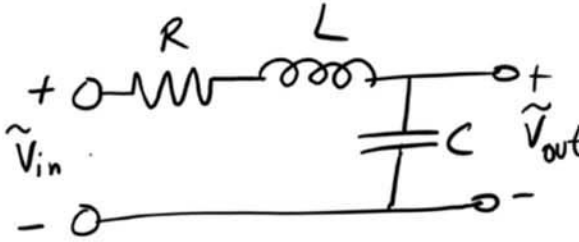
$$\lambda_2 = \frac{-A}{2} - \sqrt{\left(\frac{A}{2}\right)^2 - B}$$

$$k_2 = \frac{\lambda_1}{\lambda_1 - \lambda_2} V_0$$

or

$$v_{out}(t) = e^{\frac{-At}{2}} \left[ (k_1 + k_2) \cos\left(\sqrt{B - \left(\frac{A}{2}\right)^2} t\right) + j(k_1 - k_2) \sin\left(\sqrt{B - \left(\frac{A}{2}\right)^2} t\right) \right]$$

b) Consider the circuit below.

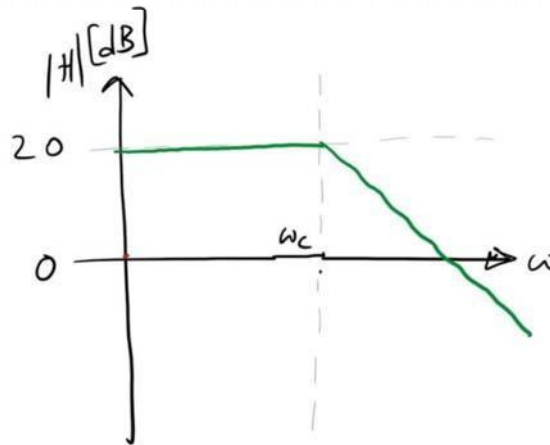


What is  $\tilde{\mathbf{H}}_{\text{out}}(\omega) = \frac{\tilde{V}_{\text{out}}(\omega)}{\tilde{V}_{\text{in}}(\omega)}$  for  $\omega \rightarrow \infty$ ? (5 points)

**Solution:**

$$\tilde{\mathbf{H}}_{\text{out}}(\omega \rightarrow \infty) = 0$$

c) Consider the Bode plot below. (5 points)



This is a Bode magnitude plot of the transfer function  $\tilde{\mathbf{H}}(\omega)$ . The expression for  $\tilde{\mathbf{H}}(\omega)$  is shown below.

$$\tilde{\mathbf{H}}(\omega) = \frac{\tilde{\mathbf{H}}_x(\omega)}{1 + j\left(\frac{\omega}{\omega_c}\right)}$$

What is  $\tilde{\mathbf{H}}_x(\omega)$ ?

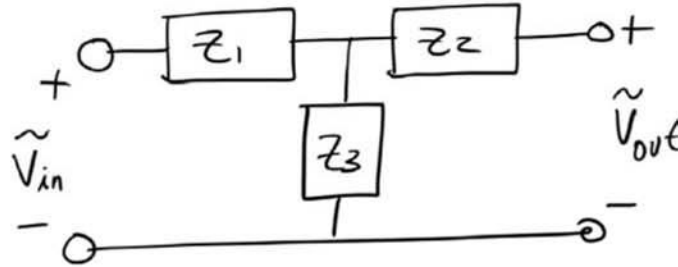
**Solution:**

$$\tilde{\mathbf{H}}_x(\omega) = \pm 10$$

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**Problem 2** *H's and Bodes...* (25 points)

Consider the circuit below. There is nothing connected to the  $V_{out}$  terminal. (5 points)



a) Provide an expression for  $\tilde{\mathbf{H}}_{out}(\omega) = \frac{\tilde{V}_{out}(\omega)}{\tilde{V}_{in}(\omega)}$

**Solution:**

$$\tilde{\mathbf{H}}_{out}(\omega) = \frac{Z_3}{Z_1 + Z_3}$$

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b) For this part of the problem, assume you have ONE capacitor, ONE inductor and ONE resistor. If  $Z_2 = 0$  for all  $\omega$ , which components would you choose for  $Z_1$  and  $Z_3$  such that the filter response is a passive low pass filter with a slope of -20 dB/decade for frequencies beyond a single cutoff frequency? (5 points)

**Solution:**

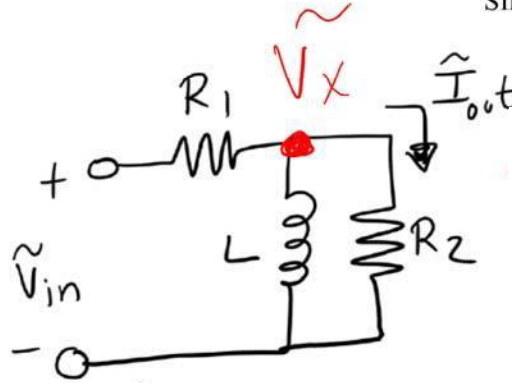
*Either combo is correct*

Circle ONE component to go into the  $Z_1$  box: Capacitor Inductor Resistor

Circle ONE component to go into the  $Z_3$  box: Capacitor Inductor Resistor

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c) Consider the following circuit:



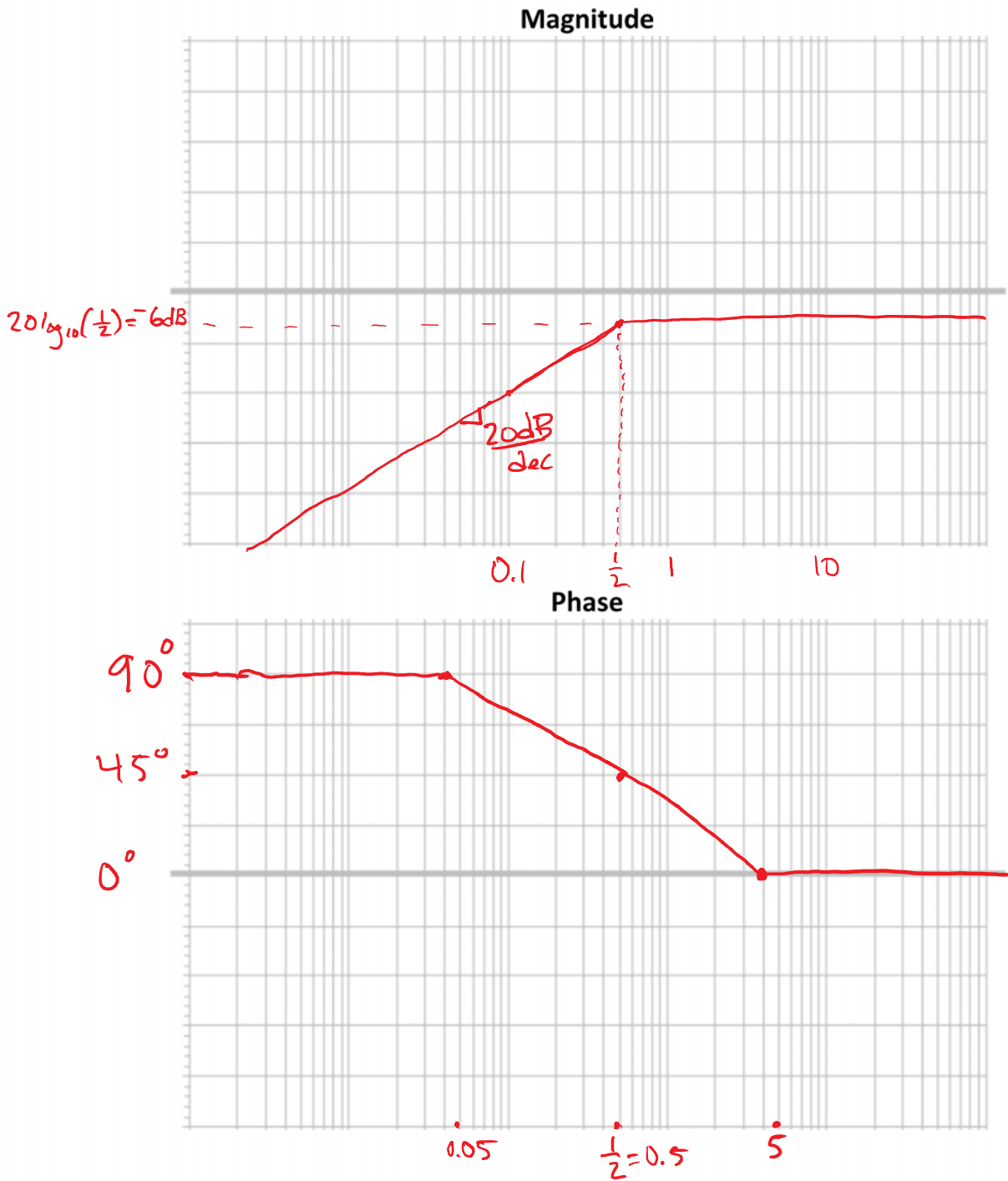
We define a transfer function  $\tilde{\mathbf{H}}_{out}(\omega) = \frac{\tilde{\mathbf{I}}_{out}(\omega)}{\tilde{\mathbf{V}}_{in}(\omega)}$ .

**LOOK AT THE DEFINITION OF THE TRANSFER FUNCTION CAREFULLY.**

Provide an expression in canonical form for  $\tilde{\mathbf{H}}_{out}(\omega)$ . (10 points)

<p><b>Solution:</b></p>	$K = \frac{1}{R_1 R_2}$	$\omega_{c1} = \frac{1}{L R_2}$
$\tilde{\mathbf{H}}_{out}(\omega) =$	$\omega_{c2} = \frac{R_1 R_2}{L(R_1 + R_2)}$	$= \frac{K j\omega/\omega_{c1}}{1 + j\omega/\omega_{c2}} ;$

d) If  $L = 1$  H and  $R_1 = R_2 = 1 \Omega$ , provide below magnitude and phase Bode plots for  $\mathbf{H}_{out}(\omega)$ . (5 points)

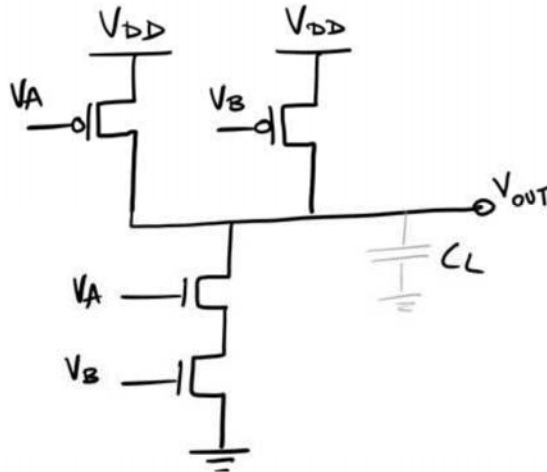




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**Problem 3** Transistors and RC's (15 points)

Consider the circuit below.



a) Fill in the truth table below for the circuit above.  $V_A$ ,  $V_B$  and  $V_{out}$  are digital voltages that can only assume values of 0 or  $V_{DD}$ . (5 points)

$V_A$	$V_B$	$V_{out}$
0	0	$V_{DD}$
0	$V_{DD}$	$V_{DD}$
$V_{DD}$	0	$V_{DD}$
$V_{DD}$	$V_{DD}$	0

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For this part, assume that  $V_A = V_B = V_{DD}$  for  $-\infty < t < 0$ . At  $t=0$ ,  $V_A$  and  $V_B$  switch instantly from  $V_{DD}$  to 0. Assume all transistors behave as resistors with the same value,  $R$ , if in the ON state and that all capacitances are already accounted for in the circuit above.

b) Provide an expression for  $V_{out}(t \geq 0)$ . **(10 points)**

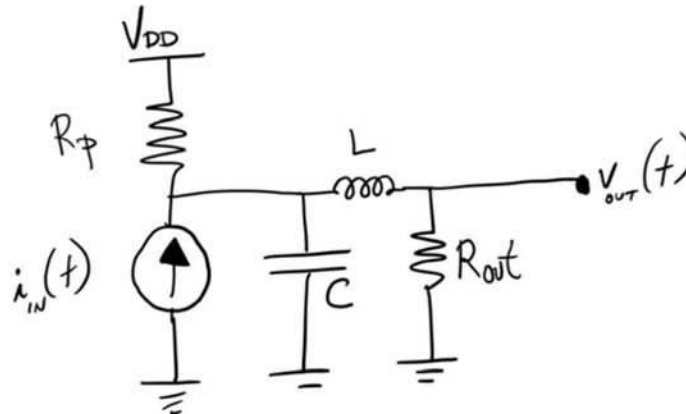
**Solution:**

$$V_{out}(t) = V_{DD} \left( 1 - e^{-t/RC} \right)$$

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**Problem 4 Phasors! (20 points)**

Consider the circuit below.



We are going to solve this circuit, which contains both a sinusoidal *and* a DC source using *superposition* and *phasors*.

a) Solve for  $v_{OUT}(t)$  if  $i_{IN}(t) = 0$  and  $V_{DD} =$  a non-zero constant. (5 points)

**Solution:**

$$V_{out}(t) = \frac{R_{out}}{R_p + R_{out}} V_{DD}$$

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b) Solve for  $v_{out}(t)$  if  $i_{in}(t) = I_0 \cos(\omega t)$  and  $V_{DD} = 0$  V. (10 points)

Solution:

$$v_{out}(t) = \frac{I_0 R_{out} R_p}{\sqrt{(R_p + R_{out} - \omega^2 L R_p)^2 + (\omega L + \omega R_p R_{out})^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L + \omega R_p R_{out}}{R_p + R_{out} - \omega^2 L R_p}\right)\right)$$

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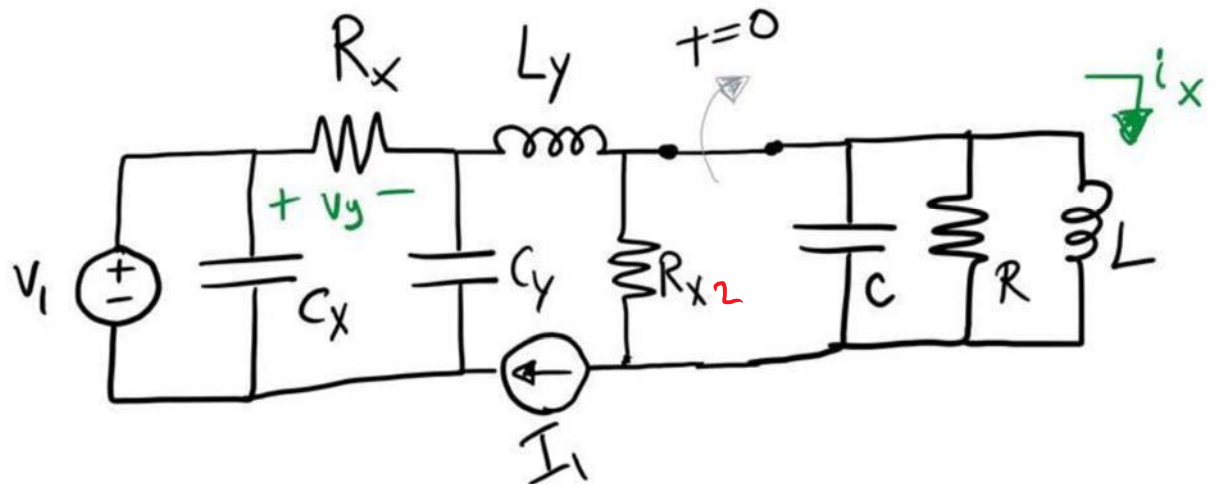
c) Solve for  $v_{OUT}(t)$  if  $i_{IN}(t) = I_0 \cos(\omega t)$  and  $V_{DD} =$  a non-zero constant. (5 points)

Solution:

$$\begin{array}{ccc} v_{out}(t) & + & v_{out}(t) \\ \text{from (a)} & & \text{from (b)} \end{array}$$

**Problem 5** (25 points)

a) Consider the following circuit. The switch is closed for  $t < 0$ , then opens at  $t = 0$ . Both of the independent sources have a DC value (i.e. they do not change with time).



a) What is  $i_x(t < 0)$ ? (5 points)

**Solution:**

$I_1$

b) What is  $v_y(t < 0)$ ? (5 points)

**Solution:**

$V_y = R_x \cdot I_1$

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c) Provide an equation in the variable  $i_x(t)$  that, when solved, would provide an expression for  $i_x(t)$  for  $t \geq 0$ .  
DO NOT SOLVE THE EQUATION. (10 points)

**Solution:**

$$i_x + \frac{L}{R} \frac{di_x}{dt} + CL \frac{d^2 i_x}{dt^2} = 0$$

d) If  $C_x = C_y = C = 1$  F and  $R_x = R = 1$   $\Omega$  and  $L_y = L = 1$  H, provide an expression for  $i_x(t)$  for  $t \geq 0$ . (5 points)

**Solution:**

$$i_x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad \begin{array}{l} \lambda_1 = -\frac{1}{2} + j\sqrt{\frac{3}{4}} \\ \lambda_2 = -\frac{1}{2} - j\sqrt{\frac{3}{4}} \end{array} \quad \begin{array}{l} C_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1} I_0 \\ C_2 = \frac{\lambda_1}{\lambda_1 - \lambda_2} I_0 \end{array}$$

or

$$i_x(t) = e^{-t/2} \left[ (C_1 + C_2) \cos\left(\sqrt{\frac{3}{4}} t\right) + j(C_1 - C_2) \sin\left(\sqrt{\frac{3}{4}} t\right) \right]$$

*Extra Space*

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Factor	Bode Magnitude	Bode Phase
<b>Constant</b> $K$	$20 \log K$ 0 dB	$\pm 180^\circ$ if $K < 0$ $0^\circ$ if $K > 0$
<b>Zero @ Origin</b> $(j\omega)^N$	0 dB slope = $20N$ dB/decade	$(90N)^\circ$ $0^\circ$
<b>Pole @ Origin</b> $(j\omega)^{-N}$	0 dB slope = $-20N$ dB/decade	$0^\circ$ $(-90N)^\circ$
<b>Simple Zero</b> $(1 + j\omega/\omega_c)^N$	0 dB slope = $20N$ dB/decade	$0^\circ$ $(90N)^\circ$
<b>Simple Pole</b> $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$	0 dB slope = $-20N$ dB/decade	$0^\circ$ $(-90N)^\circ$
<b>Quadratic Zero</b> $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$	0 dB slope = $40N$ dB/decade	$0^\circ$ $(180N)^\circ$
<b>Quadratic Pole</b> $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$	0 dB slope = $-40N$ dB/decade	$0^\circ$ $(-180N)^\circ$